Table A-9
Shear, Moment, and Deflection of Beams
(Note: Force and moment reactions are positive in the directions shown; equations for shear force $V$ and bending moment $M$ follow the sign conventions given in Sec. 3–2.)

1 Cantilever—end load

\[
\begin{align*}
R_1 &= V = F & M_1 &= Fl \\
M &= F(x - l) \\
y &= \frac{Fx^2}{6EI} (x - 3l) \\
y_{max} &= \frac{Fl^3}{3EI}
\end{align*}
\]

2 Cantilever—intermediate load

\[
\begin{align*}
R_1 &= V = F & M_1 &= Fa \\
M_{AB} &= F(x - a) & M_{BC} &= 0 \\
y_{AB} &= \frac{Fx^2}{6EI} (x - 3a) \\
y_{BC} &= \frac{Fa^2}{6EI} (a - 3x) \\
y_{max} &= \frac{Fa^2}{6EI} (a - 3l)
\end{align*}
\]

(continued)

Table A–9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force $V$ and bending moment $M$ follow the sign conventions given in Sec. 3–2.)

3 Cantilever—uniform load

$$R_1 = w/l, \quad M_1 = \frac{wl^2}{2}$$
$$V = w(l-x), \quad M = -\frac{w}{2}(l-x)^2$$
$$y = \frac{wx^2}{24EI} (4lx - x^2 - 6l^2)$$
$$y_{max} = -\frac{wl^4}{8EI}$$

4 Cantilever—moment load

$$R_1 = V = 0, \quad M_1 = M = M_B$$
$$y = \frac{M_Bx^2}{2EI}$$
$$y_{max} = \frac{M_Bl^2}{2EI}$$
Table A-9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force $V$ and bending moment $M$ follow the sign conventions given in Sec. 3–2.)

5 Simple supports—center load

$R_1 = R_2 = \frac{F}{2}$

$V_{AB} = R_1$, $V_{BC} = -R_2$

$M_{AB} = \frac{Fx}{2}$, $M_{BC} = \frac{F}{2}(l - x)$

$y_{AB} = \frac{Fx}{48EI} (4x^2 - 3l^2)$

$y_{BC} = -\frac{Fl^3}{48EI}$

6 Simple supports—intermediate load

Caution: Only for $a < b$

$R_1 = \frac{Fb}{l}$, $R_2 = \frac{Fa}{l}$

$V_{AB} = R_1$, $V_{BC} = -R_2$

$M_{AB} = \frac{Fbx}{l}$, $M_{BC} = \frac{Fa}{l}(l - x)$

$y_{AB} = \frac{Fbx}{6EI} (x^2 + b^2 - l^2)$

$y_{BC} = \frac{Fa(l - x)}{6EI} (x^2 + a^2 - 2lx)$

(continued)
Table A-9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force $V$ and bending moment $M$ follow the sign conventions given in Sec. 3–2.)

7 Simple supports—uniform load

$$ R_1 = R_2 = \frac{w l}{2} \quad V = \frac{w l}{2} - w x $$
$$ M = \frac{w x}{2} (l - x) $$
$$ y = \frac{w x}{24EI} (2l x^2 - x^3 - l^3) $$
$$ y_{max} = -\frac{5wl^4}{384EI} $$

8 Simple supports—moment load

$$ R_1 = R_2 = \frac{M_B}{l} \quad V = \frac{M_B}{l} $$
$$ M_{AB} = \frac{M_B x}{l} \quad M_{BC} = \frac{M_B}{l} (x - l) $$
$$ y_{AB} = \frac{M_B x}{6EI} (x^2 + 3a^2 - 6al + 2l^2) $$
$$ y_{BC} = \frac{M_B}{6EI} \left[ x^3 - 3lx^2 + x(2l^2 + 3a^2) - 3a^2l \right] $$
### Table A–9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force \( V \) and bending moment \( M \) follow the sign conventions given in Sec. 3–2.)

#### 9 Simple supports—twin loads

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>( R_2 = F )</td>
</tr>
<tr>
<td>( V_{AB} )</td>
<td>( M_{BC} = Fa )</td>
</tr>
<tr>
<td>( V_{BC} )</td>
<td>( M_{CD} = F(l - x) )</td>
</tr>
<tr>
<td>( V_{CD} )</td>
<td>( F )</td>
</tr>
<tr>
<td>( M_{AB} )</td>
<td>( Fx )</td>
</tr>
<tr>
<td>( y_{AB} )</td>
<td>( \frac{Fx}{6EI}(x^2 + 3a^2 - 3la) )</td>
</tr>
<tr>
<td>( y_{BC} )</td>
<td>( \frac{Fa}{6EI}(3x^2 + a^2 - 3lx) )</td>
</tr>
<tr>
<td>( y_{max} )</td>
<td>( \frac{Fa}{24EI}(4a^2 - 3l^2) )</td>
</tr>
</tbody>
</table>

#### 10 Simple supports—overhanging load

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>( \frac{Fa}{l} )</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>( \frac{F}{l}(l + a) )</td>
</tr>
<tr>
<td>( V_{AB} )</td>
<td>( -\frac{Fa}{l} )</td>
</tr>
<tr>
<td>( V_{BC} )</td>
<td>( F )</td>
</tr>
<tr>
<td>( M_{AB} )</td>
<td>( -\frac{Fax}{l} )</td>
</tr>
<tr>
<td>( y_{AB} )</td>
<td>( \frac{Fax}{6EI}(l^2 - x^2) )</td>
</tr>
<tr>
<td>( y_{BC} )</td>
<td>( \frac{F(x - l)}{6EI}[(x - l)^2 - a(3x - l)] )</td>
</tr>
<tr>
<td>( y_c )</td>
<td>( -\frac{Fa^2}{3EI}(l + a) )</td>
</tr>
</tbody>
</table>

(continued)
Table A–9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force V and bending moment M follow the sign conventions given in Sec. 3–2.)

11 One fixed and one simple support—center load

\[
R_1 = \frac{11F}{16} \quad R_2 = \frac{5F}{16} \quad M_1 = \frac{3Fl}{16}
\]

\[
V_{AB} = R_1 \quad V_{BC} = -R_2
\]

\[
M_{AB} = \frac{F}{16}(11x - 3l) \quad M_{BC} = \frac{5F}{16}(l - x)
\]

\[
y_{AB} = \frac{Fx^2}{96EI} (11x - 9l) \quad y_{BC} = \frac{F(l - x)}{96EI} (5x^2 + 2l^2 - 10lx)
\]

12 One fixed and one simple support—intermediate load

\[
R_1 = \frac{Fb}{2l^3} (3l^2 - b^2) \quad R_2 = \frac{Fa^2}{2l^3} (3l - a)
\]

\[
M_1 = \frac{Fb}{2l^3} (l^2 - b^2)
\]

\[
V_{AB} = R_1 \quad V_{BC} = -R_2
\]

\[
M_{AB} = \frac{Fb}{2l^3} [b^2l - l^3 + x(3l^2 - b^2)]
\]

\[
M_{BC} = \frac{Fa^2}{2l^3} (3l^2 - 3lx - al + ax)
\]

\[
y_{AB} = \frac{Fbxy^2}{12EI^3} [3l(b^2 - l^2) + x(3l^2 - b^2)]
\]

\[
y_{BC} = y_{AB} - \frac{F(x - a)^3}{6EI}
\]
Table A-9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force \( V \) and bending moment \( M \) follow the sign conventions given in Sec. 3-2.)

13 One fixed and one simple support—uniform load

\[
R_1 = \frac{5wl}{8} \quad R_2 = \frac{3wl}{8} \quad M_1 = \frac{wl^2}{8}
\]

\[
V = \frac{5wl}{8} - wx
\]

\[
M = -\frac{w}{8}(4x^2 - 5lx + l^2)
\]

\[
y = \frac{wx^2}{48EI}(l - x)(2x - 3l)
\]

14 Fixed supports—center load

\[
R_1 = R_2 = \frac{F}{2} \quad M_1 = M_2 = \frac{Fl}{8}
\]

\[
V_{AB} = -V_{BC} = \frac{F}{2}
\]

\[
M_{AB} = \frac{F}{8}(4x - l) \quad M_{BC} = \frac{F}{8}(3l - 4x)
\]

\[
y_{AB} = \frac{Fx^2}{48EI}(4x - 3l)
\]

\[
y_{\text{max}} = -\frac{Fl^3}{192EI}
\]
Table A–9
Shear, Moment, and Deflection of Beams (Continued)
(Note: Force and moment reactions are positive in the directions shown; equations for shear force \( V \) and bending moment \( M \) follow the sign conventions given in Sec. 3–2.)

15 Fixed supports—intermediate load

\[
R_1 = \frac{Fb^2}{l^3}(3a + b) \quad R_2 = \frac{Fa^2}{l^3}(3b + a)
\]

\[
M_1 = \frac{Fab^2}{l^2} \quad M_2 = \frac{Fa^2b}{l^2}
\]

\[
V_{AB} = R_1 \quad V_{BC} = -R_2
\]

\[
M_{AB} = \frac{Fb^2}{l^3}[x(3a + b) - al]
\]

\[
M_{BC} = M_{AB} - F(x - a)
\]

\[
y_{AB} = \frac{Fb^2x^2}{6EI^3}[x(3a + b) - 3al]
\]

\[
y_{BC} = \frac{Fa^2(l - x)^2}{6EI^3}[(l - x)(3b + a) - 3bl]
\]

16 Fixed supports—uniform load

\[
R_1 = R_2 = \frac{wl}{2} \quad M_1 = M_2 = \frac{wl^2}{12}
\]

\[
V = \frac{w}{2}(l - 2x)
\]

\[
M = \frac{w}{12}(6lx - 6x^2 - l^2)
\]

\[
y = -\frac{wx^2}{24EI}(l - x)^2
\]

\[
y_{\text{max}} = -\frac{wl^4}{384EI}
\]