So far we have discussed loading that alternately went from tension to compression with the extremes equal and opposite.

Now we look at the more general case where there could be a mean value.

This is called Fluctuating Fatigue, and is characterized by both a mean and an alternating component.

\[ \sigma_{\text{mean}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \quad \text{Average or Steady Stress} \]

\[ \sigma_{\text{alt}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \quad \text{Stress Amplitude} \]
Effect of Mean Stress

This axis is the fraction that $\sigma_{alt}$ is of the Endurance Strength, $S_e$.

As the Mean stress increases, the material breaks at a lower Alternating stress amplitude.

Circles represent failures.

This axis is the fraction that $\sigma_{mean}$ is of the Ultimate Strength, $S_u$.
Fluctuating Fatigue Diagrams

Several ways are available to characterize the Fluctuating Fatigue behavior. Two common ones are:
• Goodman line and
• Gerber line.

Both approximate the material behavior. We will use the Goodman line because it is simpler and conservative.

The Modified Goodman Diagram is the red line and is the Goodman line truncated by the Yield line.

Hamrock
Section 7.10

Drawing the Modified Goodman Diagram

The diagram is based on material properties Sut, Sy, and Se.

As with the S-N curve, Se should be derated to reflect your part:
\[ S'_{e} = k_{f} k_{s} k_{t} k_{m} S_{e} \]
(My Part) (Test Specimen)

For torsional (shear) loading, use
\[ S_{sy} = 0.577 Sy, \]
\[ S_{us} = 0.67 Sut, \] and
Se for Torsion
Drawing the Modified Goodman Diagram

Then plot your alternating and mean stress.

If your point is below the Mod Goodman line, the part should have unlimited life.

Note: This is very different from the “complete” Modified Goodman Diagram that Hamrock details on P. 178 – 179. We will not use that version – it is pretty confusing.

Factors of Safety

The Factor of Safety depends on how the stresses behave. They might:

1. Grow proportionately
2. Only grow in mean
3. Only grow in alternating

How they behave depends on the actual hardware and loading involved.
If $\sigma_{alt}$ and $\sigma_{mean}$ Increase Proportionately

To make the Goodman Line w/F.O.S. go through $(\sigma_{alt}, \sigma_A)$:

$$\sigma_A = S_e \left( \frac{1}{n} - \frac{\sigma_{alt}}{S_{alt}} \right)$$

$$\frac{\sigma_A}{S_e} = \frac{1}{n} - \frac{\sigma_{alt}}{S_{alt}}$$

$$\frac{1}{n} = \frac{\sigma_A}{S_e} + \frac{\sigma_{alt}}{S_{alt}}$$

Equations of the Goodman Line:

$$\sigma_{alt} = \frac{S_e}{S_{alt}} \sigma_{mean} + S_e = S_e \left( -\frac{\sigma_{mean}}{S_{alt}} + \right)$$

$$\sigma_{alt} = S_e \left( 1 - \frac{\sigma_{mean}}{S_{alt}} \right)$$

For a Factor of Safety of $n$:

$$\sigma_{alt} = S_e \left( \frac{1}{n} - \frac{\sigma_{mean}}{S_{alt}} \right)$$

---

If $\sigma_{alt}$ and $\sigma_{mean}$ Increase Proportionately

$$\sigma_{alt, lin} = \frac{S_e}{n} \left( \frac{1}{\sigma_A S_{alt}} \right)$$

Equation to find alternating stress when operating point hits the F.O.S. line or Goodman line.
If Only $\sigma_{\text{alt}}$ Increases

Equation of the Goodman Line:

$$\sigma_{\text{alt}} = S_e (1 - \frac{\sigma_{\text{mean}}}{S_{\text{ut}}})$$

$$\sigma_{u\text{max}} = S_e (1 - \frac{\sigma_{\text{ut}}}{S_{\text{ut}}})$$

Equation to find alternating stress when operating point hits the Goodman line.

Fatigue Exercise

Given a bar of steel with these properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>40 ksi</td>
</tr>
<tr>
<td>Ultimate Tensile</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Endurance</td>
<td>30 ksi</td>
</tr>
</tbody>
</table>

On a Goodman Diagram, predict fatigue for these loadings:

<table>
<thead>
<tr>
<th>Loading</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Alt</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>36</td>
<td>30</td>
<td>14</td>
</tr>
<tr>
<td>B</td>
<td>-27</td>
<td>37</td>
<td>14</td>
<td>32</td>
</tr>
</tbody>
</table>

\( n = \frac{\sigma_{u\text{max}}}{\sigma_{\text{A}}} \)
Fatigue Exercise

Yield: 40 ksi
Ultimate Tensile: 65 ksi
Endurance: 30 ksi

<table>
<thead>
<tr>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Alt</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>36</td>
<td>18</td>
<td>18  ksi</td>
</tr>
<tr>
<td>-27</td>
<td>37</td>
<td>5</td>
<td>32  ksi</td>
</tr>
<tr>
<td>-18</td>
<td>46</td>
<td>14</td>
<td>32  ksi</td>
</tr>
</tbody>
</table>

Fatigue Diagram for Finite Life

From EngRasp ETBX
Another Type of Fatigue Diagram

- $\sigma_{alt} = 50$ ksi
- $\sigma_{mean} = 70$ ksi

Note Finite Life curves

Stress Ratios for This Diagram

- $\sigma_{alt} = 50$ ksi
- $\sigma_{mean} = 70$ ksi

Stress ratio $R = \frac{\sigma_{min}}{\sigma_{max}}$

Amplitude ratio $A = \frac{\sigma_{alt}}{\sigma_{mean}}$
More Cantilever Beam

Ignore Stress Concentration

$$S_e = k_f k_s k' S_e = (0.63)(1)(0.87)(100) = 54.8 \text{ ksi}$$

**Details**
- 12 Gauge (0.1094" thick)
- 0.75 in. wide
- 4 in. long
- High Strength Steel, with $$S_y = 245 \text{ ksi}$$
- Machined finish
- Room Temperature

**CASE 2:** Tip is flexed between 0.075 in and 0.225 in. What is life for 95% survival?

By proportioning, the force now fluctuates between 8.631 lb and 3 x 8.631 = 25.893 lb.

Stresses go from +23.1 ksi to +69.3 ksi.

Cantilever Beam, contd.

$$\sigma_{\text{mean}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = \frac{69.3 + 23.1}{2} = 46.2 \text{ ksi}$$

$$\sigma_{\text{alt}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \frac{69.3 - 23.1}{2} = 23.1 \text{ ksi}$$

Alternating Stress (ksi)

Mean Stress (ksi)
What is the Factor of Safety?

A. If both alternating and mean stresses increase proportionately:
\[
\frac{1}{n} = \frac{\sigma_a}{S_a} + \frac{\sigma_m}{S_m} = \frac{23.1}{54.8} + \frac{46.2}{245} = 0.422 + 0.189 = 0.611 \quad n = \frac{1}{0.611} = 1.64
\]

B. If only alternating stress increases:
\[
\sigma_{a\text{max}} = S_a(1 - \frac{\sigma_m}{S_m}) = 54.8(1 - \frac{46.2}{245}) = 54.8(1 - 0.189) = 44.4 \text{ ksi}
\]
\[
n = \frac{\sigma_{a\text{max}}}{\sigma_a} = \frac{44.4}{23.1} = 1.92
\]

Impact

The energy of the falling block is transferred to stored energy in the spring:
\[
W(h + \delta_{\text{max}}) = \frac{1}{2}(k \delta_{\text{max}}) \delta_{\text{max}}
\]

Static displacement of Weight gently placed on Spring of stiffness, k
\[
\delta_s = \frac{W}{k}
\]

Impact factor is
\[
I_m = \frac{\delta_{\text{max}}}{\delta_{\text{static}}} = \frac{P_{\text{max}}}{W} = 1 + \sqrt{1 + \frac{2h}{\delta_s}}
\]

Effect is dependent on Spring Stiffness. A soft spring means large static deflection, which means smaller Impact factor.
Impact

For this situation, what is:
1. The static deflection
2. The Impact factor
3. The max deflection
4. The max force

\[ \delta_{\text{max}} = \frac{WV^2}{gk} \]

where \( g \) is the gravitational constant, 386 in/s^2 or 9.8 m/s^2. Recognizing that \( W/k = \delta_{\text{st}} \)

\[ \delta_{\text{max}} = \sqrt{\frac{\delta_{\text{st}} V^2}{g}} \]
Impact

For this setup, what velocity gives the same max force as the falling weight just did?

10Lb

100Lb/in

Impact

Read Hamrock’s Example 7.11 of a diver landing on a diving board. Note that the spring here is a beam, whose stiffness is calculated as Force/Deflection. Also note that he deflects the end of the board 92mm [3.62"], and sees a max force of 13.5kN [3035lb]!