Gear Tooth Strength Analysis

Stresses on Spur Gear Teeth

The two primary failure modes for gears are:

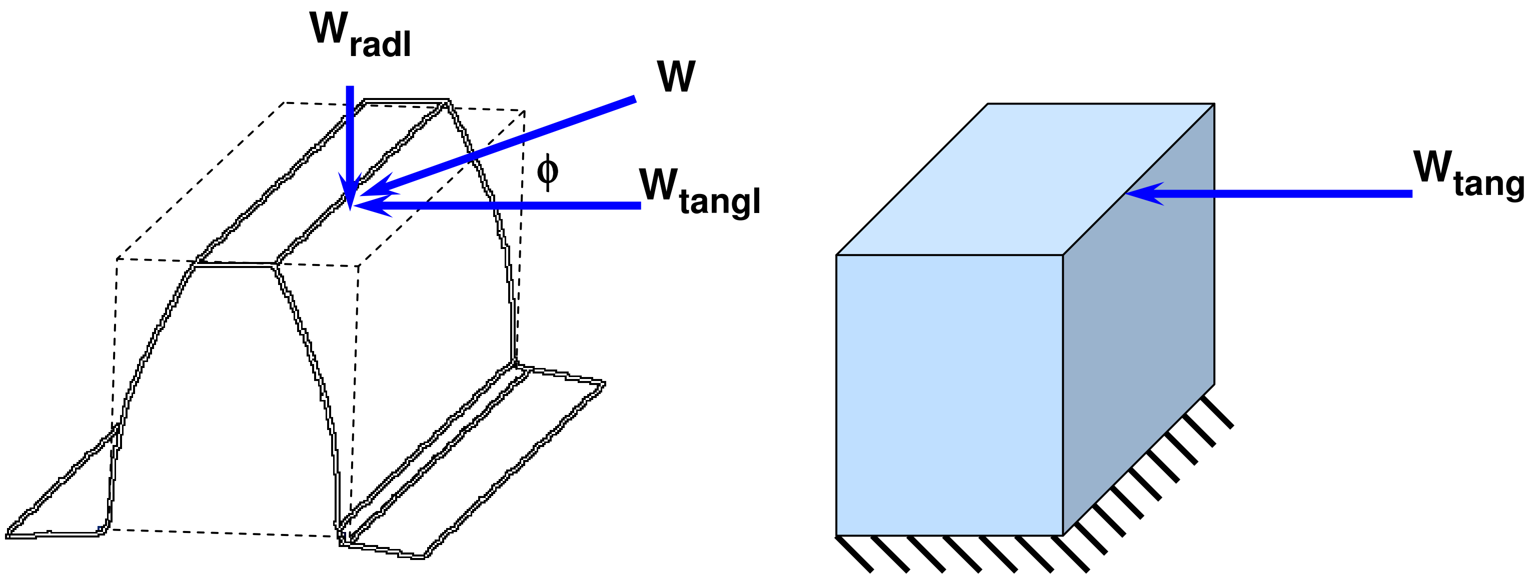
1) Tooth Breakage - from excessive bending stress, and

2) Surface Pitting/Wear - from excessive contact stress.

In both cases, we are interested in the tooth load, which we got from the torque, T. Recall that we compute the tangential force on the teeth as $W_t = T/r = 2T/D$, where D is the pitch diameter.

Bending Stress

The classic method of estimating the bending stresses in a gear tooth is the Lewis equation. It models a gear tooth taking the full load at its tip as a simple cantilever beam:



Hamrock Fig. 14.21

Lewis Bending Stress

From $\sigma = {}^{MC}\!/_{I}$, we get the maximum bending stress

$$\sigma_t = \frac{W_t P_d}{FY}$$

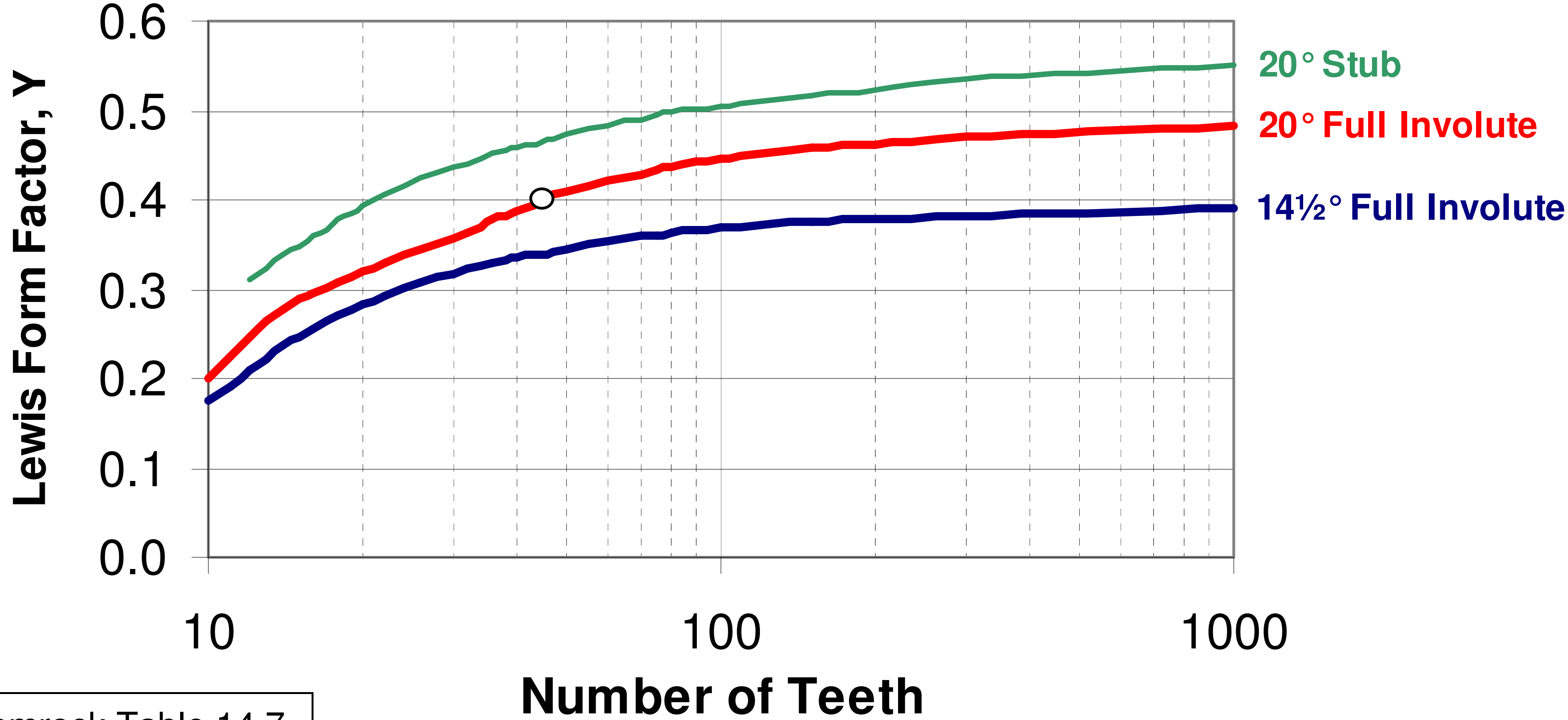
Where:
- W_t is the tangential load (lbs),
- P_d is the diametral pitch (in^{-1}),
- F is the face width (in), and
- Y is the Lewis form factor (dimensionless)

Hamrock Eqn. 14.55

The form factor, Y, is a function of the number of teeth, pressure angle, and involute depth of the gear.

It accounts for the geometry of the tooth, but does not include stress concentration - that concept was not known in 1892 when Lewis was doing his study.

Lewis Form Factor



Hamrock Table 14.7

Note that since Y is in the denominator, bending stresses are higher for the 14 ½° pressure angle teeth, and for fewer number of teeth, i.e. the pinion. Stresses are lower for stub form teeth than for full involutes.

Barth Velocity Factor

Since higher velocity gear operation results in increased stresses due to impacts at initial contact, a velocity-based factor is commonly included in tooth bending stress.

The Barth velocity factor increases the Lewis stress by approximately

$$K_V = \frac{1200 + V}{1200}$$

where V is the velocity at the pitch diameter, in feet per minute. The combined expression for tooth bending stress is then:

$$\sigma_t = \frac{W_t P_d}{FY} \frac{(1200 + V)}{1200}$$

Tooth Bending Stress Example

Given: A 43-tooth, 20° PA, full involute spur, 8 per inch diametral pitch pinion that is 0.5" wide and transmits 4 HP at 1000 RPM.
Find: Estimate the tooth bending stress

Solution: The pitch diameter D_p=Teeth/Pitch = 43/8 = 5.375 in.

Torque

$$T = \frac{4\,HP \times 550^{\,ft.lb/s}/_{HP} \times 12^{\,in.}/_{ft.}}{1000\,RPM \times (2\pi^{\,Rad}/_{\mathrm{Rev}}) \times ^{1\,\mathrm{min}}/_{60\,\mathrm{sec}}} = 252.1\,in.lb.$$

Tangential load W_t = 2T / D_p = 93.8 lb.

Pitch line velocity

V = 1000 Rev/Min x (πD_p) in./Rev x 1 ft/12 in.= 1407.2 FPM

From the graph (Slide 5), Y = 0.4. Then

$$\sigma_t = \frac{W_t P_d}{FY}\frac{(1200+V)}{1200} = \frac{(93.8)(8)(1200+1407.2)}{0.5(0.4)(1200)} = 8152\,psi$$

Allowable Bending Stress

Arriving at a safe allowable stress level for various gear materials is not straight-forward with the Lewis method - but then it is only a simplified approximation.

Unless you are given a specific material allowable value or a table of values, it is reasonable to estimate an allowable strength as S_{ut} / 3 , one third of the material's ultimate tensile strength.

See Hamrock Figs. 14.24 & 14.25

- Be aware that the teeth of gears functioning as idlers experience reversed bending because they are loaded in one direction by the driver and in the opposite direction by the driven gear.

AGMA Bending Stress (1999)

The AGMA* spur gear bending method can be viewed as a detailed refinement of the Lewis method.

Eqn. 14.58

$$\sigma_t = \frac{W_t P_d}{F Y_j} K_a K_s K_m K_v K_i K_B$$

Fig. 14.30

> Y_j is the Lewis *form factor* corrected for several geometry factors, including stress concentration effects.

Table 14.8

> K_a is the *Application factor* (1 to 2.75) that accounts for pulsation and shock in the driver and load.

> K_s is the *Size factor* (1 to 1.4) which penalizes very large or wide teeth.

§14.11.3

> K_m is the *Load Distribution factor* (1 to 2) that is a function of face width.

Fig. 14.34

> K_V is the *Dynamic factor (1 to 1.8)*, essentially a tailored Barth velocity factor that considers gear quality.

> K_i is the *Idler factor (1 or 1.42)*, accounting for reversed bending in idlers.

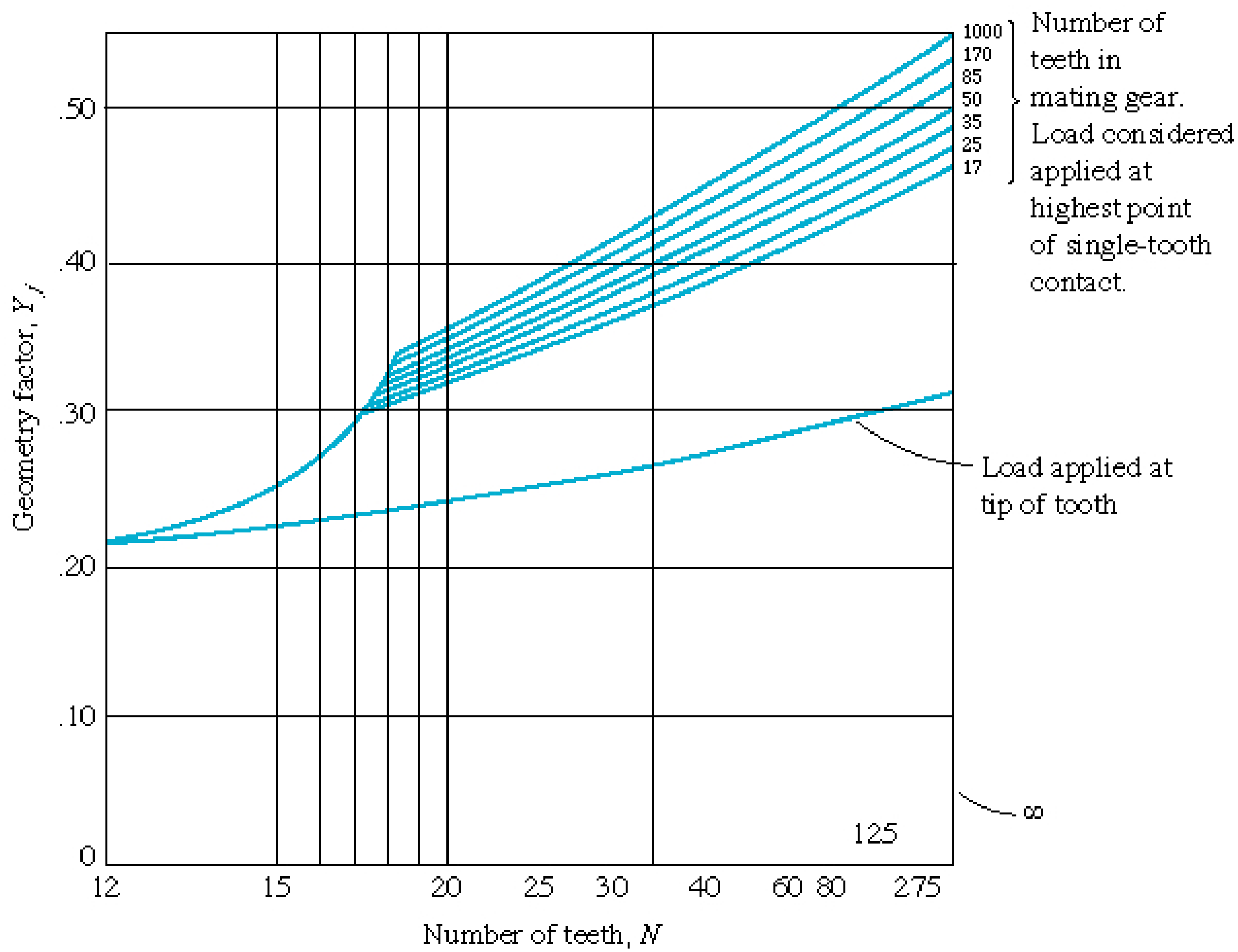
Eqn. 14.67

> K_B is the *Rim Thickness factor* which penalizes for the rim flexibility of non-solid gears.

* American Gear Manufacturers Association, Alexandria, VA.

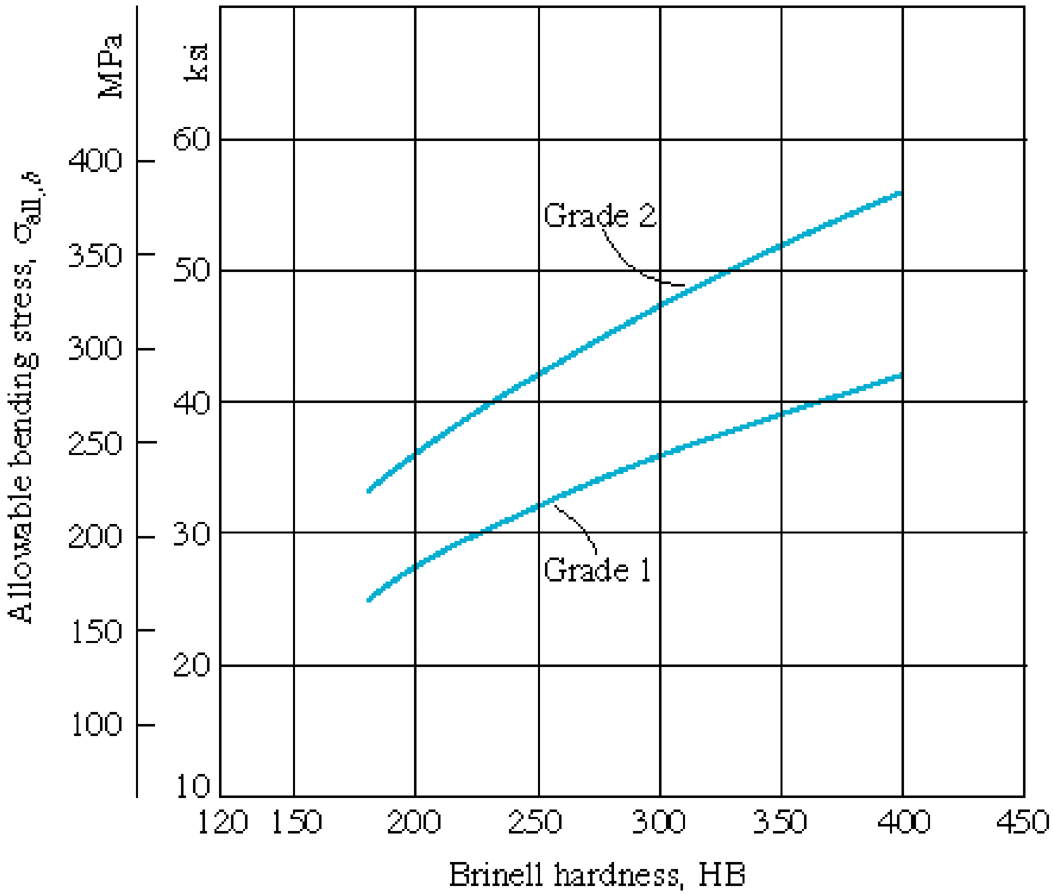
AGMA Bending Stress

These AGMA spur gear bending factors come from an extensive collection of tables and charts compiled by AGMA.



Hamrock Fig. 14.30

AGMA Bending Stress



The material allowable bending strengths also come from an array of AGMA charts that are generally a function of the material Brinell hardness.

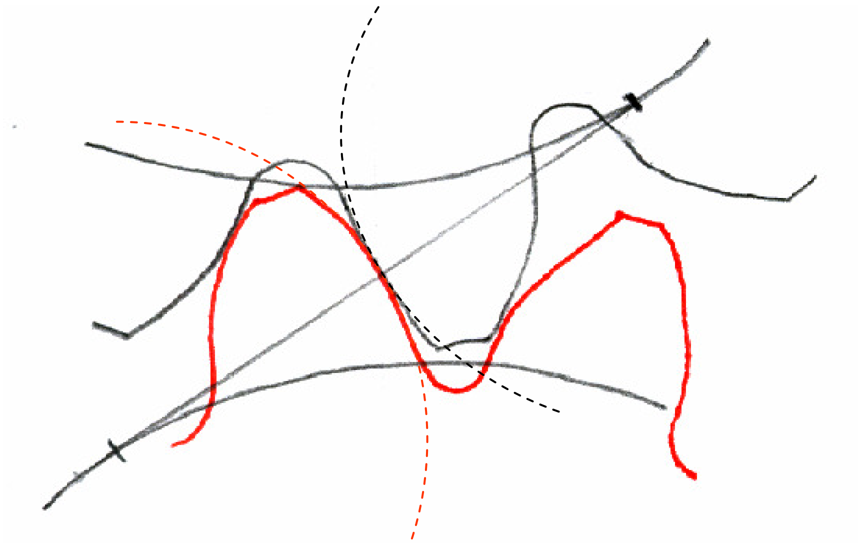
Hamrock Fig. 14.24a

These allowables are generally for 10 million cycles of tooth loading at 99% reliability, and may be adjusted downward for longer life, higher reliability, or higher operating temperatures.

Surface Stress

Even though a gear tooth may not break due to bending stresses during its life, it could develop pits on the tooth face due to high contact stresses fatiguing the surface by compression. The contact pressure is intensified near the pitch circle, where the contact is pure rolling with zero sliding velocity. There the elastohydrodynamic oil film is minimal and the load is less distributed.

This condition is modeled as a pair of cylinders in line contact, and a Hertzian contact stress analysis is used.



Hertzian Contact Pressure

The expression for maximum normal pressure, p, at the line of contact is

$$p = \sqrt{\frac{E^{*}W}{2\pi F}\left(\frac{1}{r_{eg}} + \frac{1}{r_{ep}}\right)}$$

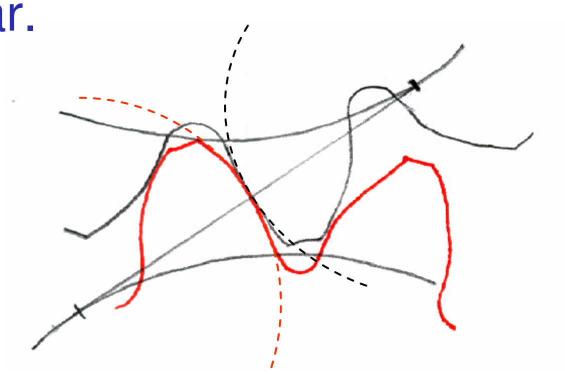
~ Hamrock Eqn. 14.71

where W is the normal tooth force = $W_t / \cos\phi$

F is the tooth face width

E^{*} is the effective modulus of elasticity, $= E / (1 - \upsilon^2)$ if gear and pinion materials are identical

r_{eg}, r_{ep} are the equivalent radii of the cylinders, equal to the pitch radius x $\sin\phi$ for each gear.



Surface Stress

In use, the maximum surface stress is proportional to this maximum pressure. AGMA further refines the stress by adding modifying factors similar to those for bending stresses.

• Be aware that pitting is likely to be more damaging in the long run than bending.

• Hardening the tooth faces increases the allowable contact stress and can help contact life approach bending fatigue life.

• Larger gears have greater radii of curvature and therefore lower stresses.

• Stresses need to be compared to representative, experimentally determined surface fatigue S-N curves.

See Hamrock Fig. 14.25 & 14.26