Rotordynamics Homework ME312

A) Get flywheel weight, W, and polar moment of inertia, J

$$W = \frac{p\, d^2}{4} t r = \frac{p(27)^2(4)(0.282)}{4} = 645.8\,LB.$$

$$J = \frac{p\, d^4 t r}{32\, g} = \frac{p\,(27)^4(4)(0.282)}{(32)(386)} = 152.5\; LB.in \sec^2$$

B) Get torsional stiffness, k

$$k = \frac{p}{32}\frac{G(OD^4 - id^4)}{l} = \frac{p(11.5\times10^6)(4^4-3^4)}{(32)\underbrace{(48)}_{\text{Length Between Disks}}} = 4.12\times10^6\,\frac{in.LB}{RAD}$$

C) Get Torsional Frequency

$$w_{TORS} = \sqrt{\frac{J_1+J_2}{J_1 J_2}k} = \sqrt{\frac{(152.5+30)}{(152.4)(30)}4.12\times10^6} = \sqrt{\frac{4.12}{25.07}}\times1000 = \sqrt{0.164}\times1000 = 405.4\,\frac{RAD}{\sec}$$

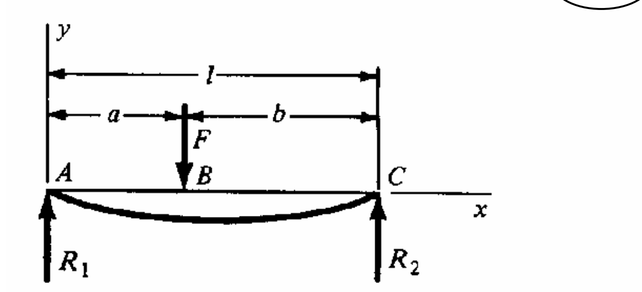
$$f_{TORS} = \frac{w_{TORS}}{2p} = 64.5\,Hz\ Torsional$$

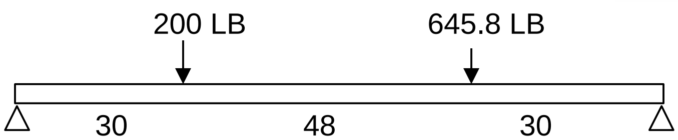
D) Get shaft deflections

$$y_{AB} = \frac{Fbx}{6EIl}(x^2+b^2-l^2)$$

$$y_{BC} = \frac{Fa(l-x)}{6EIl}(x^2+a^2-2lx)$$

6 Simple supports—intermediate load, $a < b$





a = 30, b = 78, l = 108 in.

From 200 LB load,

$$I = \frac{p}{64}(OD^4 - id^4) = 8.59\,in^4$$

$$At\ x = 30,\quad y_{left} = \frac{(200)(78)(30)}{6EI(108)}(30^2+78^2-108^2) = 0.0131\ in.$$

$$At\ x = 78,\quad y_{right} = \frac{(200)(30)(30)}{6EI(108)}[78^2+30^2-(2)(108)(78)] = 0.0106\ in.$$

From 645.8 LB load, by symmetry and scaling,

$$y_{left} = y_{right}(with\ W = 200)\times\frac{645.8}{200} = 0.0342\ in.$$

$$y_{left\,TOTAL} = 0.0131 + 0.0342 = 0.0473\ in.$$

$$y_{right} = y_{left}(with\ W = 200)\times\frac{645.8}{200} = 0.0443\ in.$$

$$y_{right\,TOTAL} = 0.0106 + 0.0443 = 0.0549\ in.$$

E) Compute Lateral Frequency by Rayleigh Ritz

$$w_{LAT} = \sqrt{g \frac{\sum W_n d_n}{\sum W_n d_n^2}} = \sqrt{g \frac{(200 \times 0.0473) + (645.8 \times 0.0549)}{200 \times (0.0473)^2 + 645.8 \times (0.0549)^2}} = \sqrt{386 \frac{44.91}{2.394}} = \sqrt{(386)(18.76)} = 85.1 \frac{RAD}{\sec}$$

$$f_{LAT} = \frac{w_{LAT}}{2p} = 13.5\,Hz\;\; Lateral$$

F) or Compute Lateral Frequency by Dunkerley

$$w_1 = \sqrt{\frac{g}{d_1}} = \sqrt{\frac{386}{0.0131}} = \sqrt{29{,}466} = 171.6 \frac{RAD}{\sec}$$

$$w_2 = \sqrt{\frac{g}{d_2}} = \sqrt{\frac{386}{0.0443}} = \sqrt{8{,}713} = 93.3 \frac{RAD}{\sec}$$

$$\frac{1}{w_{cr}^2} = \frac{1}{w_1^2} + \frac{1}{w_2^2} = \frac{1}{29{,}466} + \frac{1}{8{,}713} = 0.0001487$$ Hamrock’s Eqn. 11.58

$$w_{cr} = 82.0 \frac{RAD}{\sec}$$

$$f_{cr} = \frac{w_{cr}}{2p} = 13.05\,Hz\;\; Lateral$$

This is slightly less than the Rayleigh Ritz method, as Hamrock states.

G) Determine the dimensions of the gear.

Start with writing down what you know:

$$W = \frac{p\, d^2 t r}{4} = 200\,LB. \;\;\Rightarrow\;\; t = \frac{(200)(4)}{p d^2 r}$$

$$J = \frac{p d^4 t r}{32\, g} = 30\;LB.in\sec^2$$

Plugging the expression for t into the equation for J:

$$J = \frac{p d^4 (200)(4) r}{32\, g p\; d^2 r} = \frac{d^2 800}{32\, g} = 30\;LB.in\sec^2$$

$$d^2 = \frac{(30)(32)(387)}{800} = 463.2$$

$$or \quad d = 21.5\, in$$

$$t = \frac{(200)(4)}{p(463.2)(0.282)} = 1.95\; in$$