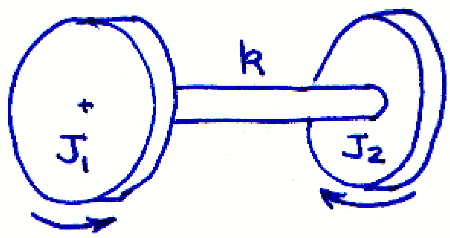
BASIC DYNAMICS OF ROTORS

TORSIONAL VIBRATION

SIMPLE SYSTEM



J = POLAR (MASS) MOMENT OF INERTIA $= \int \underset{\text{RADIUS}}{r^2} \underset{\text{MASS}}{dm}$

FOR DISKS

$$J = \frac{\pi d^4 t \rho}{32 g} \quad \text{LB.IN.SEC}^2$$

d = DIAMETER
t = THICKNESS
ρ = WEIGHT/VOLUME
g = 386 IN/S²

→ SEE TABLE 4.2, PAGE 158, AND
PLUG FORMULA FOR m INTO I_x EQUATION.

k = TORSIONAL STIFFNESS $= \dfrac{\text{TORQUE}}{\text{ANGULAR TWIST}}$

FOR ROUND SHAFTS

$$k = \frac{\pi d^4 G}{32 \ell} \quad \frac{\text{LB. IN.}}{\text{RAD}}$$

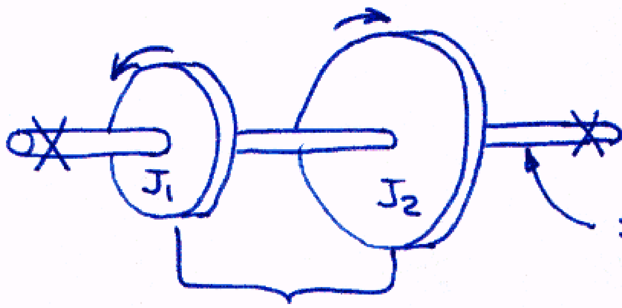
d = DIAMETER
ℓ = LENGTH
G = SHEAR MODULUS

NATURAL FREQUENCY

$$\omega_n = \sqrt{\frac{J_1 + J_2}{J_1 J_2} k} \quad \frac{\text{RAD}}{\text{SEC}} = 2\pi f_n$$

$$f_n = \frac{\omega_n}{2\pi} \quad \text{Hz} \quad \text{OR} \quad \frac{\text{VIBRATIONS}}{\text{SECOND}}$$

"REAL" SYSTEM



SHAFT "ENDS" JUST GO ALONG FOR THE RIDE, ADDING INERTIA TO ROTORS, BUT <u>NOT</u> ADDING STIFFNESS.

k IS ONLY IN SHAFT BETWEEN ROTORS

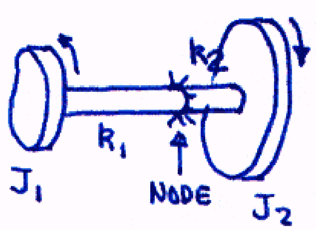
MODE SHAPE

AT RESONANCE, A SMALL INERTIA ROTOR MUST ROTATE MORE THAN A LARGER ROTOR TO BALANCE THE TORSIONAL FORCES.

$$\frac{\Theta_2}{\Theta_1} = -\frac{J_1}{J_2}$$

THE VIBRATORY "NODE" IS CLOSER TO THE LARGER INERTIA.

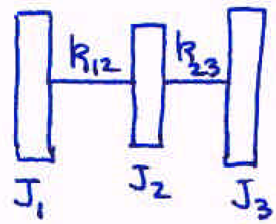
IT LOOKS LIKE 2 SYSTEMS WITH THE SAME FREQUENCY



$$\omega_n = \sqrt{\frac{k_1}{J_1}} = \sqrt{\frac{k_2}{J_2}}$$

RECALL $k = \dfrac{\pi d^4 G}{32\,\ell}$ ← SHORTER ℓ MEANS HIGHER STIFFNESS

3 ROTOR SYSTEM



$$\text{SUMM} = \frac{k_{12}}{J_1} + \frac{k_{23}}{J_3} + \frac{k_{12}+k_{23}}{J_2}$$

$$\text{PROD} = \frac{4\,k_{12}\,k_{23}}{J_1\,J_2\,J_3}(J_1+J_2+J_3)$$

$$\omega_n^2 = \frac{1}{2}\,\text{SUMM} \pm \frac{1}{2}\sqrt{\text{SUMM}^2 - \text{PROD}}$$

TWO MODE SHAPES:

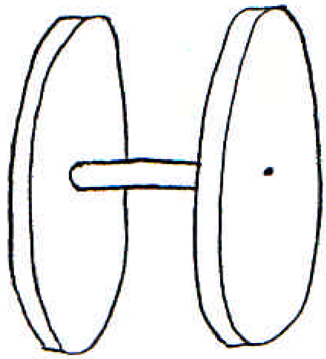
MORE THAN 3 ROTORS:

USE HOLZER TABULATION

- SEARCH FOR FREQUENCIES WHERE DYNAMIC TORQUES BALANCE

TORSIONAL VIBES

EX:



1/2" STEEL DISKS, 12" DIA.

3/8" STEEL SHAFT, 6" LONG

STEEL $\rho = .282$ LB/IN^3

$G = 11.5 \times 10^6$ PSI

$g = 386$ IN/S^2

$$J = \frac{\pi d^4 t \rho}{32 g} = \frac{\pi (12)^4 (0.5)(0.282)}{(32)(386)} = \frac{9185.3}{12352.}$$

$$J = 0.744 \text{ LB.IN.S}^2$$

$$k = \frac{\pi d^4 G}{32 \ell} = \frac{\pi (0.375)^4 (11.5 \times 10^6)}{(32)(6)} = \frac{714,452}{192}$$

$$k = 3721 \ \frac{\text{IN.LB}}{\text{RAD.}}$$

$$\omega_n = \sqrt{\frac{J+J}{J \cdot J} k} = \sqrt{\frac{2J}{J^2} k} = \sqrt{\frac{2k}{J}}$$

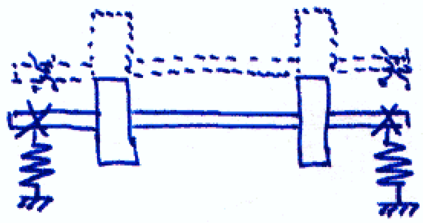
$$\omega_n = \sqrt{\frac{(2)(3721)}{0.744}} = \sqrt{10,008} = 100.0 \ \frac{\text{RAD}}{\text{SEC}}$$

$$f_n = \frac{\omega_n}{2\pi} = 15.92 \text{ Hz}$$

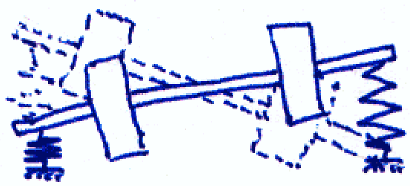
$$\times 60 = 955 \ \frac{\text{CYCLES}}{\text{MINUTE}}$$

LATERAL VIBRATION

A) "SOFT" BEARINGS
(RIGID SHAFT)

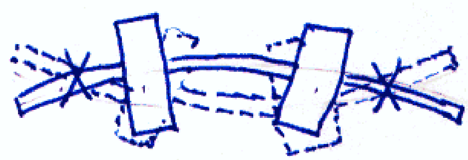


BOUNCE MODE

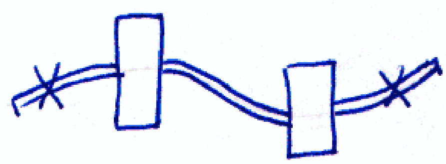


ROCK, OR ORBIT, MODE

B) "HARD" BEARINGS
(FLEXIBLE SHAFT)



FIRST BENDING MODE

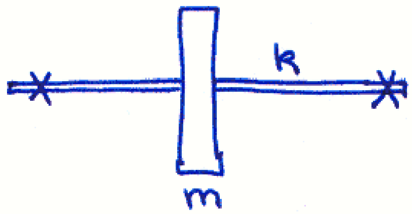


SECOND BENDING MODE

CRITICAL SPEEDS ARE CLOSE TO THE FREQUENCIES YOU WOULD GET IF YOU BANGED ON THE SYSTEM AND MEASURED THE RESPONSE.

ROTOR GYROSCOPIC FORCES WILL CAUSE DIFFERENCES.

SIMPLE SYSTEM

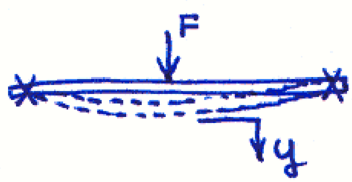


IF ROTOR IS MUCH HEAVIER THAN THE SHAFT,

EQ. 11.46 $\qquad \omega_n = \sqrt{\dfrac{k}{m}} \ \dfrac{\text{RAD}}{\text{S}}$ $\qquad m = \text{ROTOR } \underline{\text{MASS}} = \dfrac{\text{WEIGHT}}{g}$

$k = \underline{\text{LATERAL}}$ BEAM STIFFNESS OF SHAFT, $\dfrac{\text{LB}}{\text{IN}}$

EX: SIMPLE SUPPORTS, CENTER LOAD



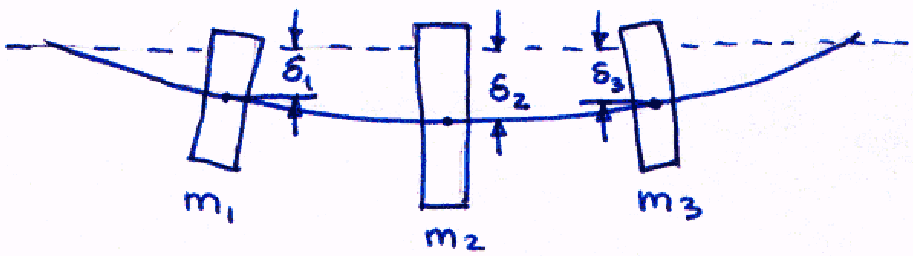
$$y_{max} = \frac{F\ell^3}{48EI}$$

$$k = \frac{F}{\delta} = \frac{F}{y_{max}} = \frac{48EI}{\ell^3} \quad \frac{\text{LB}}{\text{IN}}$$

SAME AS:

$k = \dfrac{48EI}{\ell^3} \qquad \omega_n = \sqrt{\dfrac{k}{m}}$

RAYLEIGH - RITZ METHOD
(FOR A "LIGHT" SHAFT WITH LUMPED MASSES)



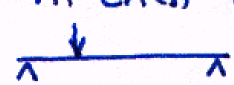
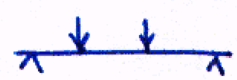
$$\omega_n = \sqrt{\frac{g \sum W_n \delta_n}{\sum W_n \delta_n^2}} \quad \frac{\text{RAD}}{\text{SEC}}$$

WHERE $g = 386$ IN/S^2

W_n = WEIGHT OF ROTOR n

δ_n = STATIC (GRAVITY) DEFLECTION AT MASS n DUE TO ALL MASSES ON SHAFT.

METHOD:

1. GET STATIC DEFLECTION OF SHAFT AT EACH MASS.
   - (USE APPENDIX E-9-6 
     E-9-9  TWIN LOADS
     E-9-10)  OVERHUNG LOAD
   - USE SUPERPOSITION TO GET TOTAL DEFLECTION AT EACH MASS

2. COMPUTE ω_n

SPECIAL CASE OF <u>ONE</u> MASS

$$\omega_n = \sqrt{g\frac{W\delta}{W\delta^2}} = \sqrt{\frac{g}{\delta}} \qquad \text{EQ. 11.50}$$

EX: 100LB MASS CENTERED ON 2" Ø SHAFT, 20" LONG.
(STEEL)

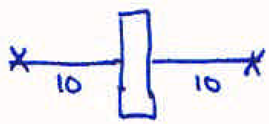


TABLE 4.1 $\quad I = \frac{\pi d^4}{64} = \frac{\pi(16)}{64} = 0.785 \text{ IN}^4$

(TABLE E-9-5)
CENTER LOAD $\quad y_{max} = \frac{F\ell^3}{48EI} = \frac{(100)(20)^3}{(48)(30\times10^6)(0.785)}$

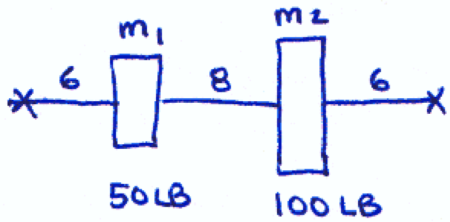
$$y_{max} = \delta = 0.0007074 \text{ IN.}$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{386}{0.0007074}} = \sqrt{545{,}660} = 738.7 \ \frac{\text{RAD}}{\text{S}}$$

$$f_n = \frac{\omega_n}{2\pi} = 117.6 \text{ HZ}$$

$$60 * f_n = 7054 \text{ RPM}$$

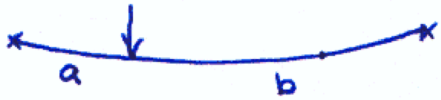
EX! 2 MASSES ON 2" Ø SHAFT



AS BEFORE,
$I = 0.785 \text{ IN}^4$

(TABLE E-9-6)
⇨ $a < b$

$\ell = a + b$



• FIRST, APPLY 50 LB LOAD

AT m_1, $x = 6$
$a = 6$
$b = 14$
$\ell = 20$

$$\delta_1 = \frac{Fbx}{6EI\ell}(x^2 + b^2 - \ell^2)$$

$$\delta_1 = \frac{(50)(14)(6)}{6EI\,20}(36 + 196 - 400) = \frac{-5880}{EI}$$

$$\delta_1 = -0.00025 \text{ IN}$$

AT m_2, $x = 14$

$$\delta_2 = \frac{Fa(\ell - x)}{6EI\ell}(x^2 + a^2 - 2\ell x)$$

$$\delta_2 = \frac{(50)(6)(6)}{6EI\,20}(196 + 36 - 560) = \frac{-4920}{EI}$$

$$\delta_2 = -0.000209 \text{ IN}$$

• THEN, APPLY 100 LB LOAD

BECAUSE OF SYMMETRY AND 2X LOAD, DEFLECTIONS ARE REVERSED (LEFT TO RIGHT) AND DOUBLED.

$$\delta_1 = -0.000418 \text{ IN}$$
$$\delta_2 = -0.00050 \text{ IN}$$

• TOTAL THE DEFLECTIONS (DROP SIGNS)

$$\delta_1 = 0.00025 + 0.000418 = 0.000668 \text{ IN}$$
$$\delta_2 = 0.000209 + 0.00050 = 0.000709 \text{ IN}$$

(MAKES SENSE, BIGGER δ IS AT BIGGER MASS)

- SUM THE PRODUCTS

$$W_1\delta_1 = 50 * 0.000668 = 0.0334$$
$$W_2\delta_2 = 100 * 0.000709 = 0.0709$$
$$\Sigma \quad 0.1043$$

$$W_1\delta_1^2 = 50 * (0.000668)^2 = 0.2231\times 10^{-4}$$
$$W_2\delta_2^2 = 100 * (0.000709)^2 = 0.5027\times 10^{-4}$$
$$\Sigma \quad 0.7258\times 10^{-4}$$

$$\omega_n = \sqrt{\frac{(386)(0.1043)}{(0.7258)10^{-4}}} = \sqrt{554{,}701} = 744.8 \ \frac{\text{RAD}}{\text{S}}$$

$$f_n = \frac{744.8}{2\pi} = 118.5 \text{ Hz}$$
$$* 60 = 7112 \text{ RPM}$$

RECALCULATE USING DUNKERLEY EQUATION [11.58]

- CRITICAL SPEED WITH ONLY LEFT MASS

$\delta_1 = 0.00025$ IN. , FROM PAGE 8

$$\omega_1 = \sqrt{g/\delta_1} = \sqrt{\frac{386}{0.00025}} = \sqrt{1{,}544{,}000} = 1242.6 \ \frac{\text{RAD}}{\text{S}}$$

- CRITICAL SPEED WITH ONLY RIGHT MASS

$\delta_2 = 0.00050$ IN.

$$\omega_2 = \sqrt{g/\delta_2} = \sqrt{\frac{386}{0.0005}} = \sqrt{772{,}000} = 878.6 \ \frac{\text{RAD}}{\text{S}}$$

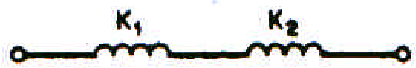
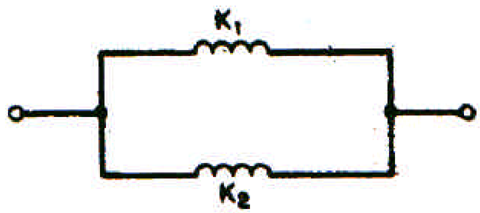
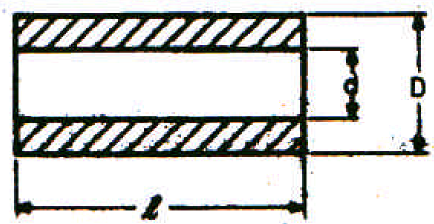
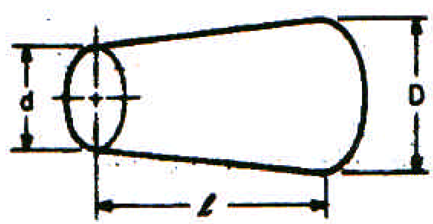
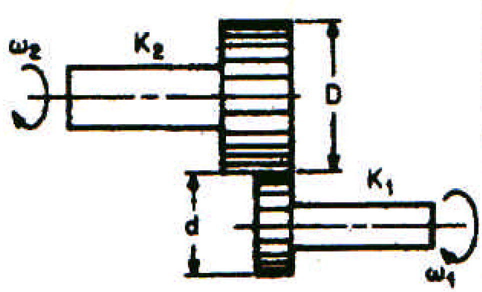
- COMBINE

$$\frac{1}{\omega_{cr}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = \frac{1}{1544000} + \frac{1}{772000} = 0.0000019$$

$$\omega_{cr}^2 = 514{,}667$$
$$\omega_{cr} = 717.4 \ \frac{\text{RAD}}{\text{S}}$$

A LOWER ESTIMATE, AS PREDICTED.

Table 38.1 Formulas for Torsional Stiffness

K = TORSIONAL STIFFNESS, LB-IN./RAD; G = SHEAR MODULUS, LB/IN^2 ω = ROTATIONAL SPEED, RAD/SEC		
SPRINGS IN SERIES		$K = \dfrac{1}{1/K_1 + 1/K_2}$
SPRINGS IN PARALLEL		$K = K_1 + K_2$
HOLLOW CIRCULAR SHAFT		$K = \dfrac{\pi}{32}\,\dfrac{G(D^4 - d^4)}{\ell}$
TAPERED CIRCULAR SHAFT		$K = \dfrac{3\pi}{32}\,\dfrac{d^4}{\ell(n + n^2 + n^3)}$ $n = \dfrac{d}{D}$
TWO GEARED SHAFTS (REFERRED TO SHAFT 1)		$K = \dfrac{K_1 K_2}{n^2 K_1 + K_2}$ $n = \dfrac{D}{d} = \dfrac{\omega_1}{\omega_2}$

From *Shock and Vibration Handbook*, McGraw-Hill, New York, C. M. Harris, 3rd Ed., 1988