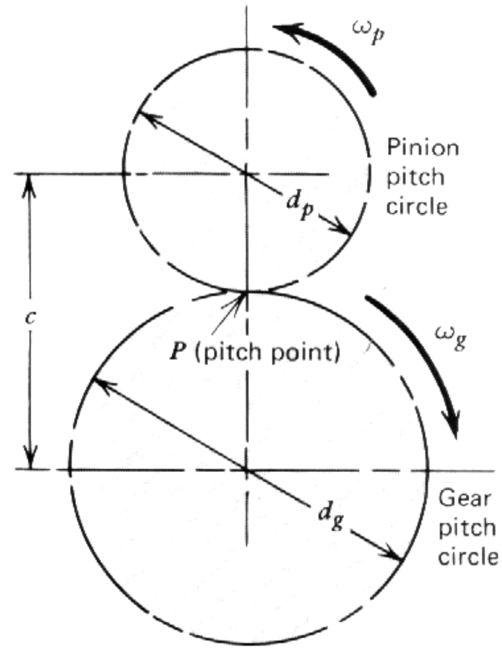
GEARS

Consider two "friction" wheels - a Pinion driving a Gear through contact of their outer diameters. The speed ratio is

$$\frac{w_p}{w_g} = -\frac{d_g}{d_p}$$

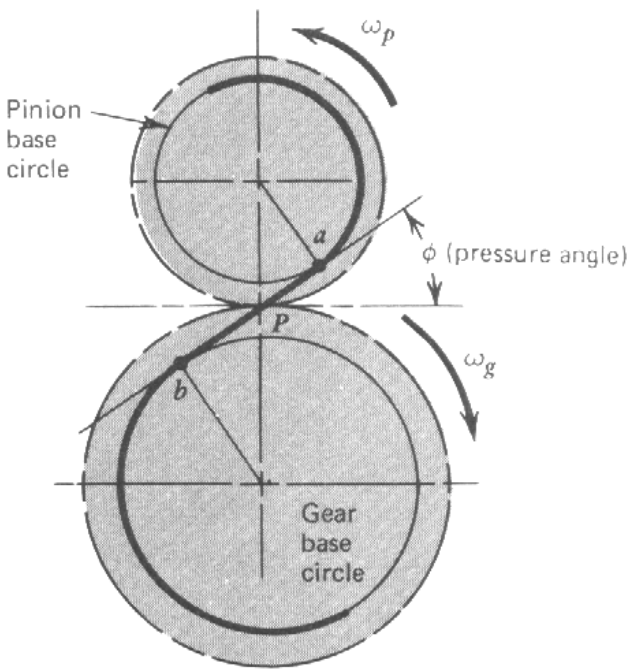
where the minus sign signifies two external gears rotating in opposite directions.

The outer diameters represent the pitch diameters of the actual gears.



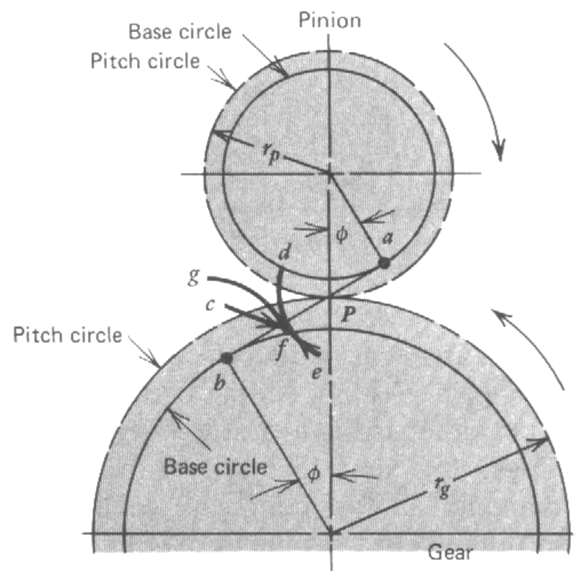
Now, add a thin belt drive running on "base" circles that have the same diametral ratio as the pitch circles.

This is essentially the way that involute-shaped gear teeth function. The teeth are "unwrapped" from the base circles of the gears, and the contact between the pinion and gear tracks the "beltline" a-b.

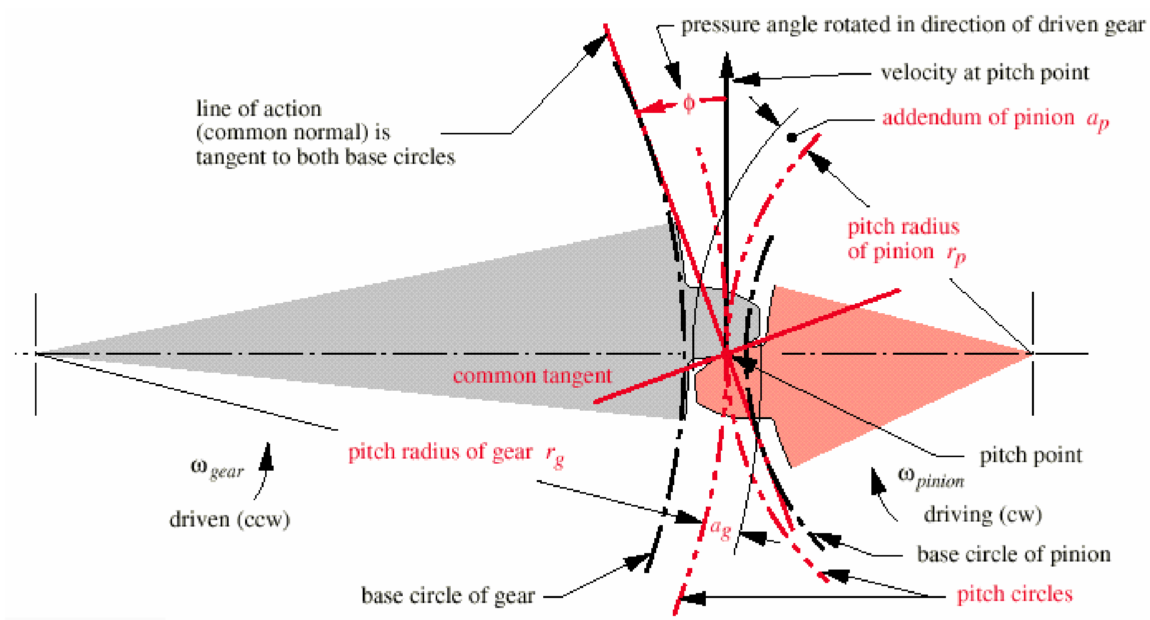


The tooth force is along the angle ϕ, called the pressure angle.

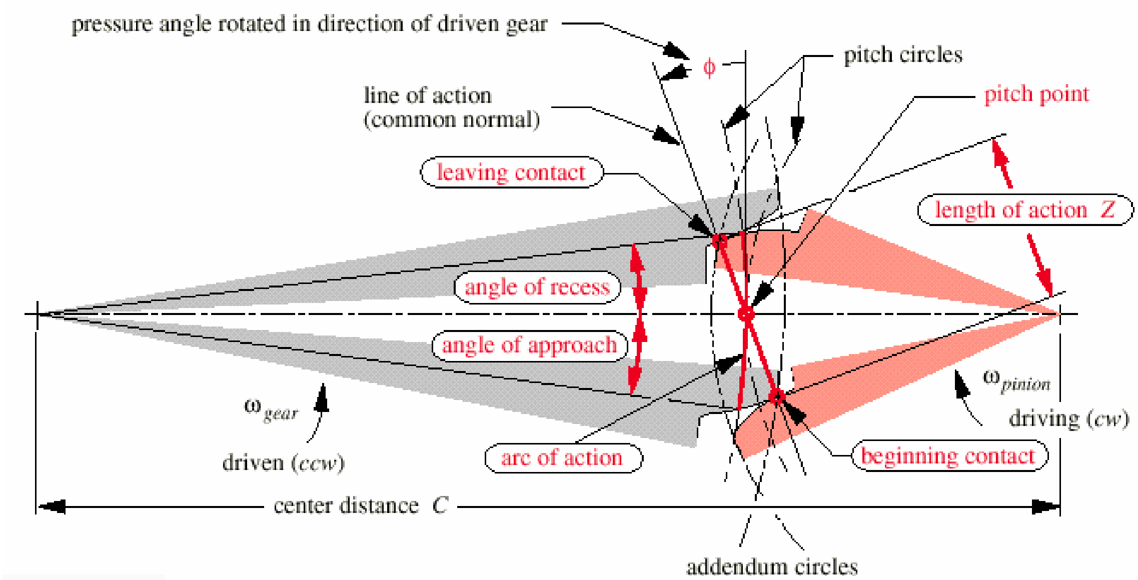
The pitch point, P, remains fixed at the intersection of the line between the two centers and the pitch circle.



Here is a view* of the tooth interaction in an actual gear. Note that the involute shape exists *only* *<u>outside</u>* *the base circle*. The base circle is determined when the gear is built. The pitch circle is dependent on the spacing of the centers of the two meshing gears.



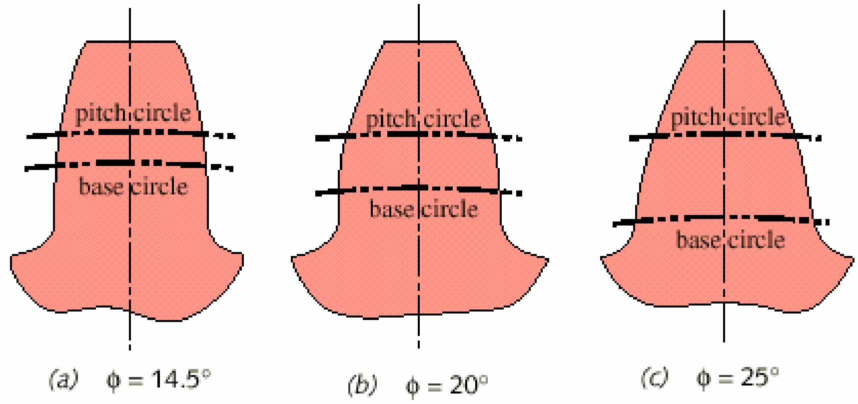
This shows a portion of rotation of the gears from beginning to end of the contact between a pair of teeth. Note the motion of the actual contact point along the line of action, at the pressure angle.



The angle of contact runs between where the addendum circle of each gear intersects the line of action.

* from **MACHINE DESIGN - An Integrated Approach, 2nd Edition by Robert L. Norton, Prentice-Hall 2000**

This shows the shape of the involute gear tooth for three pressure angles. The 20° shape is the most common today. Note again that the involute shape exists only above the base circle. No tooth contact should occur below the base circle.



Cos(14.5°) = 0.97	Cos(20°) = 0.94	Cos(25°) = 0.91

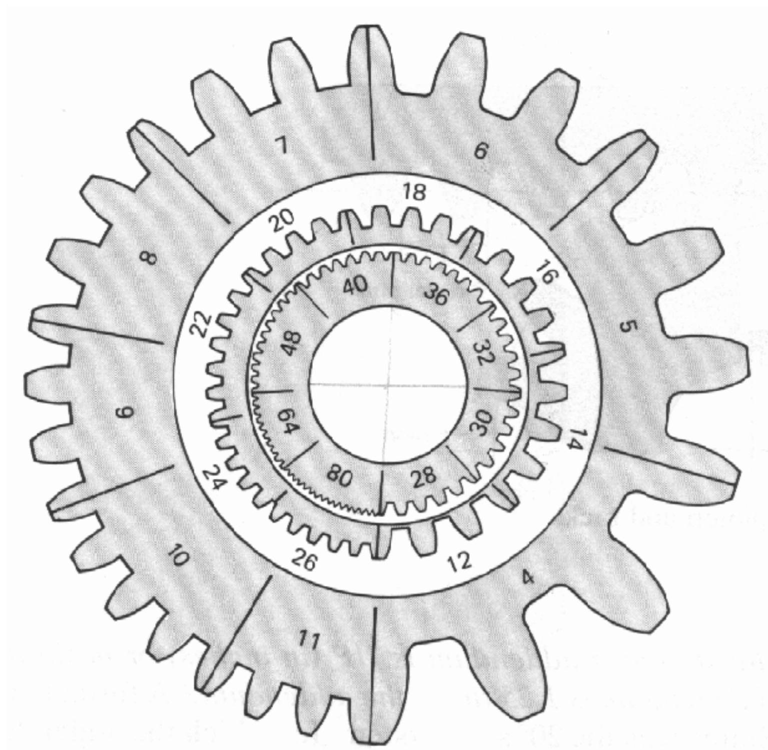
Base circle diameter = Cos ϕ x Pitch diameter

The "Primary" specification for a gear is its **diametral pitch** P_d - not a measurement, but a ratio of its number of teeth, N, to the **pitch diameter**, d.

$P_d = N / d$ in teeth per inch (of pitch circle diameter, not circumference)

Metric gears are based on their **module** $m = 1 / P_d = d / N$ (mm).

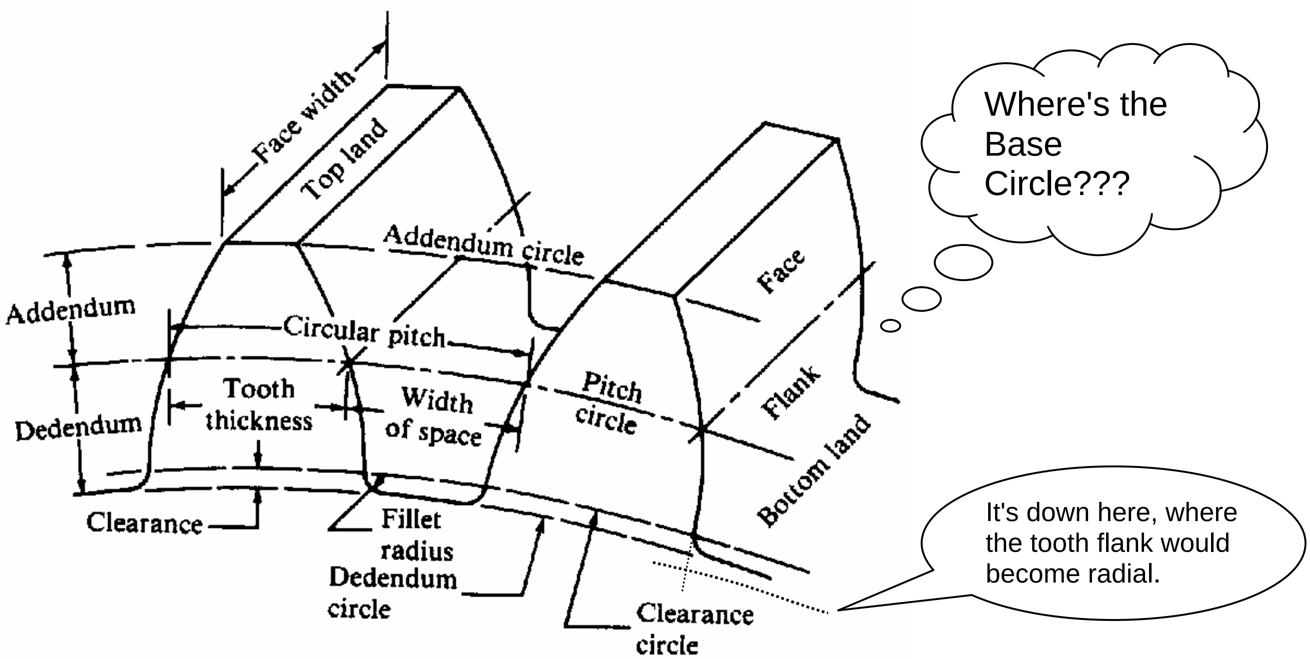
- With English units, "pitch" means diametral pitch.
- With SI units, "pitch" means circular pitch, or actual mm of pitch circumference per tooth, $= \pi$ m.



Actual sizes of gear teeth of various diametral pitches. Note: In general, fine-pitch gears have $P \geq 20$; coarse-pitch gears have $P < 20$. (Courtesy The Barber-Colman Company.)

☞ The pitch of two gears in mesh must be the same!

Gear tooth terminology



and some formulas:

Diametral pitch or "pitch" $P_d = N / d$, where d is pitch diameter,
N is number of teeth

Circular Pitch $p = \pi d / N$
inches of pitch circle <u>circumference</u> per tooth

And $P_d = \pi / p$

Base pitch (not shown) $P_b = p \cos \phi$ inches of <u>base</u> circle circumference per tooth

Face Width is usually between $9 / P_d$ and $14 / P_d$ (or 3 to 4 x the Circular Pitch)

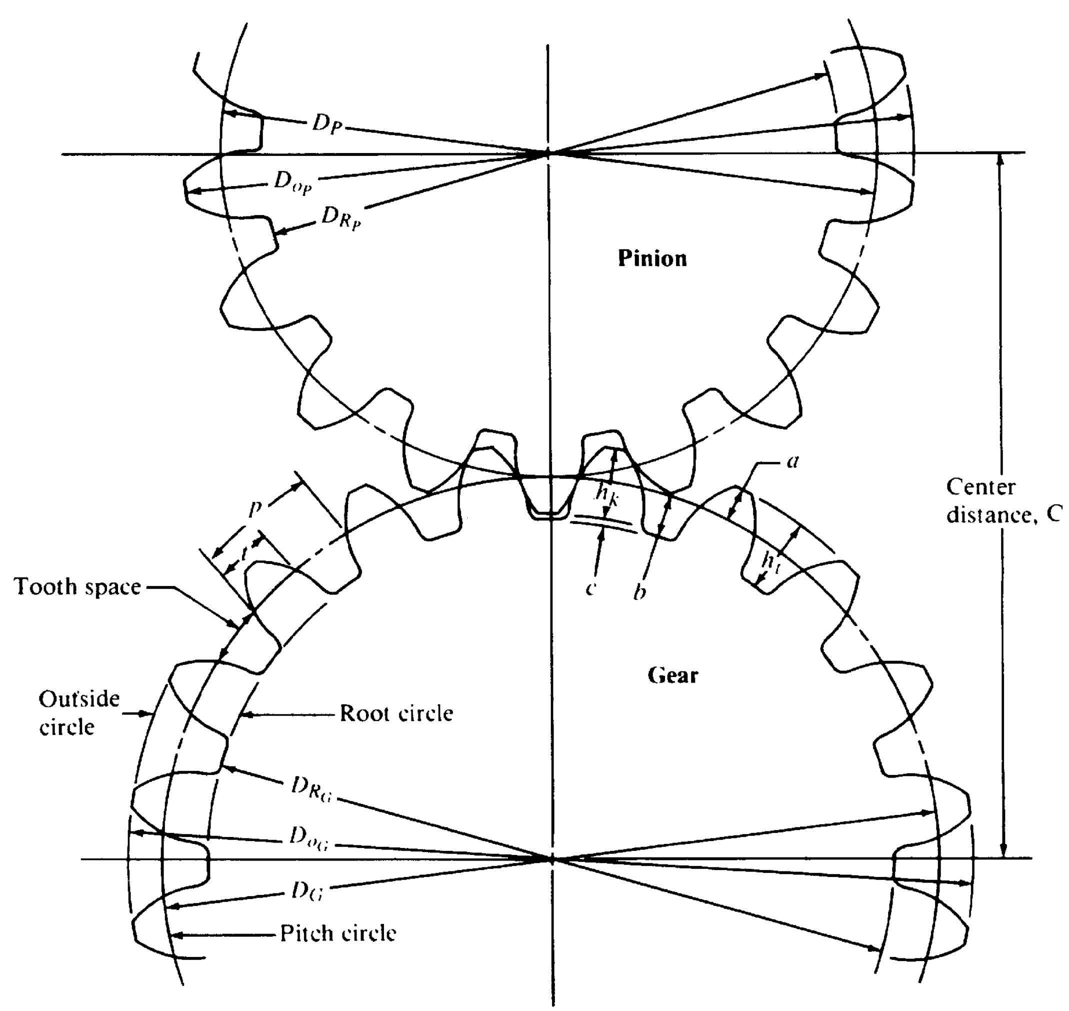
- wide enough to spread the load, but not so wide to be misaligned and concentrate load at the edge.

Pitch Line Velocity $V_{FPM} = \pi \times d \times RPM / 12$

Backlash = Width of Space - Tooth Thickness (of mating tooth) and is usually 5 to 40 mils to make room for lubrication.

Here is Hamrock's Table 14.2, AGMA tooth specifications for full involute 20° PA

		Coarse	Fine	Metric Module
Addendum	a	$1 / P_d$	$1 / P_d$	1.00 m
Dedendum	b	$1.25 / P_d$	$1.20 / P_d + 0.002$	1.25 m
Clearance	c = b - a	$0.25 / P_d$	$0.20 / P_d + 0.002$	0.25 m



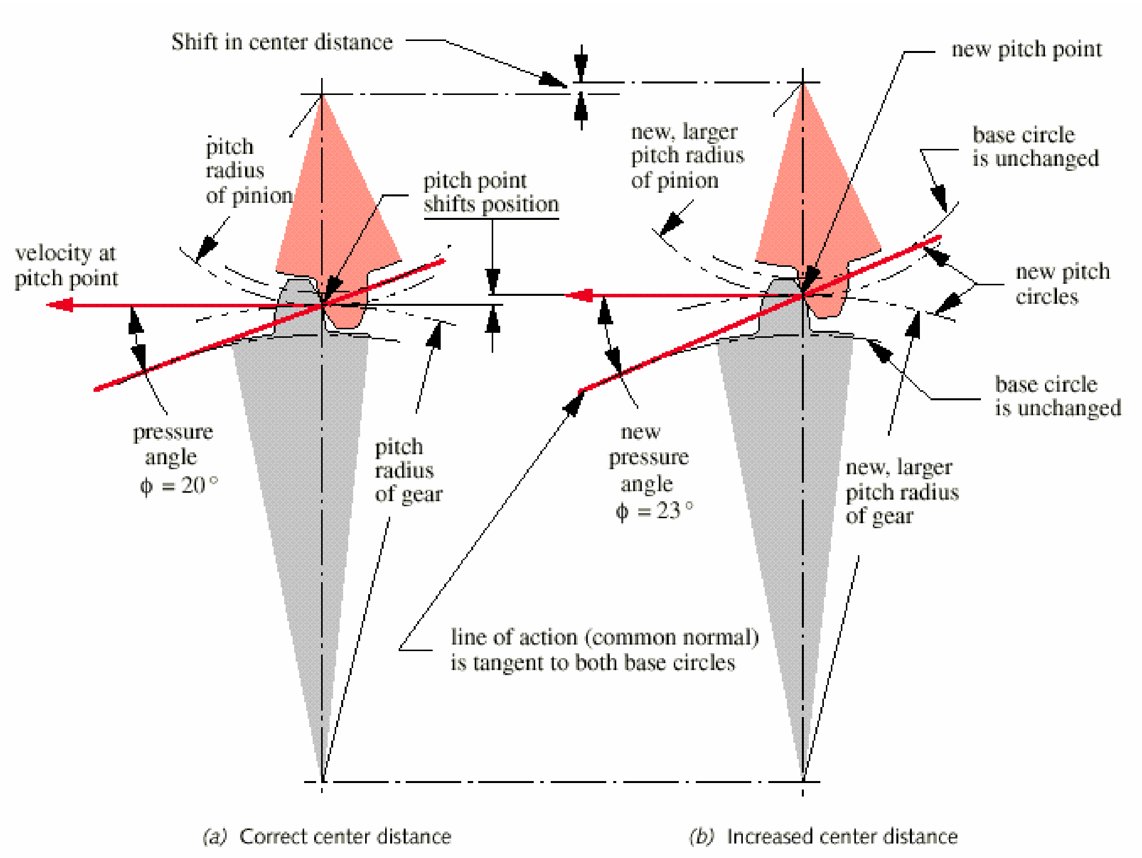
GEAR PAIR FEATURES (Here Diameters are shown as capital D's with Subscripts P for pinion and G for gear.)

Addendum, a
Dedendum, b
Clearance, c = b - a
Pitch Diameter, D
Outside Diameter, $D_O = D + 2a$
↥ Also called Addendum Diameter
Root Diameter, $D_R = D - 2b$

Whole Depth, $h_t = a + b$
Working Depth, $h_k = a + a = 2a$
Tooth thickness, t = p/2
Circular pitch, $p = \pi D / N$
Center Distance, $C = (D_G + D_P)/2$
Base Diameter = $D \cos \phi$
Pressure Angle = ϕ

Changing the center distance:

Increasing it increases the pressure angle and pitch diameter, and shortens the angle of action, but does not change the constant angular velocity ratio of the involute tooth form.



Contact Ratio

$$m_c = \frac{Length\,of\ \ Arc\,of\ \ Action}{Circular\ Pitch}$$ See the lower figure on page 2.

This can also be written in terms of the line of action:

$$m_c = \frac{L_{ab}}{p\cos f}$$ where L_{ab} is the length of the Line of Action and

p is the circular pitch (and p cos ϕ is the base pitch)

This can be expanded to:

$$m_c = \frac{\sqrt{r^2_{a_{pinion}} - r^2_{b_{pinion}}} + \sqrt{r^2_{a_{gear}} - r^2_{b_{gear}}} - C\sin f}{p\,D\cos f \big/ N}$$

where r_a is the addendum radius
C is the center distance
and r_b is the base radius

Using the terms of the figure on page 5:

$$m_c = \frac{\sqrt{\left(\frac{D_{O\,p}}{2}\right)^2 - \left(\frac{D_p}{2}\cos f\right)^2} + \sqrt{\left(\frac{D_{O\,g}}{2}\right)^2 - \left(\frac{D_g}{2}\cos f\right)^2} - C\sin f}{p\ D_g\ \cos f \big/ N_g}$$

(Note that the denominator is still the base pitch since p = π d / N and p cos ϕ is the base pitch.)

- Contact ratios should never be less than 1.1
- Higher contact ratios mean quieter operation

Interference

Happens if tooth contact occurs below the base circle of a gear where the shape is no longer involute. Unless the root is relieved, the tip will dig into the flank. This weakens the tooth.

Interference is more likely if:

1. There are few pinion teeth
2. There are a large number of gear teeth
3. The pressure angle is small (gives large base circle)

The largest addendum circle radius without interference is:

$$r_{a_{\max}} = \sqrt{r_b^2 + C^2\sin^2 f}$$ where r_b is the base circle radius and

C is the center distance.

Example Gear Geometry Worksheet in Excel

For standard gears, four inputs are all that are necessary to determine all the gear features.

Npinion	48	Teeth	Ngear	73	Teeth	
Pitch	16					
PressAng	20	°		0.349	Rad	
GearRatio	1.521	=Ngear/Npinion				
CircPitch	0.196	=PI()/Pitch				
BasePitch	0.185	=CircPitch*COS(PressAng)				
PitchDia (Pinion)	3.000	=Npinion/Pitch				
PitchDia (Gear)				4.563	=Ngear/Pitch	
CenterDist	3.781	=(PitchDiaP+PitchDiaG)/2				
Addendum	0.063	=1/Pitch				
Dedendum	0.078	=1.25/Pitch				
WholeDepth	0.141	=Addendum+Dedendum				
Clearance	0.016	=Dedendum-Addendum				
OutsideDiam (Pinion)	3.125	=PitchDiaP+2*Addendum				
OutsideDiam (Gear)				4.688	=PitchDiaG+2*Addendum	
ContactRatio	1.781	=(1/BasePitch)*{SQRT[(DOP/2)^2-((PitchDiaP/2)*COS(PressAng))^2]+SQRT[(DOG/2)^2-((PitchDiaG/2)*COS(PressAng))^2]-CenterDist*SIN(PressAng)}				
BaseDiameter	2.819	=PitchDiaP*COS(PressAng)		4.287	=PitchDiaG*COS(PressAng)	

Check out this Web site (and neighbors) at the University of Western Australia:
http://www.mech.uwa.edu.au/DANotes/gears/meshing/meshing.html#top