FRACTURE MECHANICS

ME312

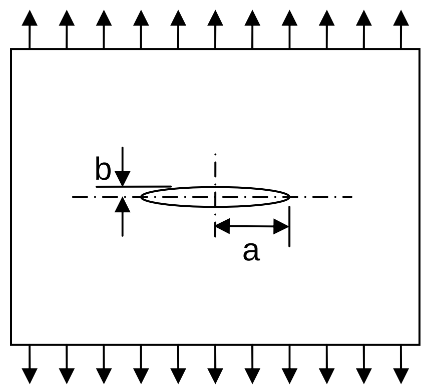
Hamrock Section 6.3 through 6.5

Just as studying Column Buckling lets us avoid rapid failure of structures by unstable bending, Fracture Mechanics (FM) lets us avoid rapid failure of structures by unstable cracking.

Examples of unstable cracking are a balloon contacted by a sharp object, and the Aloha Airlines 737 shown on page 265 of Hamrock. The "wrong" combination of materials, loads, and initiating crack can result in rapid "unzipping" of a structure.

In section 6.2, we saw that holes and notches in plates and bars caused stress concentrations above the average stress due to tension, torsion, or bending loading. For a general, ellipse-shaped hole of width = 2a and height = 2b, in a very wide plate, the stress concentration factor is:

$$K_c = 1 + 2\frac{a}{b} \qquad \text{Eqn. 6.2}$$



This is more commonly called "K_t" for "theoretical." If the hole is circular, a = b and $K_c = 3$. This agrees with Hamrock's Fig. 6.2a for a wide plate with a central hole.

For cracks, however, "b" is very, very small compared to "a", and K_c calculated by this equation gets huge. Right at the crack tip, the stress is high enough to actually cause a small volume of the material to yield and redistribute the stress in ductile materials, or cause local microfractures in brittle materials.

> ☞ If stresses are high enough at the tip of a crack of sufficient length, even a ductile material will "unzip" – have a sudden "brittle-like" fracture.

Rather than deal with giant stress concentrations, Fracture Mechanics uses a **stress intensity factor**, **K**. This represents the effective local stress at the crack tip. The stress intensity factor is calculated for a given geometry and load, and compared with a threshold value of K above which cracks will propagate in the given material. This threshold value of K is called the **fracture toughness** or **critical stress intensity factor, K_{IC},** and is a characteristic of the material measured by testing.

Hamrock gives some sample fracture toughness values in Table 6.1 on page 239. The materials with low fracture toughness that are susceptible to rapid crack growth failure are relatively brittle materials, such as:

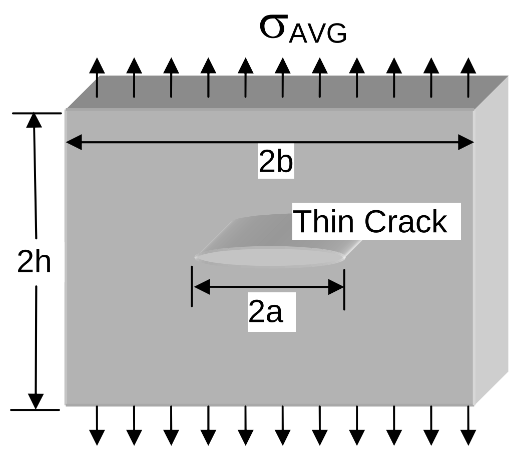
- Glass and ceramics
- Hard steels
- Strong aluminum alloys
- Titanium alloys
- Soft steels below the Ductile → Brittle Transition Temperature

STRESS INTENSITY

For the special case shown here of a long plate (h >> b), with a small crack of length 2a in a relatively wide (b >> a) plate, the stress intensity factor is:

$$K_0 = \sigma_{AVG}\sqrt{\pi a}$$

$$psi\sqrt{in} \ \ or \ \ MPa\sqrt{m}$$



☞ The σ_{AVG} here is P/A, where A is the gross cross section area of the plate with no crack there.

☞ This is different from the σ_{AVG} from Stress Concentrations (K_c), where P/A was calculated for the net area, with the area of the hole subtracted.

For a plate of realistic dimensions, there are tables of stress intensity modification factors (K_I/K_0) given in Hamrock's Appendix C (he calls them "Y"). Multiplying the appropriate K_I/K_0 times the K_0 calculated above from crack half-length and the average (gross) tensile stress, will give the actual stress intensity K_I. As long as K_I is below the K_{IC} for the material, the crack will not unzip.

It is called K_I (pronounced K "one") because it is for Mode I crack propagation where the load tends to open the crack. This is shown in Hamrock's Figure 6.9a on page 237. The other two modes are In-Plane Shear and Out-Of-Plane Shear, and are less used. (Hamrock incorrectly converts the Roman numeral "I" to a little "i" in his sections, and uses K_i.)

Also: Watch out for Hamrock's equation C.2 and C.3. If you can figure out what "a" is in them, please let me know. Use the figures, but ignore the equations.

Hamrock's Table 6.1 gives $psi\sqrt{in}$ and $MPa\sqrt{m}$ as units for stress intensity.

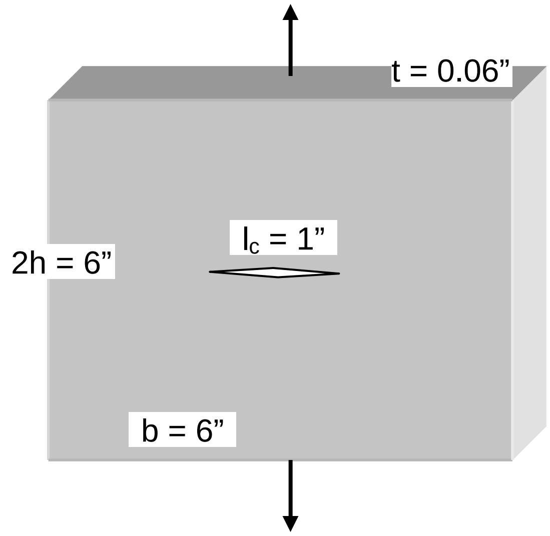
The conversion is $MPa \cdot \frac{1\,ksi}{6.89\,MPa}\sqrt{m \cdot \frac{39.37\,in}{m}} = \frac{6.27}{6.89}ksi\sqrt{in} = 0.91\,ksi\sqrt{in}$.

To summarize, the stress intensity analysis is as follows:

1. Compute the average stress on the uncracked part. It could be tensile or bending.
2. Compute $K_0 = \sigma_{AVG}\sqrt{\pi a}$, where "a" is <u>usually</u> half the length of the crack. Note: For edge cracks, "a" is the <u>whole</u> length of the crack. (This equation derives from analysis of an infinite plate, but is used for all configurations.)
3. Determine the stress intensity modification factor (K_I/K_0) for the actual geometry that you have.
4. Multiply K_0 by K_I/K_0 to get K_I, the actual stress intensity for your geometry and loading.
5. Compare K_I to the fracture toughness or critical stress intensity factor, K_{IC}, for the material that you have. If K_I doesn't exceed K_{IC}, the crack will not propagate.
6. Compute the factor of safety $n = \frac{K_{IC}}{K_I}$.
7. For completeness, check for yielding using the net area (the cross section minus the crack area).

TO STUDY FURTHER:
www.efunda.com/formulae/solid_mechanics/fracture_mechanics/fm_intro.cfm
Good web site on Engineering Fundamentals

EXAMPLE



You have a 7075-T651 Aluminum plate of
Width b = 6”
Length 2h = 6”
Thickness t = 0.06”
With a central crack of length l_c = 1”.

What is the highest load that can be applied without causing a sudden fracture?

- Gross area is (6” x 0.06”) = 0.36 in^2.
- Average tensile stress is F/A = F/0.36 = 2.78F psi.
- From Table 6.1, K_{IC} = 26 $ksi\sqrt{in}$.
- From Hamrock Appendix C, Figure C.1,
    - o h/b = 3/6 = 0.5
    - o l_c/b = 1/6 = 0.167
    - o K_I/K_0 = 1.15
- $K_I = 1.15K_0 = 1.15\sigma_{AVG}\sqrt{\pi a} = 1.15(2.78F)\sqrt{\pi \cdot 0.5}$, or K_I = 4.01F
- And K_I must not exceed the fracture toughness, so set
  K_I = 4.01F = 26 $ksi\sqrt{in}$, from which F = 6,584 lb.
- As a check, compute P/A on the net area
  $$\sigma = \frac{6584\,lb}{(0.06)(6-1)} = 21.945\ psi$$
  This is less than the 73 ksi yield strength from Table 6.1.
  (Note that the column heading says Yield <u>stress</u>, S_y, when it really should say Yield <u>strength</u>.)

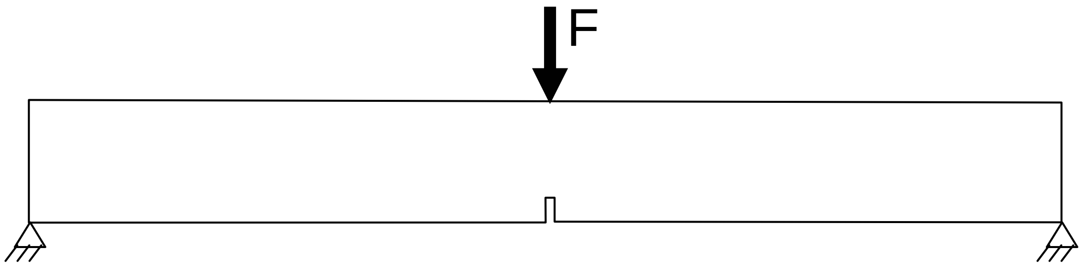
If the problem had given us an applied load and had asked for the maximum allowable crack size, we would have used:

$$\sqrt{\pi a} = \frac{K_0}{\sigma_{AVG}} = \frac{K_{IC}/(K_I/K_0)}{\sigma_{AVG}}$$

$$\pi a = \left[\frac{K_{IC}/(K_I/K_0)}{\sigma_{AVG}}\right]^2$$

$$a = \frac{1}{\pi}\left[\frac{K_{IC}/(K_I/K_0)}{\sigma_{AVG}}\right]^2$$

A BENDING EXAMPLE



A 6” length of 1” x 0.25” 2024-T351 Aluminum bar has a 0.1” deep edge cut in it, and is loaded as shown above. What is the maximum load, F, that can be applied to the bar without failure?

First we compute the local bending stress on the uncracked part. The area moment of inertia is $bh^3/12 = (0.25)(1.0)^3/12 = 0.0208\ in^4$. The distance from the neutral axis to the outer fiber, c, is 0.5”. The moment at the center of the bar is (0.5F)(3) = 1.5F in.lb. The stress is Mc/I = (1.5F)(0.5)/(0.0208) = 36.06 F psi.

From Table 6.1, K_{IC} = 33 $ksi\sqrt{in}$ for 2024-T351.

Going to Appendix C, we see that Figure C.4 represents the geometry and loading that we have.

$l_c/b = 0.1/1.0 = 0.1$

$a/b = 3.0/1.0 = 3.0$

Reading between the lines, it looks like $K_I/K_0 = 1.0$.

We can write $K_I = K_I/K_0 \times K_0 = K_I/K_0 \times \sigma_{AVG}\sqrt{\pi a}$ and set this equal to the fracture toughness K_{IC} = 33 $ksi\sqrt{in}$ to determine the maximum load. Since $K_I/K_0 = 1.0$, this is just

$$33000 = \sigma_{AVG}\sqrt{\pi a} = 36.06F\sqrt{0.1\pi}$$

$$F = \frac{33000}{36.06\sqrt{0.314}} = \frac{33000}{(36.06)(0.5605)} = \frac{33000}{20.21} = 1632\,lb$$

Now to check yielding, we analyze the cracked bar. The height of the cracked section is (1.0 – 0.1) = 0.9 in, and c is half that, or 0.45 in. Our new moment of inertia is $(0.25)(0.9)^3/12 = 0.0152\ in^4$. The stress is

Mc/I = (1.5)(1632)(0.45)/(0.0152) = 72.47 ksi.

But this is greater than the 47 ksi Yield Strength, per Table 6.1, so the bar would actually yield in bending before the stress intensity got high enough to cause a crack propagation failure. The maximum load would actually be 1058 lb before the bar would begin to yield. Good thing we checked.