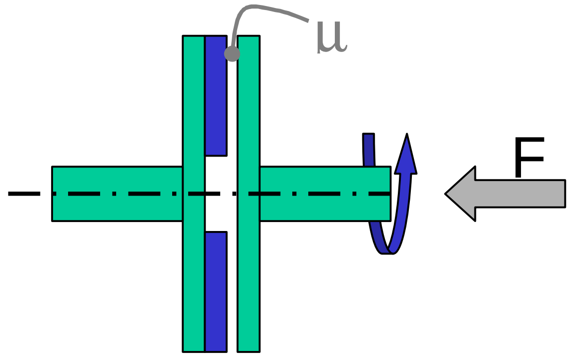
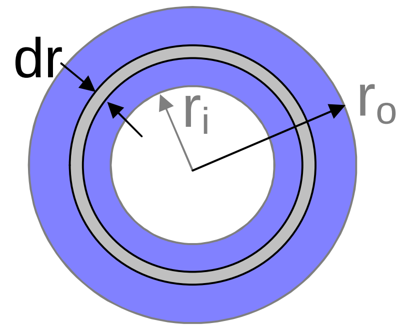
AXIAL DISK CLUTCHES   **Hamrock §18.2**





When it's new, things will be flat and the pressure will be uniform.

$$F = p \times A = p \times p\,(r_o^2 - r_i^2)$$ [Eqn. 18.4]

Can also integrate dA = 2πrdr :

$$\int p\,dA = \int_{r_i}^{r_o} p\,2prdr = 2pp\,\tfrac{1}{2}r^2 \Big|_{r_i}^{r_o} = pp(r_o^2 - r_i^2)$$

This is useful because we need to integrate to get the Torque:

dT = p dA μ r , where μ is the coefficient of friction

dT = 2πr^{2}p μ dr

$$T = \int_{r_i}^{r_o} 2ppmr^2 dr = 2ppm\,\tfrac{1}{3}r^3 \Big|_{r_i}^{r_o} = \tfrac{2}{3}ppm(r_o^3 - r_i^3)$$

[Eqn. 18.5]

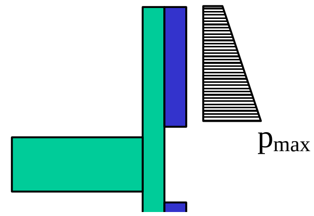
Since $p = \dfrac{F}{p\,(r_o^2 - r_i^2)}$

$$T = \tfrac{2}{3}mF\,\frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$ [Eqn. 18.5]

If the clutch has multiple plates, multiply this Torque by the N friction interfaces.

Since wear is proportional to pressure times surface velocity, and surface velocity increases linearly with radius, r, wear is maximum at the outer radius. Wear reduces the pressure at the rim, and the geometry changes to a uniform wear condition.
If pv is constant and v increases linearly with r, then p must decrease with r:

$$p = p_{\max} \frac{r_i}{r}$$



now computing the normal force

$$F = \int_{r_i}^{r_o} 2p \; prdr = \int_{r_i}^{r_o} 2p \; p_{\max} \frac{r_i}{r} rdr \; = 2pp_{\max} r_i (r_o - r_i)$$

[Eqn. 18.10]

Similarly,

$$T = \int_{r_i}^{r_o} 2p \; pmr^2 dr = \int_{r_i}^{r_o} 2p \; p_{\max} m \frac{r_i}{r} r^2 dr \; = pmp_{\max} r_i (r_o^2 - r_i^2)$$

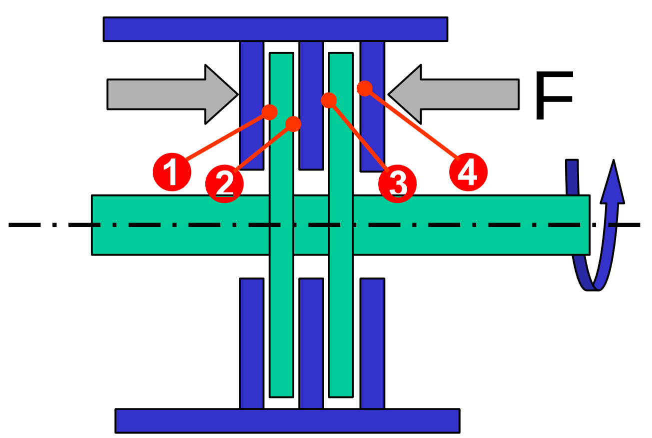
[Eqn. 18.12]

Or, in terms of applied force, F

$T = NmF \dfrac{(r_o + r_i)}{2}$, where N is the number of friction faces. [Eqn. 18.13]

Example of multiplate axial clutch with N = 4.

- Four gaps close up on actuation.
- Components slide axially so that each contact surface transmits the actuating force, F.
- N is usually even.



For a given applied force, F, there is an optimal ratio of r_o to r_i.

$$T = pmp_{\max} r_i (r_o^2 - r_i^2)$$

$$T = k \times r_i (r_o^2 - r_i^2) \qquad where\ k = pmp_{\max}$$

$$T = k r_i r_o^2 - k r_i^3$$

$$\frac{d}{dr_i} T = k r_o^2 - 3k r_i^2 = 0\ at\ \max\ or\ \min$$

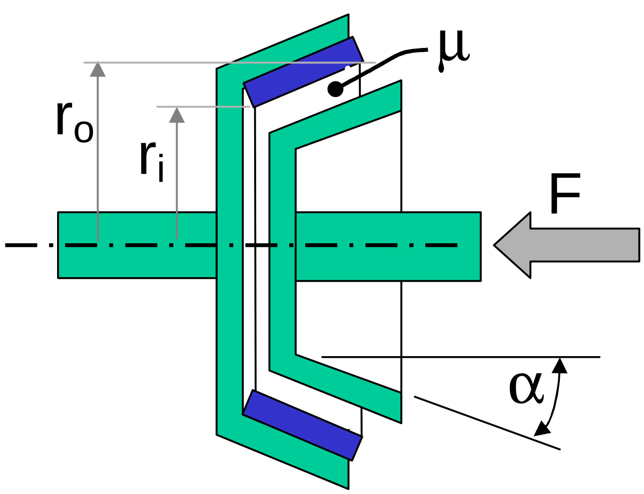
$$k r_o^2 = 3k r_i^2$$

$$r_o^2 = 3 r_i^2$$

$$r_o = \sqrt{3} r_i \quad or\ r_i = 0.577 r_o$$

Generally r_I is between .45 and .80 of r_o.

CONE CLUTCHES **Hamrock §18.3**

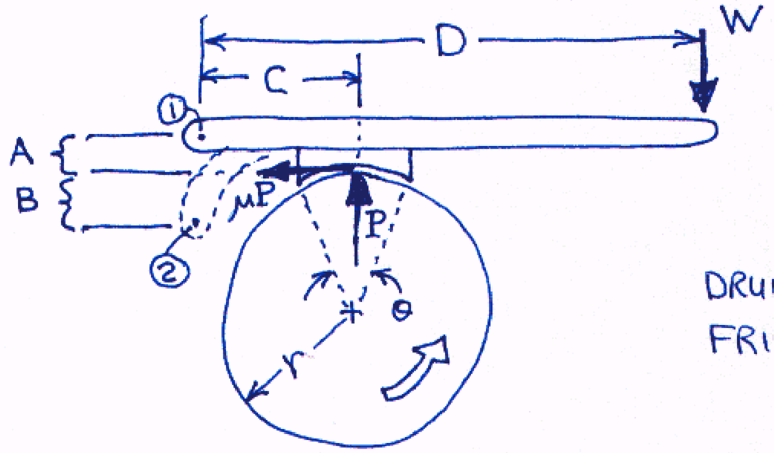


$$T_{Unif\ Pres} = \frac{1}{\sin(a)} \frac{2}{3} mF \frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

$$T_{UnifWear} = \frac{mF}{\sin(a)} \frac{(r_o + r_i)}{2}$$

- Simply take the torque equations for Axial clutches, and multiply them by 1/(sinα)
- Since 8° < α < 15° usually, the multiplier ~ 7.2 to 3.9 .
- N usually is 1 -- cone clutches are not typically ganged.

DRUM BRAKE – SHORT SHOE



DRUM WIDTH = b
FRICTION COEF. = μ

FOR A LINING PRESSURE OF p_a,

THE NORMAL FORCE IS $P = p_a r \theta b$ (θ in RADIANS)

THE TORQUE IS $T = \mu P r$

CASE 1: PIVOT AT ①

ΣM ABOUT PIVOT: (CW = CCW)

$$WD + \mu PA = PC$$

$$WD = PC - \mu PA = P(C - \mu A)$$

$$W = P\frac{(C-\mu A)}{D}$$

THE FRICTION TORQUE μPA ADDS TO THE ACTUATION TORQUE.

IF $\mu A > C$ THEN IS SELF-LOCKING

$$T = \frac{\mu r W D}{(C - \mu A)} \qquad \text{[EQ. 18.28]}$$

CASE 2: PIVOT AT ②

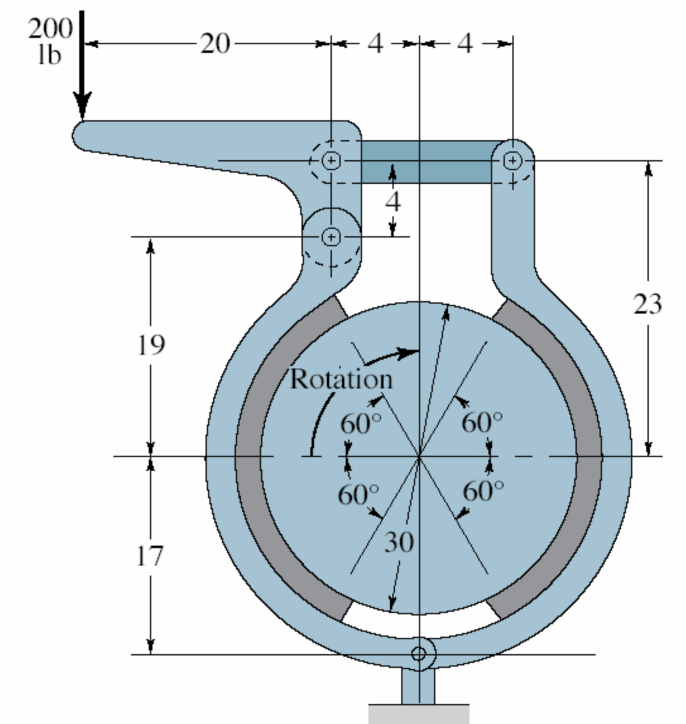
ΣM:

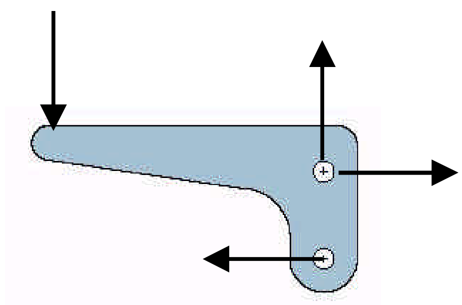
$$WD = PC + \mu PB = P(C + \mu B)$$

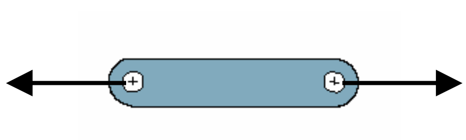
$$W = P\frac{(C+\mu B)}{D}$$

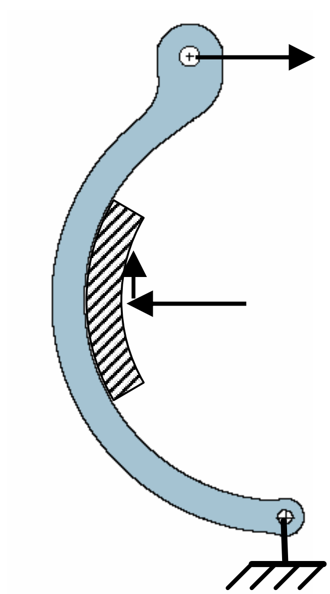
$$T = \frac{\mu r W D}{(C + \mu B)} \qquad \text{[EQ. 18.30]}$$

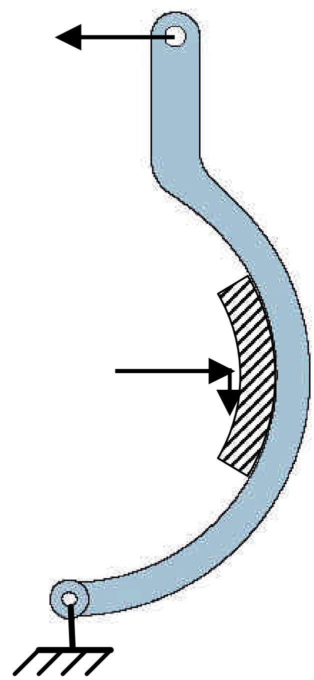
Problem 18.27 Done As SHORT Shoe





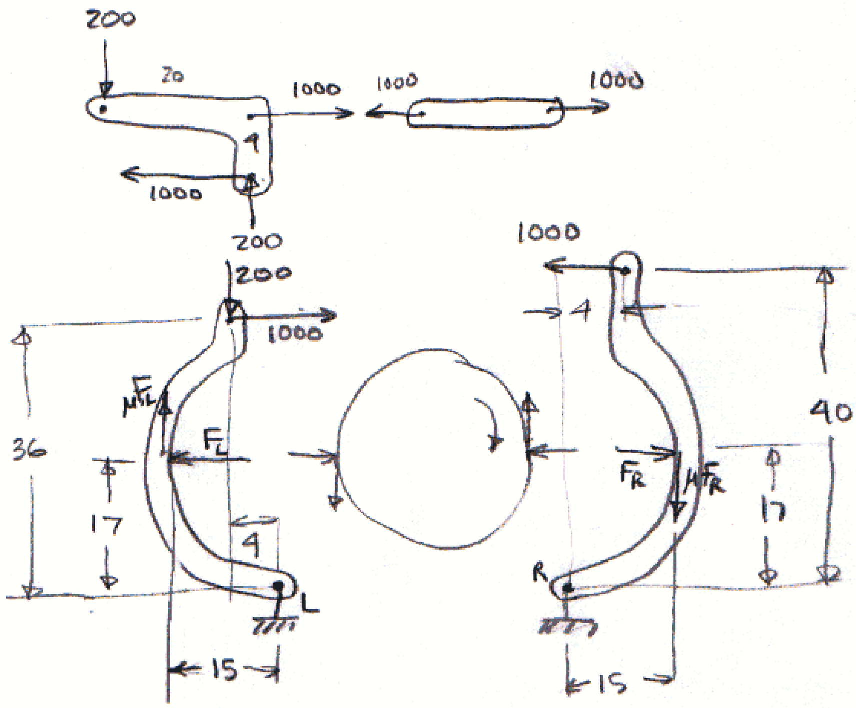






PROB. 18.27

AS SHORT SHOE



ΣM_L

cw ccw

$$(1000)(36) + \mu F_L(15) = (200)(4) + F_L(17)$$

$$F_L(15\mu - 17) = 800 - 36000$$

0.25

$$F_L(-13.25) = -35{,}200$$

$$F_L = 2656.6 \text{ LB.}$$

ΣM_R

cw ccw

$$F_R(17) + \mu F_R(15) = (1000)(40)$$

$$F_R(17 + 3.75) = 40{,}000$$

$$F_R = 1927.7 \text{ LB.}$$

PAD: $L = \frac{1}{3}(\pi)(30) = 31.42''$ CONTACT

$$P_L = \frac{F_L}{L \cdot b} = \frac{2656.5}{(31.42)\,b} = 100 \text{ PSI}$$

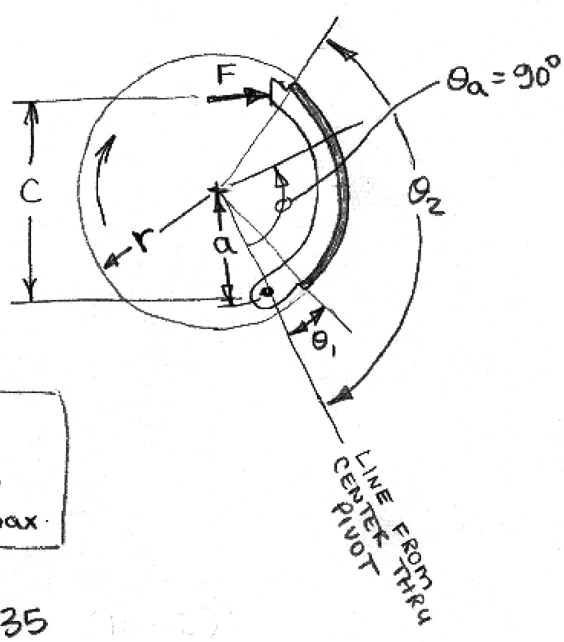
$$b = \frac{2656.5}{3142} = 0.845 \text{ IN.}$$

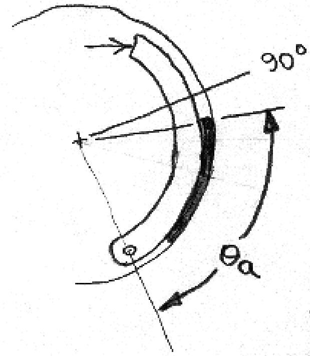
$$P_R = \frac{F_R}{L \cdot b} = \frac{1927.7}{(31.42)(0.845)} = 72.6 \text{ PSI}$$

$$\text{TORQUE} = \mu r(F_L + F_R) = (0.25)(15)(2656.6 + 1927.7)$$

$$T = 17{,}191 \text{ IN. LB.}$$

DRUM BRAKE - LONG SHOE





EXAMPLE OF ANGLE OF MAX PRESSURE NOT = 90°

FIG. 18.8

$a = d_7$
$C = d_6$
$p_a = p_{max}$

EQ. 18.35

$$M_f = \frac{\mu p_a b r}{\sin\theta_a}\left[-r(\cos\theta_2 - \cos\theta_1) - \frac{a}{2}(\sin^2\theta_2 - \sin^2\theta_1)\right]$$

MOMENT ON SHOE FROM CIRCUMFERENTIAL FRICTION

EQ. 18.34

$$M_p = \frac{p_a b r a}{\sin\theta_a}\left[\frac{1}{2}(\theta_2 - \theta_1)\frac{\pi}{180} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right]$$

MOMENT FROM PERPENDICULAR FORCES

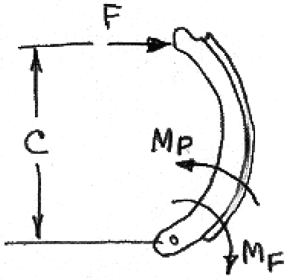
ΣM @ PIVOT

$$FC = M_p \mp M_F$$

SIGN DEPENDENT ON DIRECTION OF ROTATION

TORQUE:

$$T = \frac{\mu p_a b r^2}{\sin\theta_a}(\cos\theta_1 - \cos\theta_2) \qquad [18.38]$$



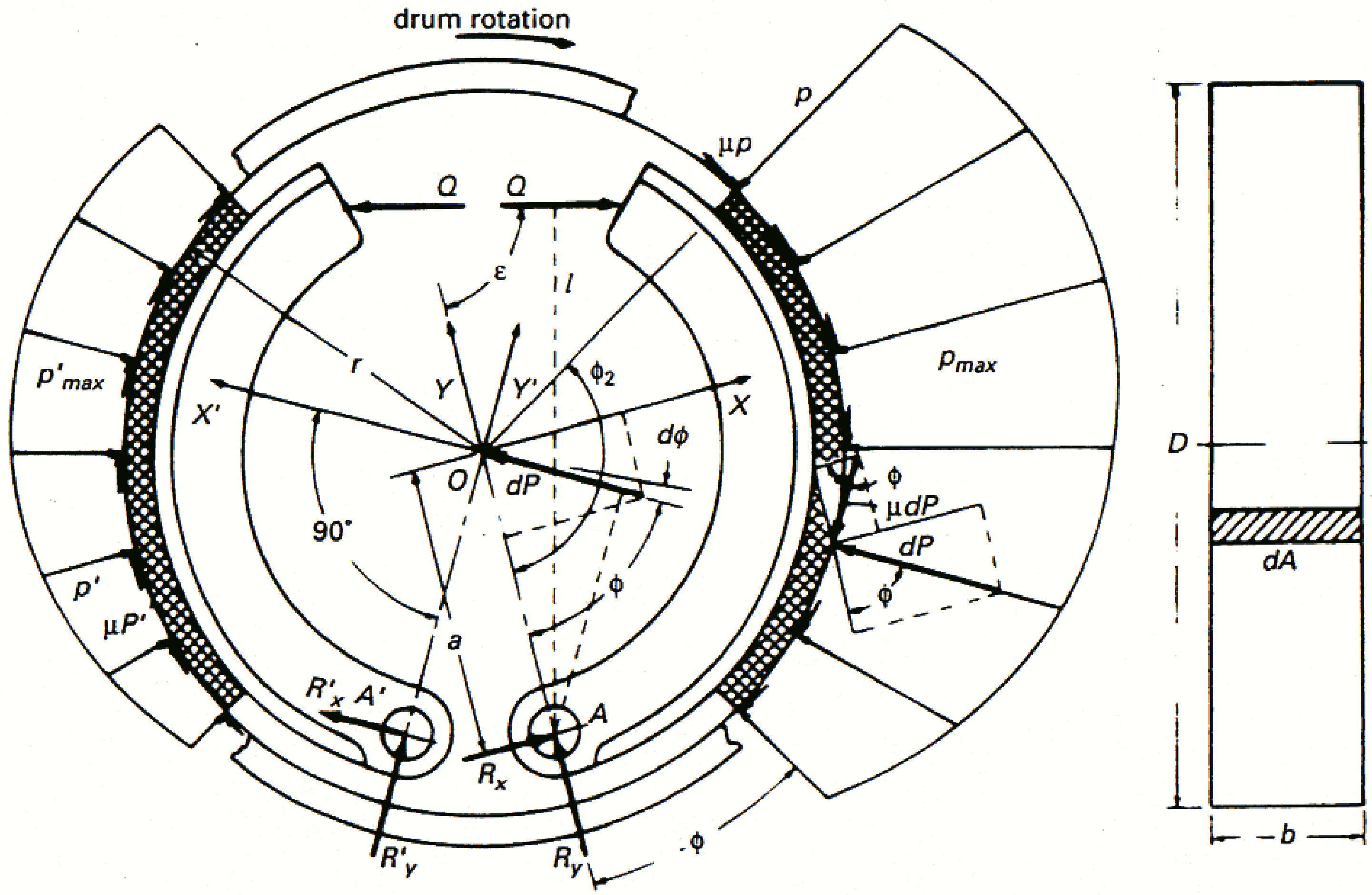
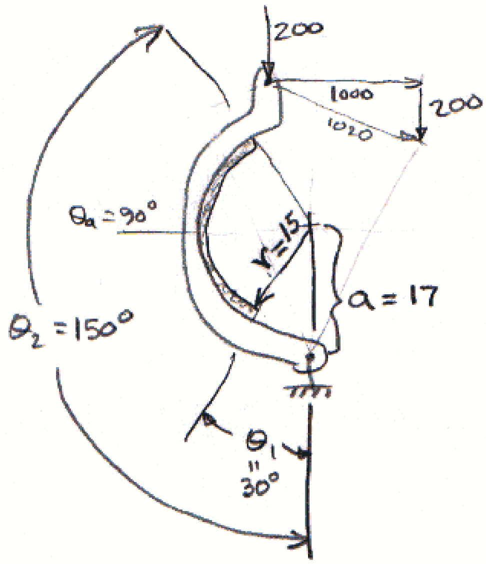


Figure 3.7 Free-body force diagrams for two shoes of an internal brake. Actuating force Q, anchor-pin reactions R, and distribution and relative magnitude of pressure and friction forces are shown.

From "Mechanical Analysis and Design" 2nd Edition, Arthur H. Burr and John B. Cheatham, Prentice-Hall, Upper Saddle River, NJ, 1995, ISBN 0-02-317265-7

PROB. 18.27

AS LONG SHOE



MOMENT ABOUT PIVOT

$\Sigma FC = (1000)(36) - (200)(4)$

$FC = 36{,}000 - 800 = 35{,}200$ LB. IN.

SET MAX SHOE PRESSURE ON SELF-ACTUATING SHOE AND SOLVE FOR SHOE WIDTH, b.

$$M_f = \frac{\mu P_a b r}{\sin\theta_a}\left[-r(\cos\theta_2 - \cos\theta_1) - \frac{a}{2}(\sin^2\theta_2 - \sin^2\theta_1)\right]$$

(.5², .5²)

$$= (0.25)(100)b(15)\left[-15(-0.866 - 0.866) - 8.5(0.25 - 0.25)\right]$$

(0)

$$= 375\, b\,[25.98]$$

$$M_f = 9742.5\, b$$

$$M_p = \frac{P_a b r a}{\sin\theta_a}\left[(\theta_2 - \theta_1)\frac{\pi}{360} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right]$$

(120°, −.866)

$$= (100)b(15)(17)\left[1.0472 - (0.25)(-0.866 - 0.866)\right]$$

$$= 25{,}500\, b\,[1.0472 + 0.433]$$

$$M_p = 37745.1\, b$$

$$FC = M_p - M_F$$

$$35{,}200 = 37745.1\, b - 9742.5\, b$$

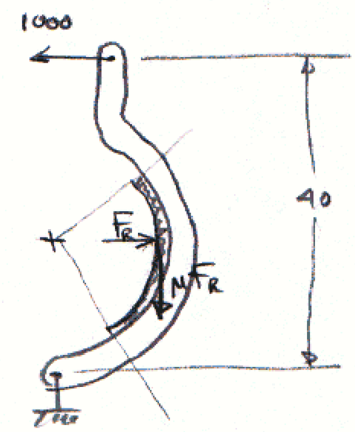
$$b = \frac{35{,}200}{28{,}003} = 1.26 \text{ IN.}$$

$$T_{LEFT} = \mu P_a b r^2(\cos\theta_1 - \cos\theta_2)$$

$$= (0.25)(100)(1.26)(15)^2(0.866 - (-0.866))$$

$$T_{LEFT} = 12{,}276 \text{ LB. IN.}$$

SOLVE FOR PRESSURE ON NON-SELF ACTUATING SHOE.



RATIO FROM LEFT SHOE, WHERE P = 100 PSI

$$M_f = \frac{(9742.5)(1.26)}{100} P_{RIGHT}$$

$$M_F = 122.76\, P_{RIGHT}$$

$$M_P = \frac{(37745.1)(1.26)}{100} P_{RIGHT}$$

$$M_P = 475.59\, P_{RIGHT}$$

$$FC = M_P + M_F$$

$$(40)(1000) = 475.59\, P_{RIGHT} + 122.76\, P_{RIGHT}$$

$$P_{RIGHT} = \frac{40{,}000}{(475.59 + 122.76)} = \frac{40{,}000}{598.35}$$

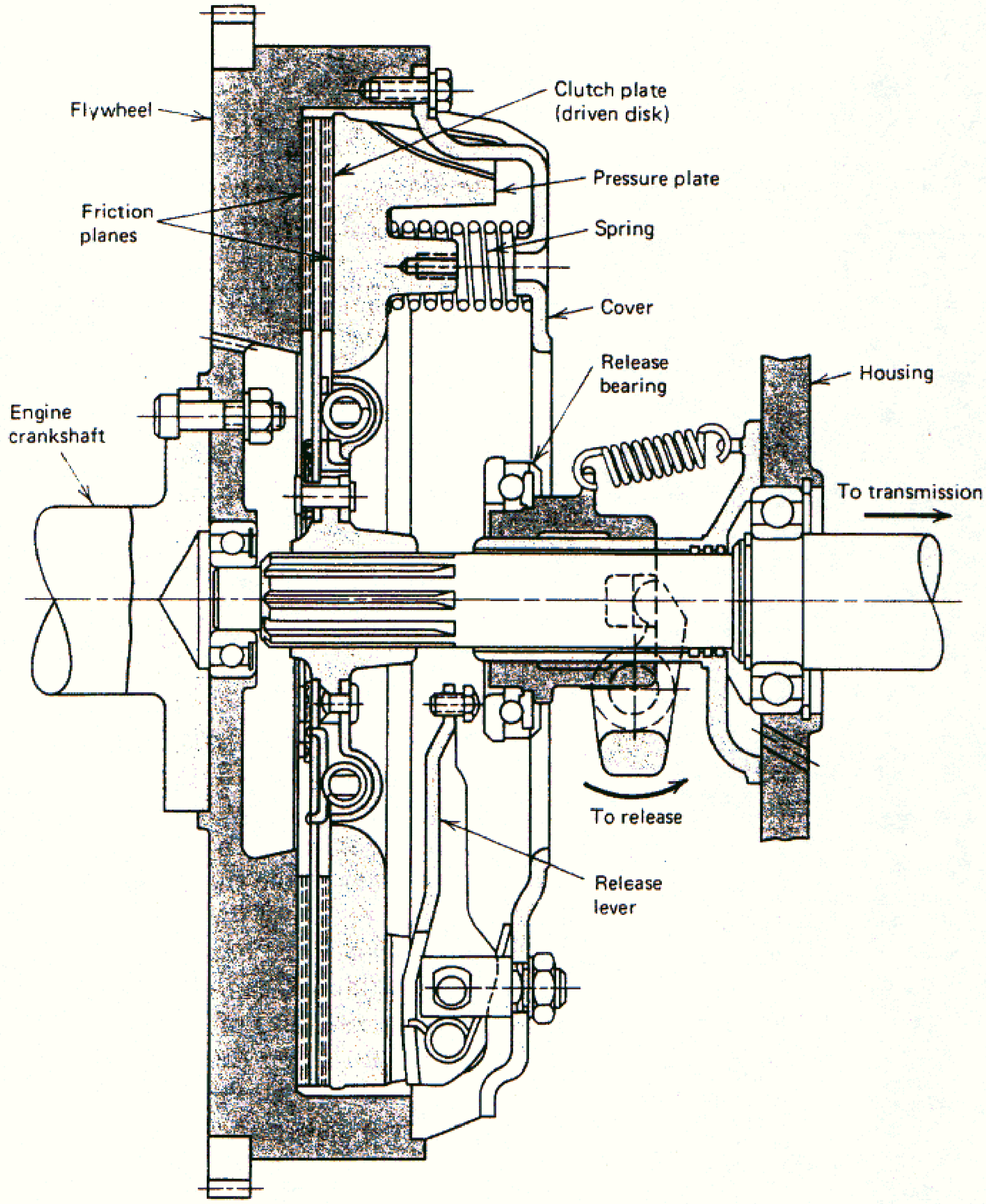
$$P_{RIGHT} = 66.85 \text{ PSI}$$

$$T_{RIGHT} = \mu P_{RIGHT}\, b r^2 (\cos\theta_1 - \cos\theta_2)$$

$$= (0.25)(66.85)(1.26)(15)^2(2)(0.866)$$

$$T_{RIGHT} = 8206. \text{ LB. IN.}$$

$$T_{TOT} = T_{LEFT} + T_{RIGHT} = 20{,}482 \text{ LB. IN.}$$



Automotive-type disk clutch. (Courtesy Borg-Warner Corporation.)