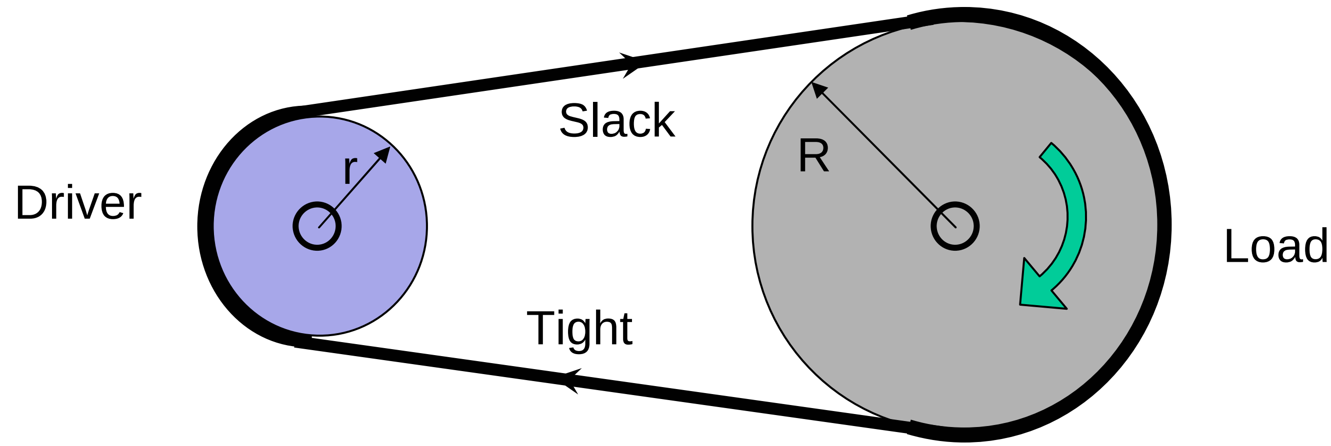
# Machine Design

## Flexible Power Transmission: Belts

# Belt Basics

To transmit power: need Torque.

*Recall: Power = Torque x Rot'l Speed*



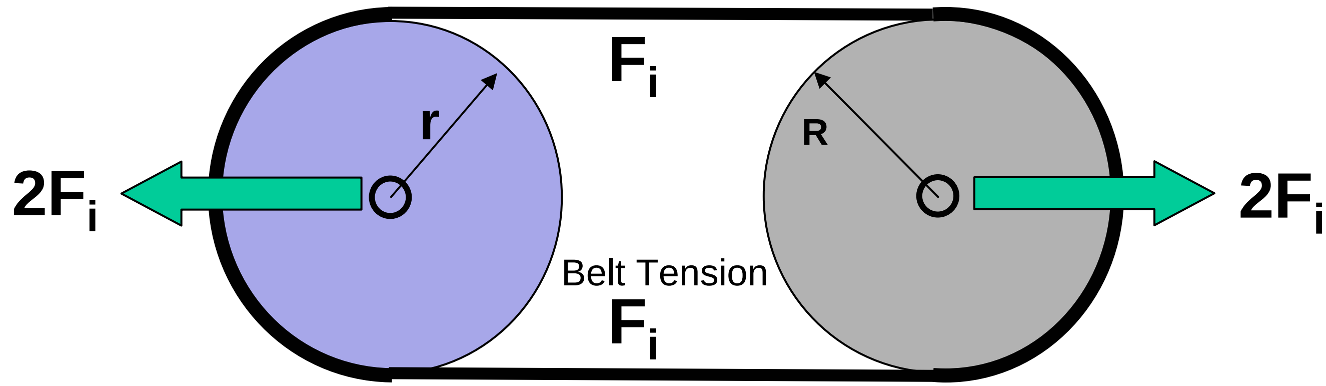

And since Torque = $(F_{tight} - F_{slack}) \times R$ [ ΣMoment ]

need $F_{tight} > F_{slack}$ ,

where $F_{tight}$ and $F_{slack}$ are the tensions in the two legs of the belt.

# Belt Tension

Flat and V-Belts depend on friction, so they need a normal force due to an initial preload.




Before moving, $F_{\mathbf{tight}} = F_{\mathbf{slack}} = F_{\mathbf{i}}$ .

As rotation begins and torque builds,

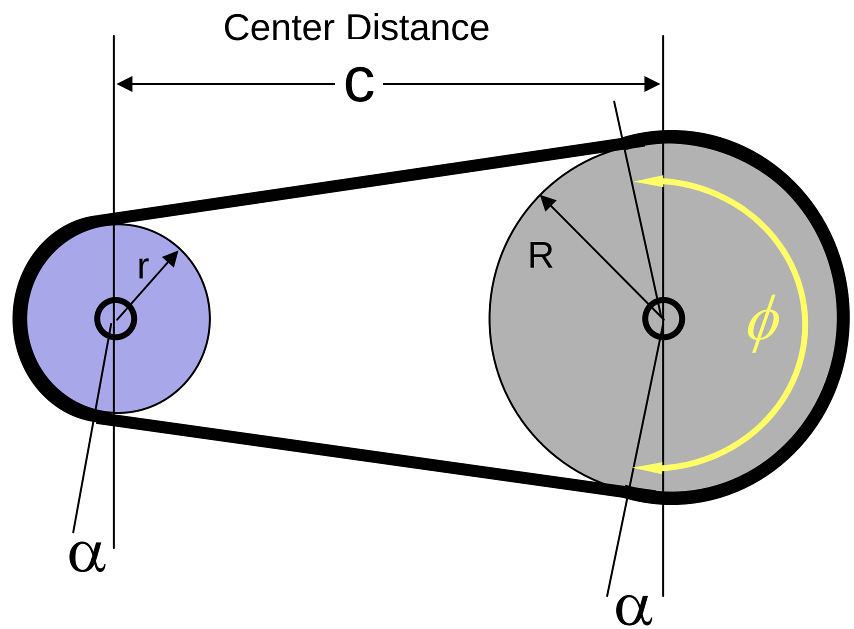
$$F_{\mathbf{tight}} = F_{\mathbf{tight}} + \Delta F = F_i + \Delta F$$ and

$$F_{\mathbf{slack}} = F_{\mathbf{slack}} - \Delta F = F_i - \Delta F$$ , from which

$$F_{\mathbf{tight}} + F_{\mathbf{slack}} = 2\, F_{\mathbf{i}}$$

# Belt Wrap Angles

If both pulleys are the same size, the "wrap" angles, $\phi$ , are both 180°.




If not, calculate

$$\alpha = \sin^{-1}\left(\frac{R-r}{C}\right)$$

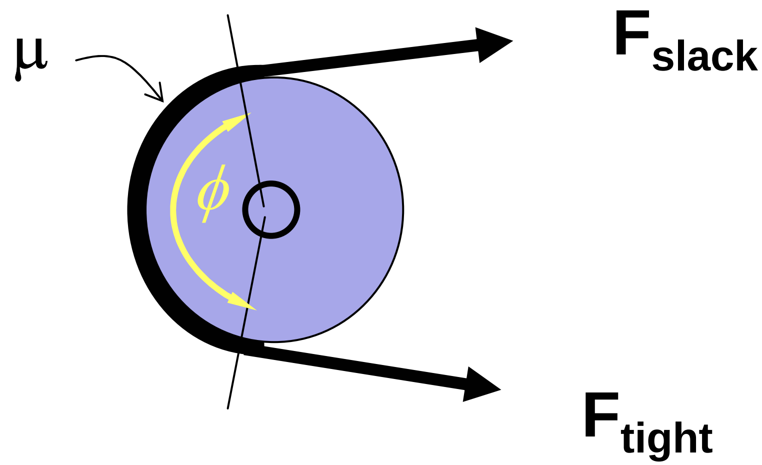
and the wrap angles are:

$\phi = 180° - 2\alpha$ for the small pulley, and

$\phi = 180° + 2\alpha$ for the big pulley.

# Weightless Belts

The operating belt tension ratio is constrained by the friction, μ, and the wrap angle, $\phi$ :

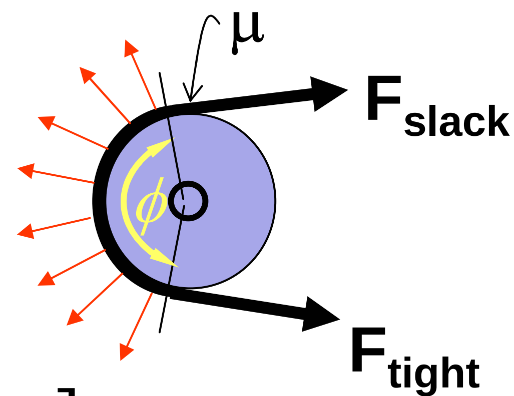



$$\frac{F_{tight}}{F_{slack}} = e^{\mu\phi}$$

$\phi$ is in Radians

Note: Since the wrap angle is <u>less</u> on the smaller pulley, it is the limiting component.

# Real Belts




For a real belt that weighs W lb/in (or has a mass of m kg/m), there is a centrifugal force ($mv^2$) of

$$F_c = \frac{Wv^2}{g} \left[ lb \right] \quad or \quad F_c = mv^2 \left[ N \right]$$

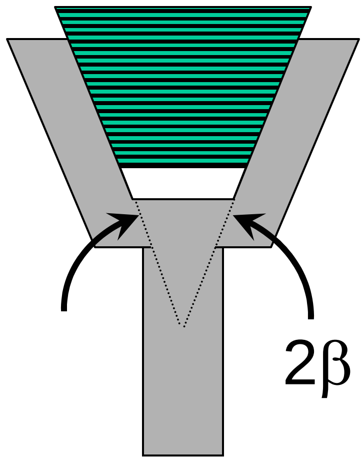
**( g = 386 in/s$^2$ )**

This force reduces the belt tension that the pulley feels, and the operating tension ratio becomes:

$$\frac{F_{tight} - F_c}{F_{slack} - F_c} = e^{\mu\phi}$$

# V-Belts

The wedging action of a V-Belt with Vee angle = 2β will increase the belt-to-pulley friction to be effectively




$$\mu_{eff} = \frac{\mu}{\sin(\beta)}$$

This lets the operating tension ratio, and therefore the torque, be much greater.

For a typical Vee angle of 2β = 36°, 1/sin(18°) = 3.2 , more than tripling friction.

# Belt Calculations

Frequently, you will know the initial belt tension, $F_i$ , and you will use these two equations:

1) $F_{tight} + F_{slack} = 2\,F_i$

2) $$\frac{F_{tight} - F_c}{F_{slack} - F_c} = e^{\mu\phi}$$

to solve for whichever quantity is unknown.

Once you know $F_{tight}$ and $F_{slack}$ , you can compute the operating Torque T = ( $F_{tight}$ - $F_{slack}$ ) x R , and also the

operating Power $P_{max}$ = T x Rot'l Speed [Rad/sec] or
$P_{max}$ = ( $F_{tight}$ - $F_{slack}$ ) x Linear Belt Speed

# Belt Power Limit

As a sanity check, recall $F_{belt} = F_i \pm \Delta F$ .
When torqued so that $\Delta F$ increases to $F_i$ , then

$$F_{tight} = 2\,F_i \;\; and \;\; F_{slack} = 0$$

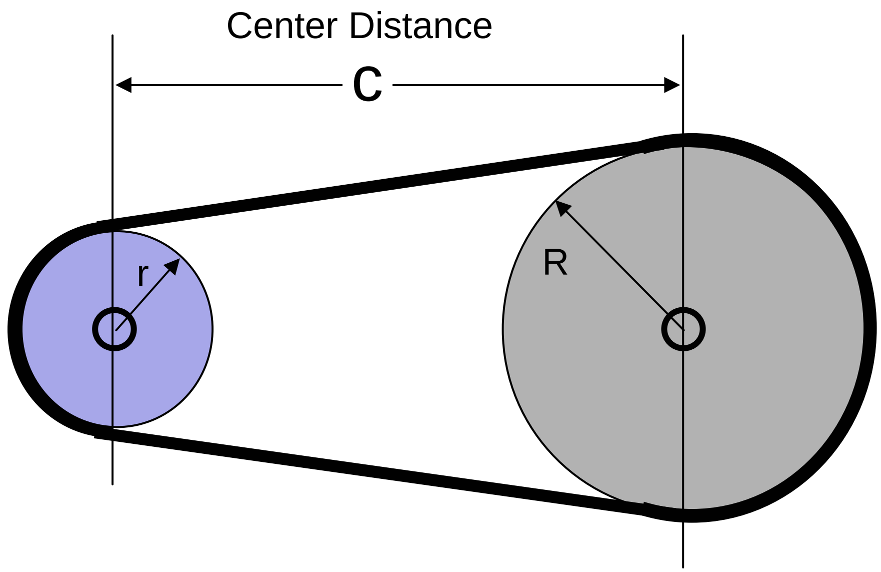
Then the limit of power transmitted is

$$P_{HP} = \frac{2F_i\left[lb\right]\times V\left[ft/s\right]}{550}\ HorsePower$$

*(This tension ratio = 2 $F_i$ / 0 will be greater than the friction and wrap angle allow, so actual power must be something less than this result.)*

# Belt Length

Knowing the pulley radii and their center spacing, the belt length can be calculated:




$$L \approx 2C + \pi(R + r) + \frac{(R - r)^2}{C}$$

# Belts

- Belts may be "rated" by:

    1) A max. tension, $F_{tight}$

    2) A max. stress (which, times cross section area, gives max tension)

    3) A max power (esp. V-Belts; is function of speed and pulley diameter)

- Timing belts and roller-chain carry all the torque in the tight side of the loop.