For Homework 6 Handout

Problem 7.40 – Now Including Steady Stress

A 20-mm-diameter shaft transmits a variable torque of 500 ± 400 N-m. The frequency of the torque variation is 0.1 s^{-1}. The shaft is made of high-carbon steel (AISI 1080). Find the endurance life of the shaft.

Notes: Now we include the steady component, but want to find the mean and alternating stresses to proportionately reduce the actual stresses to for a Factor of Safety of 1.5.

Solution: For AISI 1080, the inside front cover gives S_y = 380 MPa and S_{ut} = 615 MPa. Then, from Eq. (7.6), one obtains:

$$S_e^{'} = 0.29 S_{ut} = 0.29(615MPa) = 178\,MPa$$

The alternating shear stress amplitude is

$$\tau_{alt} = \frac{\tau_{\max} - \tau_{\min}}{2} = \frac{(T_{\max} - T_{\min})r}{2J} = \frac{(900Nm - 100Nm)(0.01m)}{2\frac{\pi}{32}(0.02m)^4} = 254.65MPa$$

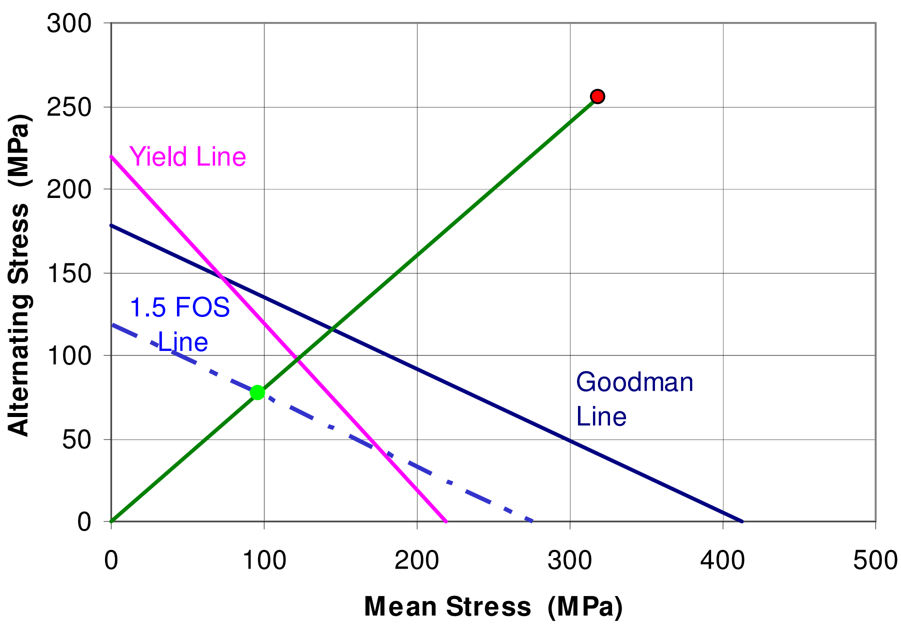
The mean stress amplitude is

$$\tau_{mean} = \frac{\tau_{\max} + \tau_{\min}}{2} = \frac{(T_{\max} + T_{\min})r}{2J} = \frac{(900Nm + 100Nm)(0.01m)}{2\frac{\pi}{32}(0.02m)^4} = 318.3MPa$$

Note that this is also simply 500/400 x 254.65MPa.

These are shear stresses, so to make the Modified Goodman diagram, we need to use the Shear Ultimate Strength for the Goodman line, and the Shear Yield Strengths for the Yield line. From Slide 6 in Lecture 6: "For shear loading, use Ssy = 0.577 Sy and Sus = 0.67 Sut."
So Ssy = (0.577)(380) = 219.3MPa, and Sus = (0.67)(615) = 412.1 MPa.

Here's the Goodman diagram with our point way outside the safe zone:



Now we want to reduce the stresses (mean and alternating) proportionately to get a Factor of Safety of 1.5 – where the green load line crosses the dashed 1.5 FOS line.

To do this, we use two equations, Eqn. 7.28 (ignoring the Kf) which is the equation of the dashed blue line when n is set to 1.5:

$$\frac{1}{n} = \frac{\tau_A}{S_e} + \frac{\tau_M}{S_{us}}$$

and the equation of the Load Line

$$\tau_A = \frac{\tau_{alt}}{\tau_{mean}}\tau_M = \frac{254.65}{318.3}\tau_M = 0.80\tau_M$$

We want to be at the intersection of these lines, so can substitute the second equation in for σ_A in the first equation and solve for σ_M.

$$\frac{1}{1.5} = \frac{0.8\tau_M}{178} + \frac{\tau_M}{412} = 0.00692\tau_M$$

$$\tau_M = 96.44\,MPa$$

Then

$$\tau_A = 0.80\tau_M = 77.16\,MPa$$

These values agree with the intersection on the Goodman diagram.

Problem 7.41

For the shaft in Problem 7.40 determine how large the shaft diameter has to be for infinite life. Ans. D = 25.0 mm.

Notes: The stress amplitude must make the Factor of Safety be just 1.0 for this case. This solution uses the results of Problem 7.40.

Solution: From Problem 7.40, we saw that the alternating stress would need to be reduced to 77.16 MPa for a FOS of 1.5. For a FOS of 1.0, the alternating stress would be 1.5 x 77.16 = 115.73 MPa. Then we can write the torsional shear stress equation

$$\tau_{alt} = \frac{T_{alt}r}{J} = \frac{400Nm\left(d/2\right)}{\frac{\pi}{32}(d)^4} = \frac{2037.2\ Pa}{d^3} = 115.73MPa$$

$$d^3 = \frac{2037.2}{115.73\times10^6} = 17.60\times10^{-6}m^3,\ \ and\ d = 0.0260m,\ or\ 26.0mm$$

Problem 7.63

A toy with a bouncing 50-mm-diameter steel ball has a compression spring with a spring constant of 100,000 N/m. The ball falls from a 3-m height down onto the spring (which can be assumed to be weightless) and bounces away and lands in a hole. Calculate the maximum force on the spring and the maximum deflection during the impact. The steel ball density is 7840 kg/m^3.
Ans. 1743 N.

Notes: The weight of the ball is calculated from its volume and density, and then Eq. (7.54) solves the problem.

Solution: The weight of the ball is the product of its volume and density, or

$$W = \left(\frac{4}{3}\pi r^3\right)(\rho g) = \left(\frac{4}{3}\pi (0.025\ \mathrm{m})^3\right)(7840\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2}) = 5.03\ \mathrm{N}$$

We compute the static spring deflection

$$\delta_{st} = \frac{W}{k} = \frac{5.03}{100{,}000} = 5.032\times 10^{-5}\, m$$

and the Impact factor

$$I_m = \frac{\delta_{\max}}{\delta_{static}} = \frac{P_{\max}}{W} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{(2)(3)}{5.023\times 10^{-5}}}$$
$$= 1 + \sqrt{119450.5} = 1 + 345.6 = 346.6$$

Then

$$\delta_{\max} = I_m \delta_{static} = (346.6)(5.023\times 10^{-5}) = 0.01743 m = 17.43 mm$$

And

$$P_{\max} = I_m W_{static} = (346.6)(5.03) = 1743 N$$

Wrench Fatigue Problem A

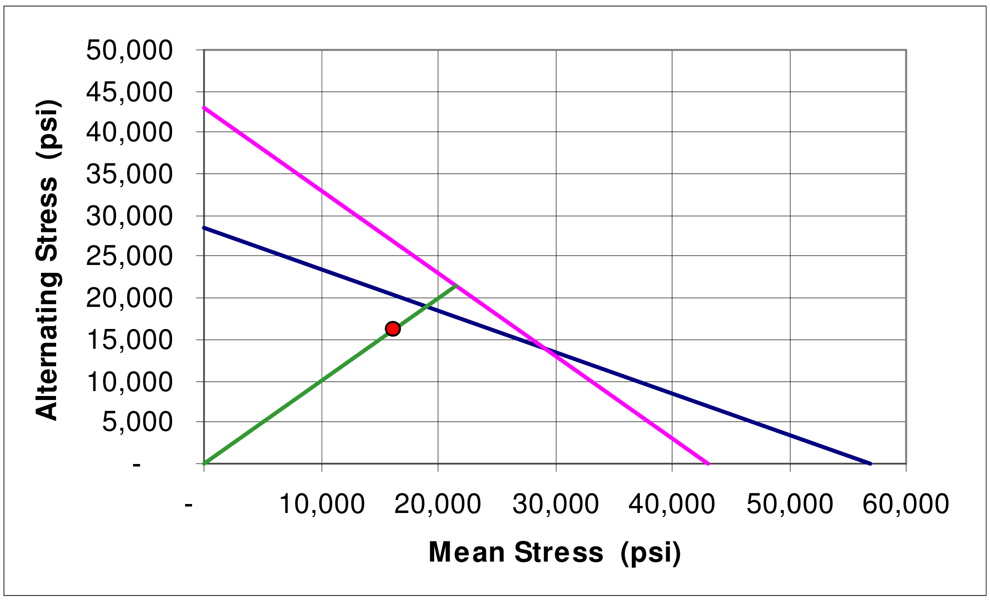
The wrench only sees torque in one direction, so the bending stress in it goes from zero to max and back to zero. This is a fluctuating fatigue case, so we will use a Goodman diagram.

The bending stress is MC/I, and the bending moment is the applied 65 ft.lb. of torque.

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(65\,ft.lb.)(12\,in/ft)\left(\frac{5/8}{2}\right)}{\frac{\pi}{4}\left(\frac{5/8}{2}\right)^4} = 32{,}543\,psi$$

The minimum stress is zero, so both the alternating and mean stresses are 16,271 psi.

For our Goodman diagram, we get from inside the front cover that Sut = 57ksi and Sy = 43ksi. Because this is bending, Se = 0.5Sut = 28.5ksi. Then we can draw the Goodman



For proportional changes in the alternating and mean stress, the Factor of Safety equation is

$$\frac{1}{n} = \frac{\sigma_A}{S_e} + \frac{\sigma_M}{S_{ut}} = \frac{16.27}{28.5} + \frac{16.27}{57} = 0.571 + 0.285 = 0.856$$

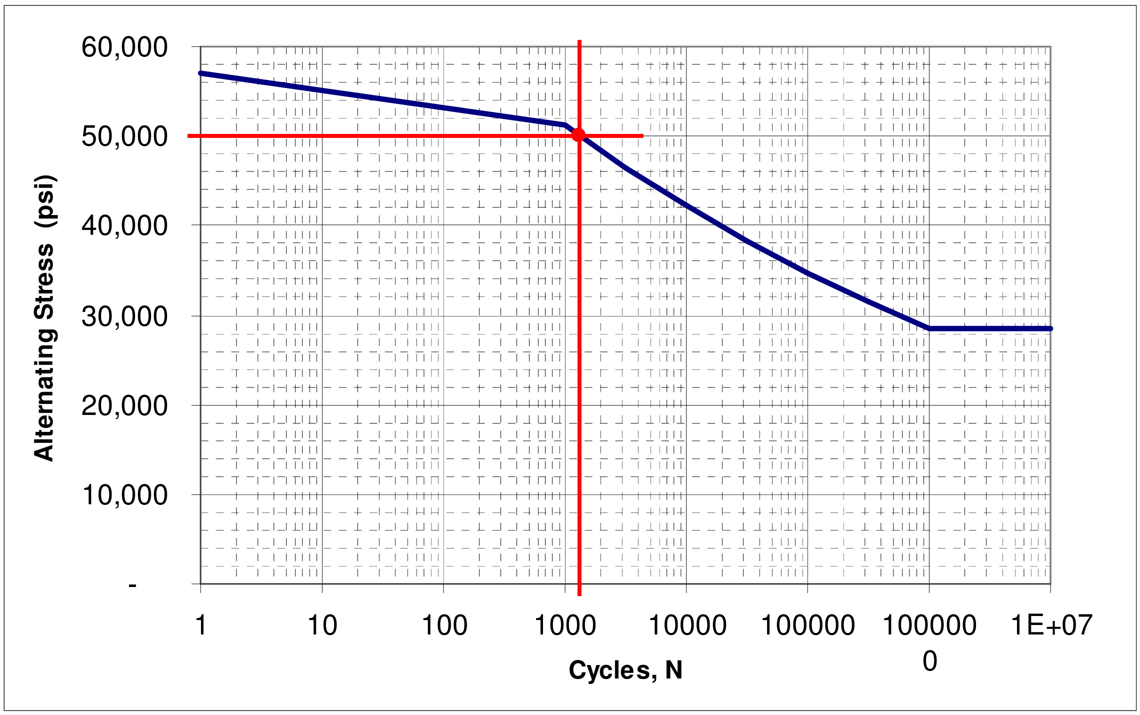
$$n = \frac{1}{0.856} = 1.168$$

Wrench Fatigue Problem B

The wrench now sees a fully reversing torque of 100 ft.lb, which results in a reversing bending stress of

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(100\,ft.lb.)(12\,in/ft)\left(\frac{5/8}{2}\right)}{\frac{\pi}{4}\left(\frac{5/8}{2}\right)^4} = 50{,}066\,psi$$

This is now a S-N problem, so we use Se = 0.5Sut = 28.5ksi from above, but also need S_L = 0.9Sut = 51.3ksi. Then we can draw the S-N diagram:



We calculate the factors for the curve between 1000 and 1 million cycles:

$$a = \frac{(S_L^{'})^2}{S_e{'}} = \frac{(51.3)^2}{28.5} = 92.34\,ksi$$

$$and \quad b = -\tfrac{1}{3}\log_{10}\left(\frac{S_L^{'}}{S_e{'}}\right) = -\tfrac{1}{3}\log_{10}\left(\frac{51.3}{28.5}\right) = -(0.333)\log_{10}(1.8) = -(0.333)(0.255) = -0.0851$$

Then the cycles for an alternating stress of 51.3ksi are

$$N = \left(\frac{\sigma_{alt}}{a}\right)^{1/b} = \left(\frac{50.066}{92.34}\right)^{1/-0.0851} = (0.542)^{-11.752} = 1{,}331\,cycles$$

This is reasonable, because the 50.066ksi stress is only slightly lower than the LCF stress of 51.3ksi, so the cycles are only slightly more than 1000.