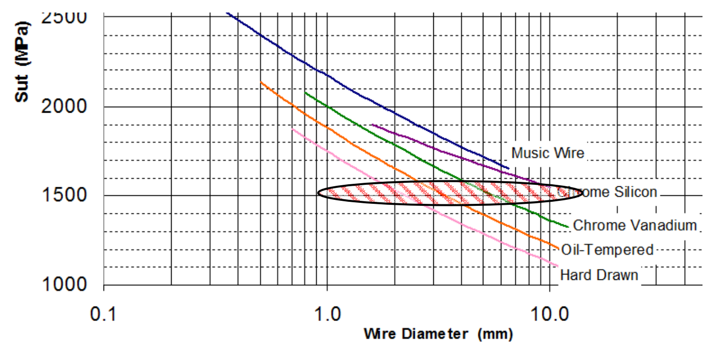
Solving the Cam Compression Spring Problem

1. Investigate the loading and determine a max allowable shear stress
Pmax = 600N, Pmin = 300N

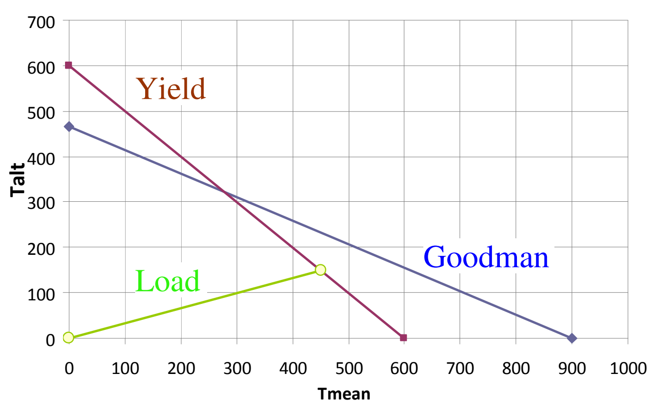
Pmean = 0.5 (Pmax + Pmin) = 0.5(900) = 450N
Palt = 0.5 (Pmax - Pmin) = 0.5(300) = 150N
Ratio of Pmean/Palt = 450/150 = 3

For a shot peened steel spring, fatigue endurance strength, Sse = 465MPa
We don't know the wire diameter to determine Sut, so we will pick a mid-range value to start with and adjust it later. It looks like a value of 1500MPa is midrange.



Then
Shear Ultimate, Ssu = 0.6 Sut = 900MPa
and
Shear Yield, Ssy = 0.4 Sut = 600MPa

Now we can draw a Goodman diagram with a load line where τ mean = 3 τ alt:



We can see that the Yield Line actually limits the max shear stress. The intersection of the Load Line and the Yield Line gives

$$\frac{1}{FOS} = \frac{\tau_{alt}}{Ssy} + \frac{\tau_{mean}}{Ssy}$$ and because

FOS = 1 and τ mean = 3 τ alt

$$1 = \frac{\tau_{alt}}{600} + \frac{3\tau_{alt}}{600} = \frac{4\tau_{alt}}{600} = \frac{\tau_{alt}}{150}$$, or

τ_{alt} = 150MPa

2. Next, we see that the spring rate, k = ΔP/ΔX = (600-300)/25 = 300/25 =12 N/mm.
We pick a midrange value for spring constant C = D/d of 8, and because this is cyclic loading we use the Bergstrasser factor

$$K_b = \frac{4C+2}{4C-3} = \frac{32+2}{32-3} = \frac{34}{29} = 1.172$$

Then we can write the equation for Shear Stress - I'll use alternating force and stress, but I could also have used means, maxes, or mins.

$$\tau_{alt} = \frac{8DK_bP_{alt}}{\pi d^3} = \frac{8CK_bP_{alt}}{\pi d^2} \; or \; d^2 = \frac{8CK_bP_{alt}}{\pi \tau_{alt}} = \frac{(8)(8)(1.172)(150N)}{\pi(150MPa)} = 23.875mm^2$$

so $\boxed{d = 4.886mm}$

Now that we know d, we can go back and calculate Sut, but it turns out it is ~1540MPa.

Now we can calculate the mean diameter D = Cd = (8)(4.886) = **39.1mm**

3. Next, we can solve the spring rate equation to get Na:

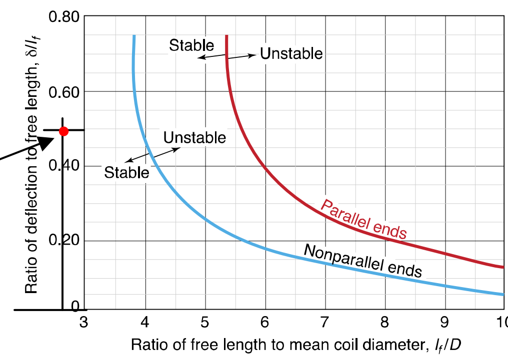
$$k = \frac{Gd}{8C^3 Na(1+0.5/C^2)}$$ and solve that for

$$Na = \frac{Gd}{8kC^3(1+0.5/C^2)} = \frac{(79{,}300MPa)(4.886)}{(8)(12)(8)^3(1+0.5/64)} = \frac{387{,}459.8}{(49{,}152)(1.0078)} = 7.82$$

and being Squared and Ground means that Ntot = Na + 2 = **9.82 turns**,
and that Lsolid = dNtot = (4.886)(9.82) = 48.0mm

4. Add 10% of Lsolid for margin, and add to that the max spring deflection at 600N = 600/12 = 50mm:
Lfree = (1.1)(48) + 50 = 52.8 + 50 = **102.8mm**

5. For buckling, calculate
Lfree/D = 102.8/39.1 = 2.63 and
ΔXmax/Lfree = 50/102.8 = 0.49
so we are definitely stable, per Fig. 17.8



6. Finally, for steel springs, the natural frequency is

$$f = \frac{353{,}000d}{N_{tot}D^2} = \frac{(353{,}000)(4.886)}{(9.82)(39.09)^2} = 114.9\text{ Hz}$$

The cam is running at 650 RPM = 650/60 = 10.8Hz, so that should not be problem.

ASSIGNMENT

1. Design your own spring for this application. You can use the Spring Design spreadsheet calculator on the ME311 web page.

2. Find a spring from an online catalog (any manufacturer) that you could buy for this application. You may need to tweak some dimensions from those calculated.