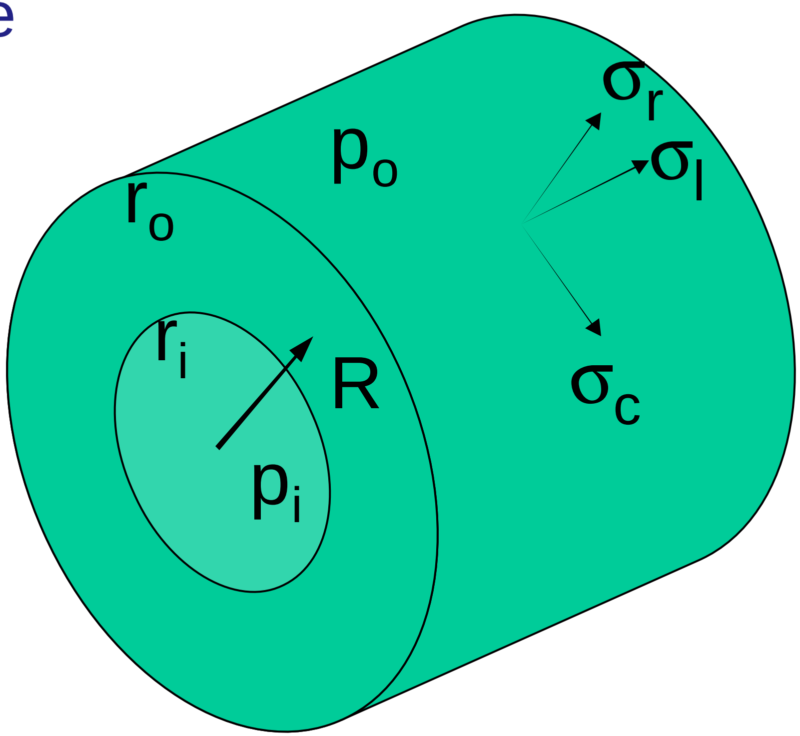
# Thick-Walled Cylinders and Press Fits

# Stresses in Thick-Walled Cylinders

- Thick-Walled cylinders have an average radius less than 20 times the wall thickness.

- They are pressurized internally and/or externally.

- The principal stresses are circumferential (hoop) $\sigma_c$, radial $\sigma_r$, and longitudinal (axial) $\sigma_l$.



# Circumferential & Radial Stresses

For the general case of <u>both</u> internal and external pressure, the circumferential and radial stresses at radius R in the wall are:

$$s = \frac{r_i^2 p_i - r_o^2 p_o \pm (p_i - p_o) r_i^2 r_o^2 / R^2}{r_o^2 - r_i^2}$$

Eqns 10.20/10.22

**Where the ± is: + for circumferential, and**
**- for radial stress.**

For the special case of only internal pressure, $p_o = 0$, and the stresses at radius R are:

$$s = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

Eqns 10.23/10.24

**The sign convention is the same.**

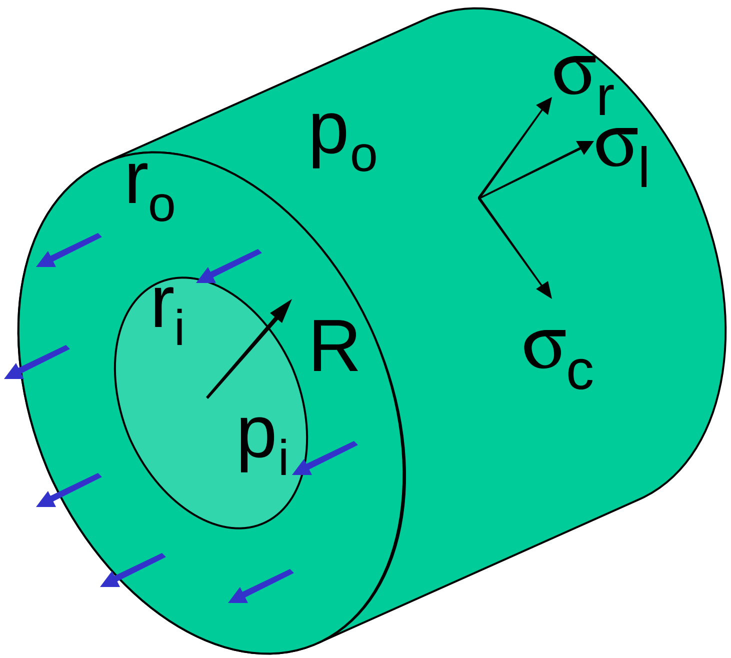
# Longitudinal Stresses

The longitudinal stress is simply given by a Force/Area, where the Force is $p_i$ times the circular inside area $\pi r_i^2$ , and the Area is the annular area of the cylinder cross section, $\pi( r_o^2 - r_i^2)$ , or:

$$s_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

Un-numbered Equation
just below Eqn. 10.8

This is generally only considered for the case of internal pressurization ( $p_o = 0$).



# Stresses vs. Radius

First, the easy observation: Radial stresses at the inner and outer surfaces are equal to minus the pressurization.

- If a surface is unpressurized, the radial stress there is zero.
- If a surface is pressurized, the radial stress there = - p, because it is in compression.

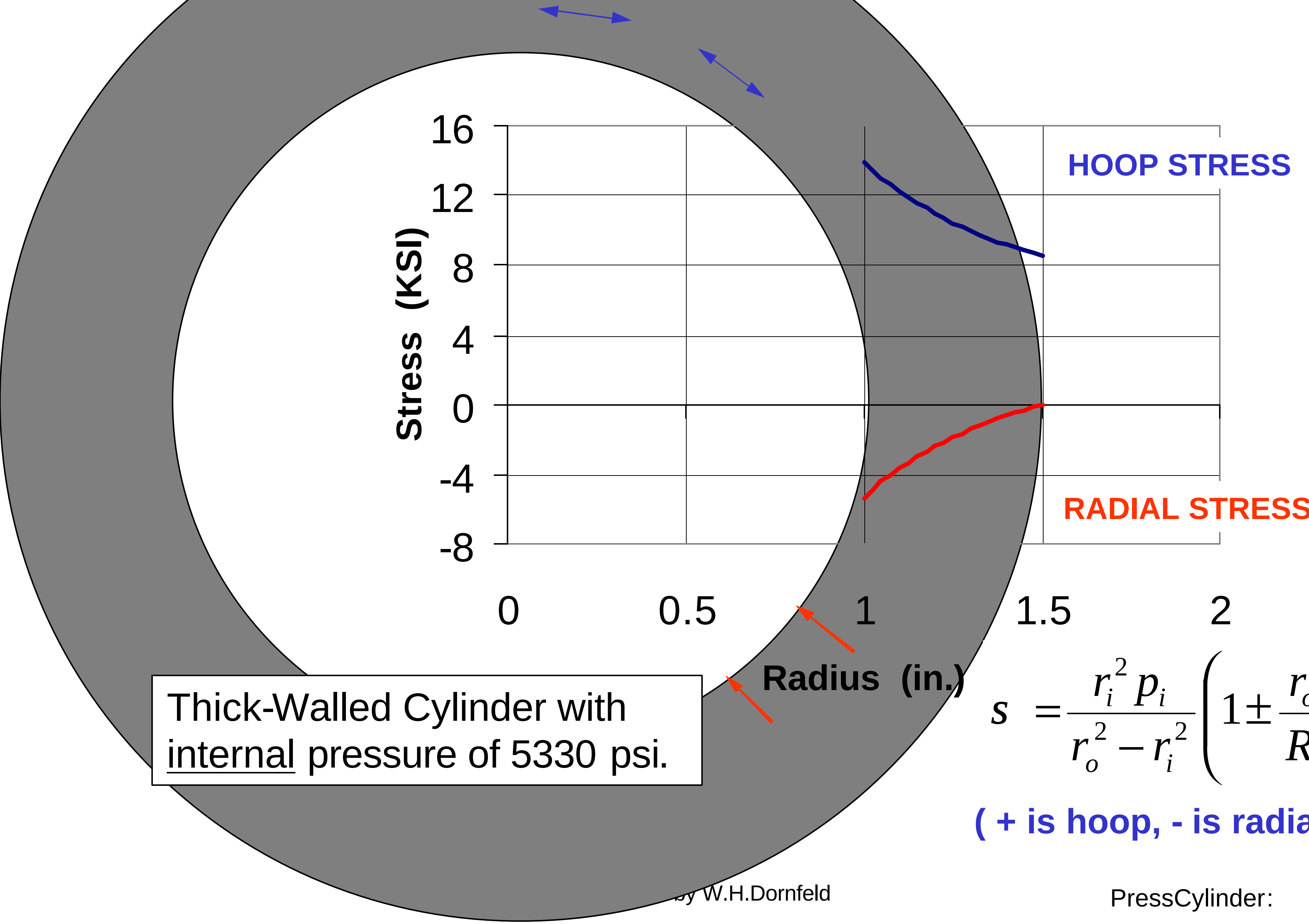
Now let's look at an internally pressurized cylinder, and how the radial and circumferential stresses vary across the wall thickness at radius R.

$$s = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

Eqns 10.23/10.24

( + is circumferential, - is radial )

# Stresses for Internal Pressurization




Thick-Walled Cylinder with
internal pressure of 5330 psi.

$$s = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

**( + is hoop, - is radial )**

# Stresses vs. Radius - Internal Pressure

**Radial** stress is as predicted:

- -5330 psi at the inner, pressurized surface.
- 0 at the unpressurized outer surface.

**Hoop** stress is:

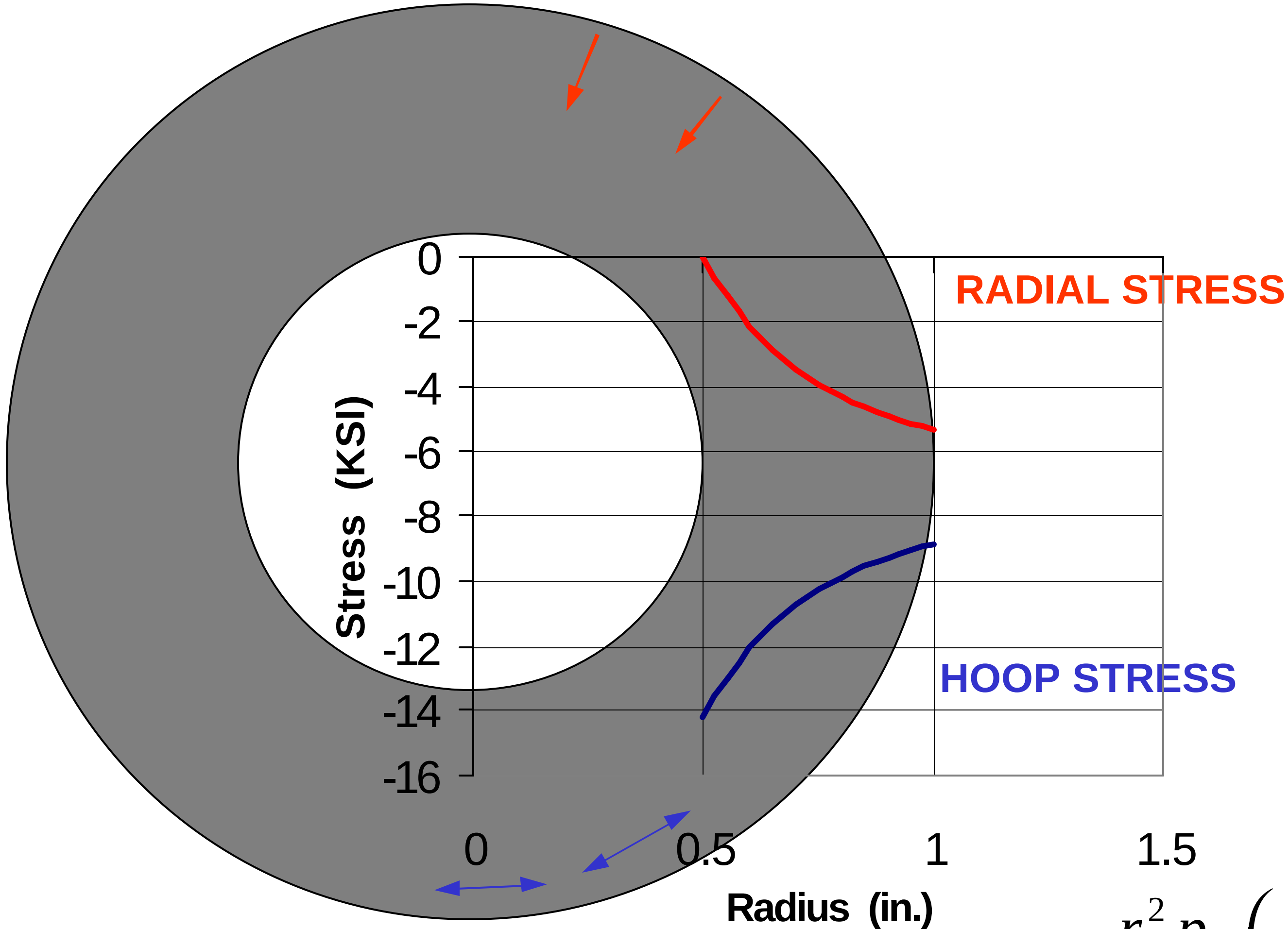
- Maximum at the inner surface, 13.9 ksi.
- Lower, but not zero, at the unpressurized outer surface, 8.5 ksi.
- Larger in magnitude than the radial stress

**Longitudinal** stress is (trust me):

- 4.3 ksi, considered as a uniform, average stress across the thickness of the wall.

| |
|---|
| Now let's look at an externally pressurized cylinder. |

# Stresses for External Pressurization




Thick-Walled Cylinder with
<u>external</u> pressure of 5330 psi.

$$s = \frac{-r_o^2 p_o}{r_o^2 - r_i^2}\left(1 \pm \frac{r_i^2}{R^2}\right)$$

**( + is hoop, - is radial )**

# Stresses vs. Radius - External Pressure

**Radial** stress is as predicted:

- 0 at the unpressurized inner surface.
- -5330 psi at the outer, pressurized surface.

**Hoop** stress is:

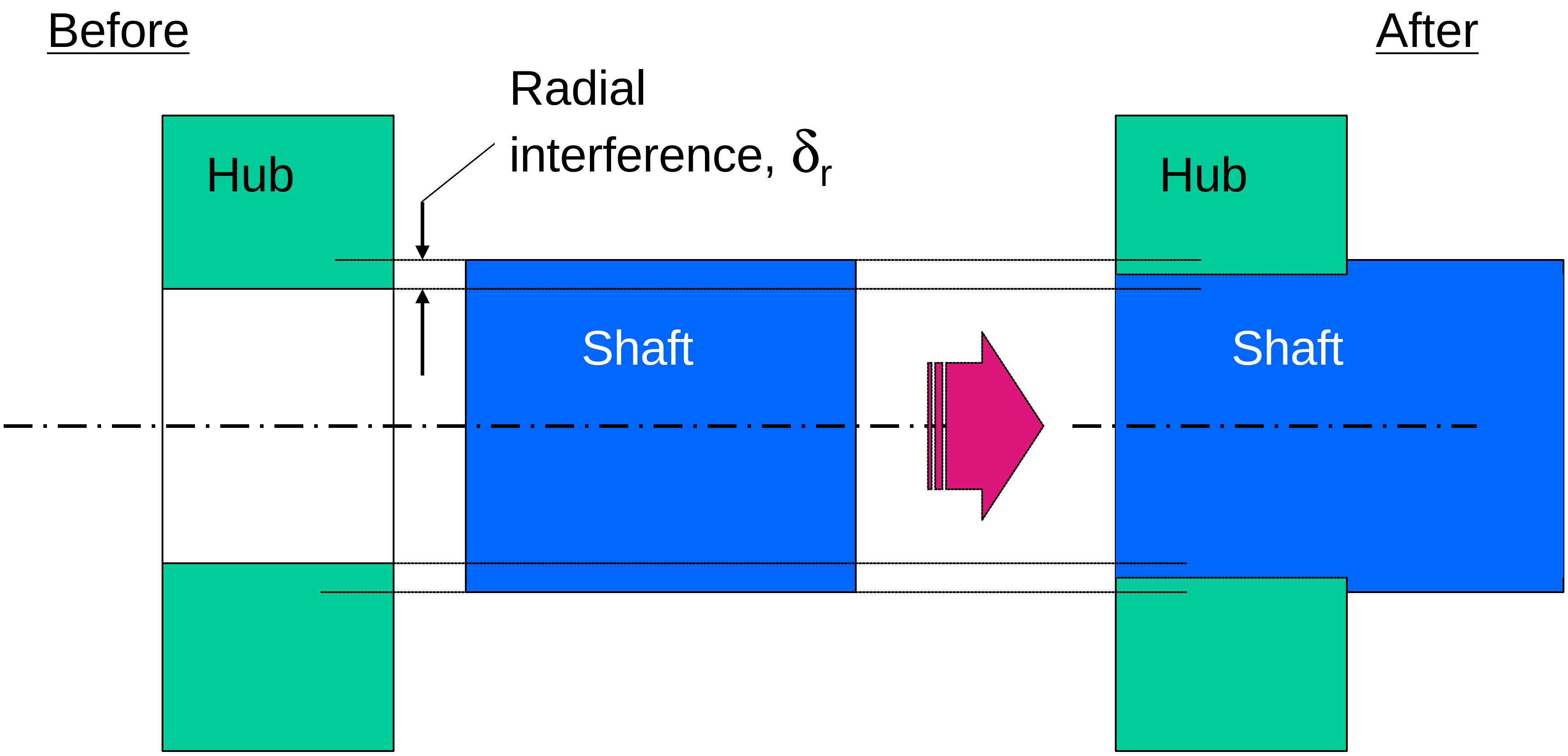
- Minimum at the outer surface, -8.9 ksi.
- Maximum at the (unpressurized) inner surface, -14.2 ksi.
- Larger than the radial stress

**Longitudinal** stress is:

- Not usually considered for external pressurization.

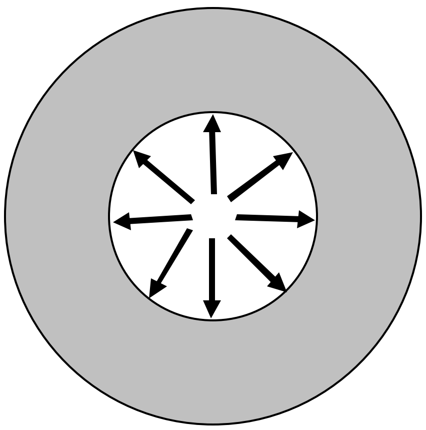
# Press Fits

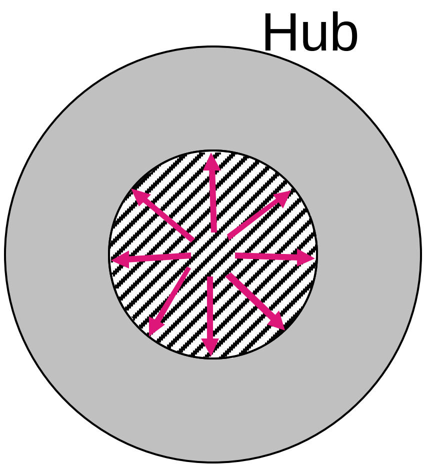
**In a press fit, the shaft is compressed and the hub is expanded.**



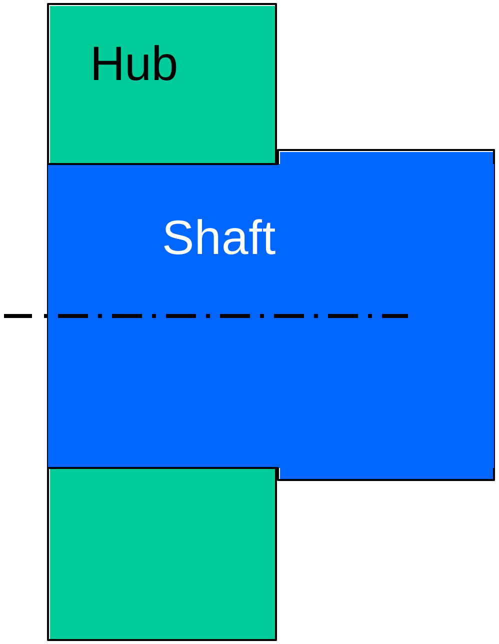
# Press Fits

**Press fits, or interference fits, are similar to pressurized cylinders in that the placement of an oversized shaft in an undersized hub results in a radial pressure at the interface.**





# Characteristics of Press Fits




**1) The shaft is compressed and the hub is expanded.**

**2) There are equal and opposite pressures at the mating surfaces.**

**3) The relative amount of compression and expansion depends on the stiffness (elasticity and geometry) of the two pieces.**

**4) The sum of the compression and the expansion equals the interference introduced.**

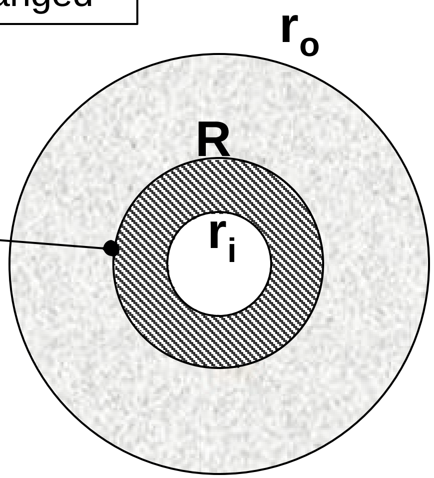
**5) The critical stress location is usually the inner diameter of the hub, where max tensile hoop stress occurs.**

# Analysis of Press Fits

**Start by finding the interface pressure.**

$$p = \frac{E\delta_r}{R}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{2R^2\left(r_o^2 - r_i^2\right)}\right]$$

Eqn 10.52, rearranged



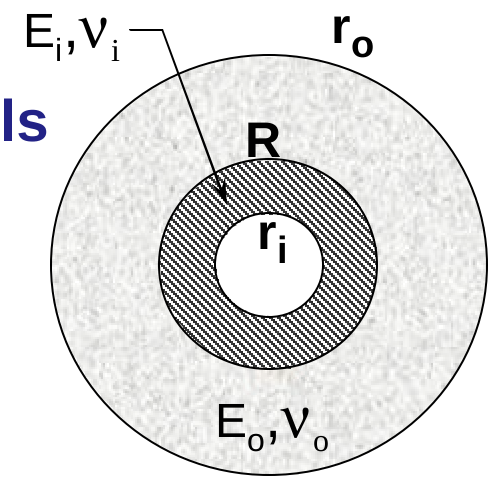

**Where $\delta_r$ is the RADIAL interference for hub and shaft of the same material, with modulus of elasticity, E.**

**If the shaft is solid, $r_i = 0$ and**

$$p = \frac{E\delta_r}{2R}\left[1 - \frac{R^2}{r_o^2}\right]$$

Eqn 10.53, rearranged

# Analysis of Press Fits




**If the shaft and hub are of different materials**

$$p = \frac{d_r}{\frac{R}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + n_o\right) + \frac{R}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - n_i\right)}$$

Eqn 10.51, rearranged

$n_{i,o}$ **= Poisson**

**Once we have the pressure, we can use the cylinder equations to compute the hoop stresses at the interface.**

A) The ID of the hub is tensile:

$$s_{o_c} = p\,\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

Eqn 10.45

B) The OD of the shaft is compressive:

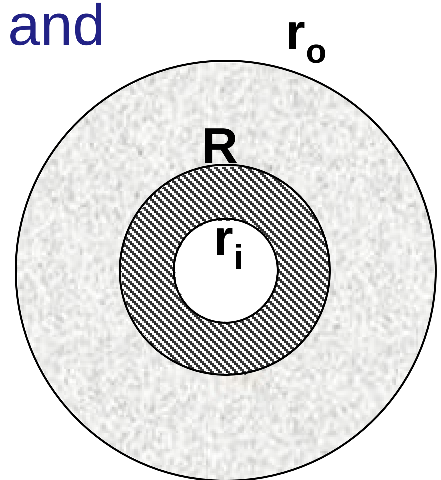
$$s_{i_c} = -p\,\frac{R^2 + r_i^2}{R^2 - r_i^2}$$

Eqn 10.49

= -p if shaft is solid

# Strain Analysis of Press Fits

The press fit has no axial pressure, so $\sigma_l$ = 0, and it is a biaxial stress condition.




The circumferential strain $$e_c = \frac{s_c}{E} - \frac{n\, s_r}{E}$$

which equals the radial strain (because C = 2πr).

Because the radial change $\delta = R\,\varepsilon_r$ , we get the <u>increase in Inner</u> <u>Radius</u> of the <u>outer member</u> (hub):

$$d_o = \frac{pR}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + n_o\right)$$

Eqn 10.46

And the <u>decrease in Outer</u> <u>Radius</u> of the <u>inner member</u> (shaft):

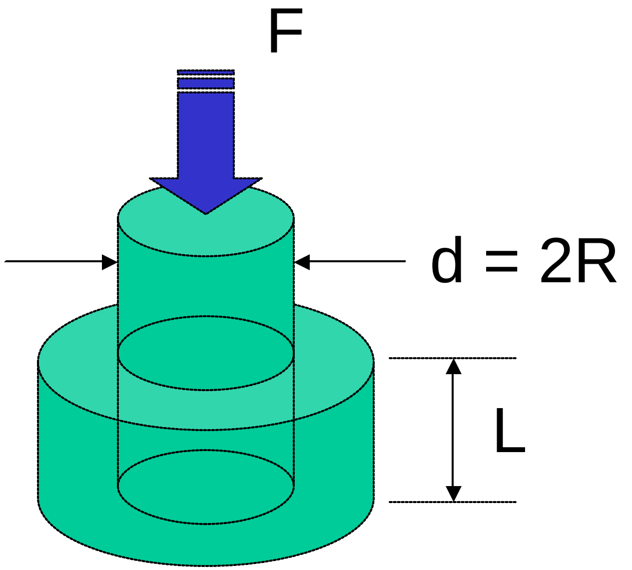
$$d_i = -\frac{pR}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - n_i\right)$$

Eqn 10.50

# Notes on Press Fits

As a check, make sure that

$$|d_i| + |d_o| = d_r$$




The assembly force required will be

$$F_{max} = \pi d L p \mu$$

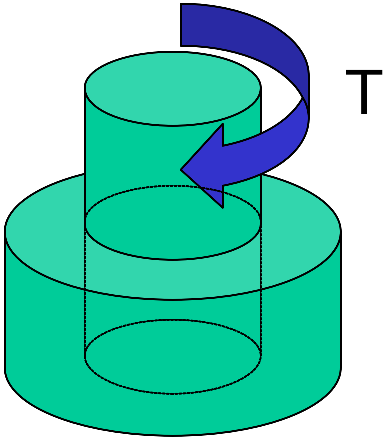
where p = the interface pressure

$\mu$ = the coefficient of friction

The torque capacity available is

$$T = FR = R\pi d L p \mu$$

where R = the interference radius, as before.




We conveniently know the interface pressure for these equations!

# Shrink Fits

If heating or cooling a part to achieve a shrink fit, the required radial interference is:

$$\Delta R = d_{\mathbf{r}} = R\alpha\Delta T$$

where R is the interface radius
$\alpha$ is the coefficient of thermal expansion
$\Delta T$ is the temperature change

To select an amount of interference see ANSI/ASME tables for class FN1 (light) to FN5 (Heavy-drive) fits.

- They give interference in 0.001" on diameter for a range of diameters

Ex: FN4 for 0.95 to 1.19" diameter, interference = 1 to 2.3 mils on diameter.