MEEG3311 Machine Design

Lecture 9: Power Screws
(Chapter 16)

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16Nov2023

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School of Engineering

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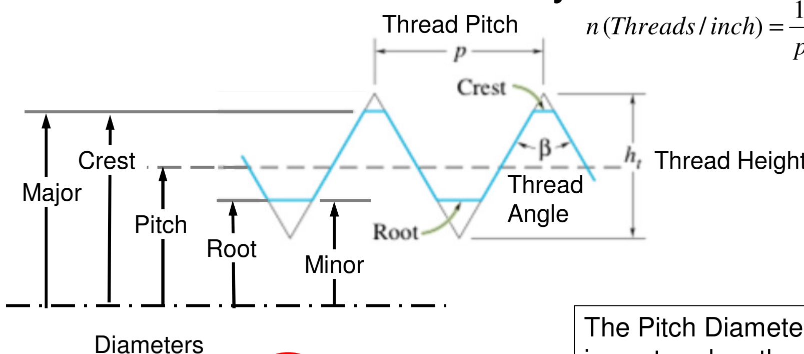
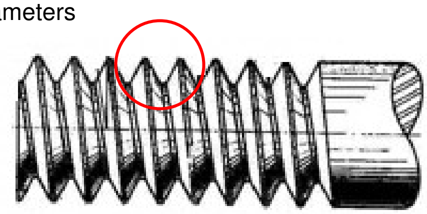
Thread Geometry

$$n\,(Threads\,/\,inch) = \frac{1}{p}$$



The Pitch Diameter is centered on the Thread Height, h_t.



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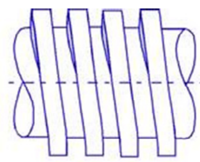
Thread Types

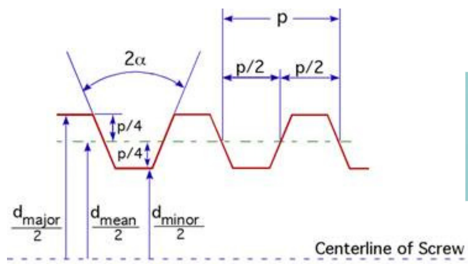
Square Thread

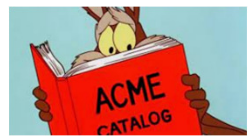


0° Thread Angle

Acme Thread

29° Thread Angle





Acme threads are used in C-Clamps, vices, and cartoons.

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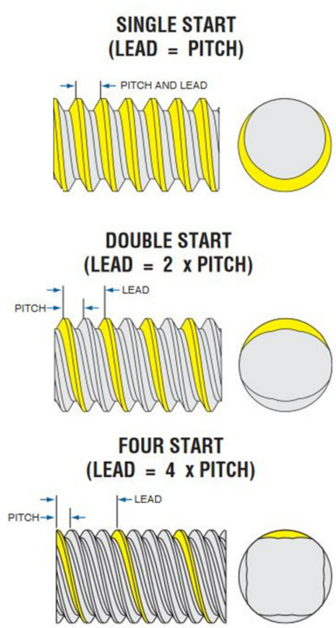
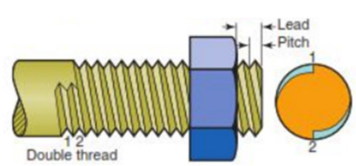


Thread Types

Single-, double-, and quadruple-threaded screws.

Also called single-, double-, and four-start.

Lead = Screw advance per turn



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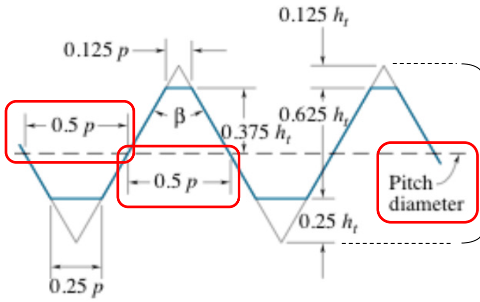
Details of 60° Thread Profile



Thread Height

$$h_t = \frac{0.5p}{\tan(30°)} = 0.8660\,p$$

Relationships for M (metric) and UN (unified = US) screw threads.

Example:
UN: ¼-20, means 0.25in. Major diameter & 20 threads/inch.
M: M8x1.25, means 8mm Major diameter & pitch of 1.25mm

Pitch Diameter = Crest Diameter – ¾*Thread Height
It's where the thread thickness = the space between threads.

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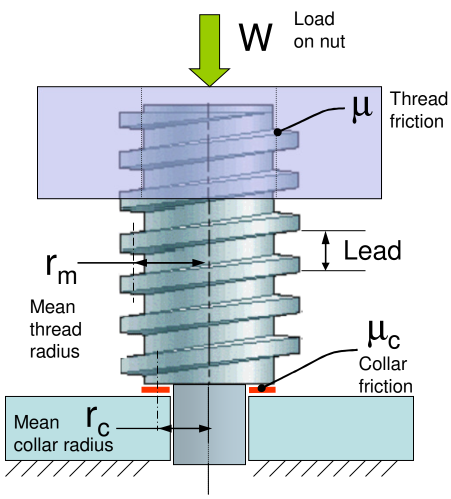
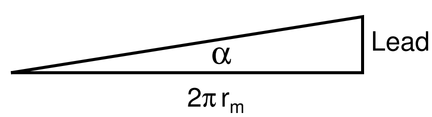
Power Screws



The Mean Radius is midway between the Crest and Root Radii.

Looking at a square thread screw, we unwind one turn:



This shows an inclined ramp with angle

$$\alpha = \tan^{-1}\frac{Lead}{2\pi\, r_m}$$

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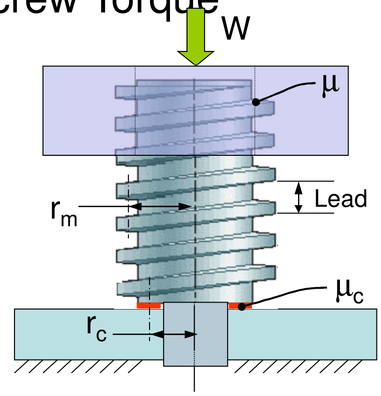
Square Thread Screw Torque



The torque required to
raise the load W is

$$T_{raise} = W\left[r_m \frac{\mu + \tan\alpha}{1 - \mu\tan\alpha} + \mu_c r_c \right]$$

and to lower the load, we
flip two signs:

$$T_{lower} = W\left[r_m \frac{\mu - \tan\alpha}{1 + \mu\tan\alpha} + \mu_c r_c \right]$$

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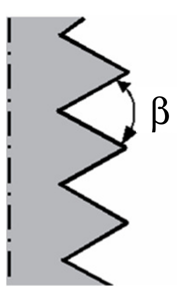
Power Screw Thread Angle

If the thread form is not square but has an angle β,
replace the thread friction μ with the effective friction

$$\mu_{eff} = \frac{\mu}{\cos(\beta/2)}$$



The effect:

- Square: $\beta = 0$, $\beta/2 = 0$, $1/\cos(0°) = 1.0$
- Acme: $\beta = 29°$, $\beta/2 = 14.5°$, $1/\cos(14.5°) = 1.033$
- Unified: $\beta = 60°$, $\beta/2 = 30°$, $1/\cos(30°) = 1.15$

The thread angle effectively increases surface friction between 3 and 15%

Note: Instead of β/2, Hamrock uses $\theta_n = \tan^{-1}(\cos\alpha\tan\beta/2)$
The difference is negligible.

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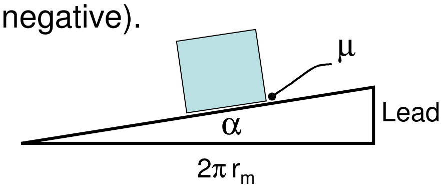
Power Screws - Overhauling

If the collar friction is small (e.g., it may have a ball thrust bearing), too small a thread friction may let the weight screw down on its own.

This can happen when $\mu < \tan\alpha = \dfrac{Lead}{2\pi\, r_m}$

(the numerator $\mu_{eff} - \tan\alpha$ goes negative).

$$T_{lower} = W\left[r_m \frac{\mu_{eff} - \tan\alpha}{1 + \mu_{eff}\,\tan\alpha} + \overset{0}{\mu_c}\, r_c \right]$$



This is the same case for a weight sliding down a ramp when the incline angle α exceeds $\tan^{-1}\mu$.

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Various Standard Screws

These could be English or Metric.

All have 60° thread angles.

Other types include Torx drive, Button head, Pan head and more.

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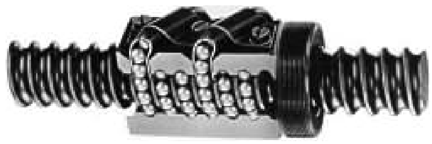
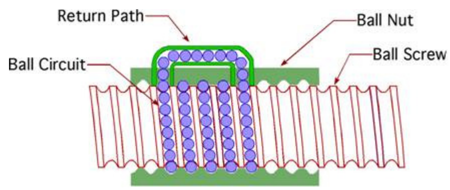
Ball Screws Have Low Friction



Recirculating balls roll between ball screw and ball nut to minimize friction.



These almost always overhaul.

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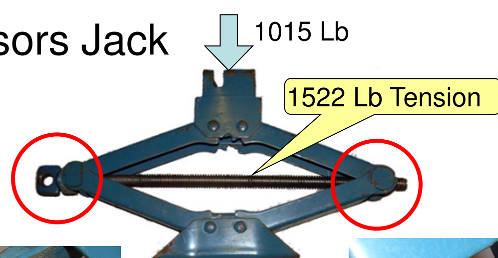
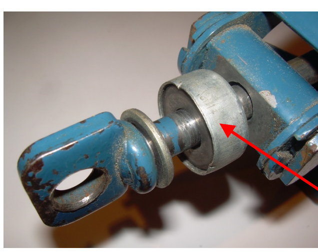
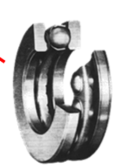
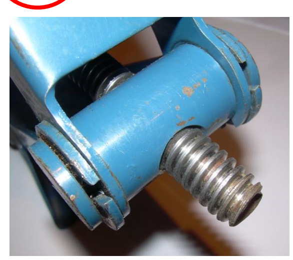
Our Scissors Jack







Handle End with ball thrust bearing



End with nut

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C-Clamp Analysis

Thread ID = 0.391 in.
Thread OD = 0.480 in.
Handle length = 3 in.
N = 8 Threads/Inch
Thread angle $\beta = 60°$
Guess $\mu = 0.15$
$\mu_c = 0$ to simplify things
W = 500 Lb.

What torque is required to cause the 500 Lb. squeeze?

Note: If Acme, could use Eqn. 16.5

$$d_p = d_c - 0.5p - 0.01 = 0.48 - (0.5)(0.125) - 0.01 = 0.4075\,in.$$

But with a 60° thread angle, this is NOT an Acme.

Estimate d_p= (ID+OD)/2 = (0.390+0.480)/2 = 0.436 in.

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Using Dornfeld Lecture Equations

d_p= 0.436 in.
N = 8 Threads/Inch
Lead = 1/N = 0.125 in.

Thread angle b = 60°
$\mu = 0.15$
W = 500 Lb.

$$\alpha = \tan^{-1}\frac{Lead}{2\pi\, r_m} = \tan^{-1}\frac{0.125}{2\pi(0.436/2)} = \tan^{-1}(0.09126) = 5.21°$$

Because this is not a square thread, must use <u>effective</u> coefficient of friction = $\mu/\cos(\beta/2) = 0.15/\cos(30°) = 0.15/0.866 = 0.1732$

$$T_{raise} = W\left[r_m \frac{\mu_{eff} + \tan\alpha}{1 - \mu_{eff}\tan\alpha} + \overset{0}{\cancel{\mu_c}} r_c\right] = 500\left[\frac{0.436}{2}\;\frac{0.1732 + 0.09126}{1 - (0.1732)(0.09126)}\right]$$

$$= (500)(0.218)\frac{0.26446}{0.9842} = 29.29\,Lb.In.$$

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Using Hamrock Equations

d_p= 0.436 in.
N = 8 Threads/Inch
Lead = 1/N = 0.125 in.

Thread angle b = 60°
μ = 0.15
W = 500 Lb.

$$\alpha = \tan^{-1}\frac{Lead}{2\pi r_m} = 5.21°;\quad \tan(\alpha) = 0.09126$$

$$\theta_n = \tan^{-1}(\cos\alpha \tan\frac{\beta}{2}) = \tan^{-1}(\cos 5.21° \tan 30°) = \tan^{-1}(0.9959 \times 0.57735)$$

$$\theta_n = \tan^{-1}(0.57496) = 29.897°$$

How close is this to β/2 = 30°?

$$T_{raise} = W\left[\frac{(d_p/2)(\cos\theta_n \tan\alpha + \mu)}{\cos\theta_n - \mu\tan\alpha} + \mu_c r_c\right] = 500\left[\frac{(0.436/2)(\cos 29.9° \tan 5.21° + 0.15)}{\cos 29.9° - 0.15\tan 5.21°}\right]$$

0

$$= (500)(0.218)\frac{(0.866)(0.09126)+0.15}{0.866-(0.15)(0.09126)} = (109)\frac{0.22903}{0.85231} = 29.29\,Lb.In.$$

[Eqn. 16.13]

The equations are equivalent. Pick whichever one suits you best.

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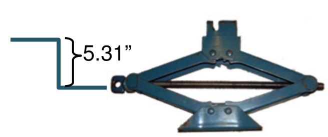
Scissors Jack Analysis

Thread ID = 0.398 in.
Thread OD = 0.468 in.
Estimate d_p= (0.398+0.468)/2
= 0.433 in.
Handle Radius = 135/25.4 = 5.31 in.

Lead = 0.10 in.
Thread angle β = 29°
Guess μ = 0.20
μ_c = 0 due to bearing
W = 1522 Lb.

What torque is required to raise the jack?
What force is required on the handle?



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Overhauling Revisited

• Power screws can lower all by themselves if the friction becomes less than the tangent of the lead angle, α.
• This corresponds to the numerator in the T_{lower} equation going negative, with the transition being where the numerator is Zero.
• You can use either Dornfeld or Hamrock equation, but remember that the Dornfeld equation is Effective friction, and you must multiply by $\cos(\beta/2)$ to get the actual friction.

Hamrock:

$$T_{lower} = W\left[\frac{(d_p/2)(\mu - \cos\theta_n \tan\alpha)}{\cos\theta_n + \mu\tan\alpha} + \mu_c r_c\right]$$

Transition when:

$$\mu = \cos\theta_n \tan\alpha$$

Dornfeld:

$$T_{lower} = W\left[r_m \frac{\mu_{eff} - \tan\alpha}{1 + \mu_{eff}\tan\alpha} + \mu_c r_c\right]$$

$$\mu_{eff} = \tan\alpha$$

$$\mu = \mu_{eff}\cos(\beta/2) = \cos(\beta/2)\tan\alpha$$

The equations are equivalent. Pick whichever one suits you best.

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Failure Modes: Tensile Overload

When the tensile stress on a bolt exceeds the material's Proof Strength, the bolt will permanently stretch.

$$\sigma = \frac{P}{A_t}$$

Where A_t is the Tensile Stress Area for the bolt – the equivalent area of a section cut through the bolt.

For UN threads,

$$A_t = (0.7854)\left(d_c - \frac{0.9743}{n}\right)^2$$

d_c = Crest Dia (in.)
n = threads/in.

For M threads,

$$A_t = (0.7854)(d_c - 0.9382\,p)^2$$

d_c = Crest Dia (mm)
p = pitch (mm)

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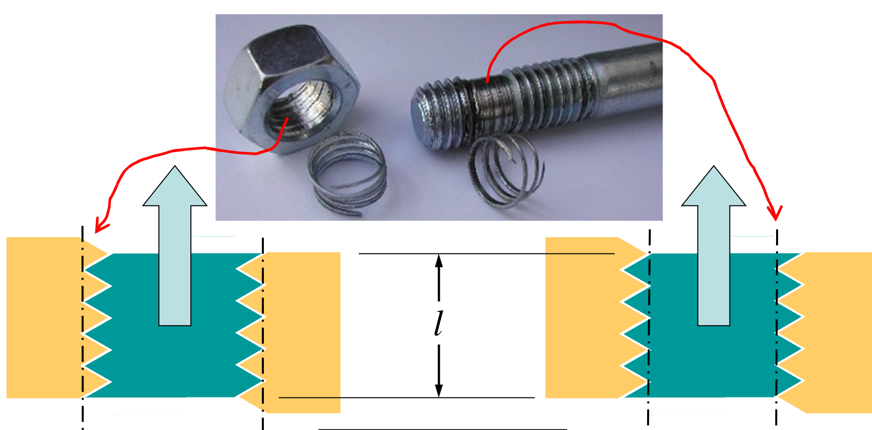
Tensile Stress Area

Table 16.8 Dimensions and Tensile Stress Areas for UN Coarse and Fine Threads

		Coarse threads (UNC)			Fine threads (UNF)		
	Crest diameter, d_c, in.	Number of threads per, inch, n	Root diameter, d_r, in.	Tensile stress area, A_t, in.^{2}	Number of threads per, inch, n	Root diameter, d_r, in.	Tensile stress area, A_t, in.^{2}
#0	0.0600	—	—	—	80	0.04647	0.00180
#1	0.0730	64	0.05609	0.00263	72	0.05796	0.00278
#2	0.0860	56	0.06667	0.00370	64	0.06909	0.00394
#3	0.0990	48	0.07645	0.00487	56	0.07967	0.00523
#4	0.1120	40	0.08494	0.00604	48	0.08945	0.00661
#5	0.1250	40	0.09794	0.00796	44	0.1004	0.00830
#6	0.1380	32	0.1042	0.00909	40	0.1109	0.01015
#8	0.1640	32	0.1302	0.0140	36	0.1339	0.01474
#10	0.1900	24	0.1449	0.0175	32	0.1562	0.0200
#12	0.2160	24	0.1709	0.0242	28	0.1773	0.0258
¼-20	0.2500	20	0.1959	0.0318	28	0.2113	0.0364
	0.3125	18	0.2523	0.0524	24	0.2674	0.0580
	0.3750	16	0.3073	0.0775	24	0.3299	0.0878
	0.4750	14	0.3962	0.1063	20	0.4194	0.1187
	0.5000	13	0.4167	0.1419	20	0.4459	0.1599
	0.5625	12	0.4723	0.182	18	0.5023	0.203
	0.6250	11	0.5266	0.226	18	0.5648	0.256
	0.7500	10	0.6417	0.334	16	0.6823	0.373
	0.8750	9	0.7547	0.462	14	0.7977	0.509
	1.000	8	0.8647	0.606	12	0.9098	0.663
	1.125	7	0.9703	0.763	12	1.035	0.856
	1.250	7	1.095	0.969	12	1.160	1.073
	1.375	6	1.195	1.155	12	1.285	1.315
	0.500	6	1.320	1.405	12	1.140	1.581
	1.750	5	1.533	1.90	—	—	—
	2.000	4.5	1.759	2.5	—	—	—

Failure Modes: Thread Shear



Shear of Nut Threads

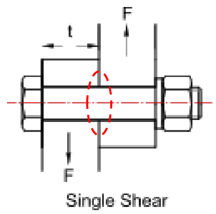
$$A_{shear} = \pi d_{crest} l$$

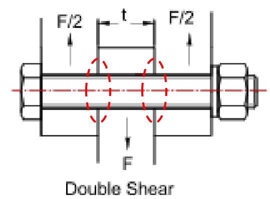
The shear strength of the bolt and nut material might not be the same.

Shear of Bolt Threads

$$A_{shear} = \pi d_{root} l$$

Failure Modes: Shank Shear

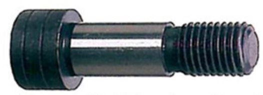




$$A_{shear} = \frac{\pi d_{shank}^2}{4}$$

$$A_{shear} = 2 \times \frac{\pi d_{shank}^2}{4} = \frac{\pi d_{shank}^2}{2}$$

Bolts are not really intended to be used this way unless they are Shoulder Bolts:



Typically, the preload from tightening the bolt clamps the joint, and the friction between the members holds the joint.

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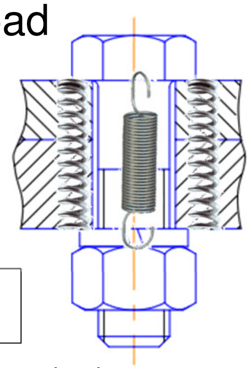
Bolt Preload



JH Bickford explains :

'When we tighten a bolt,

(a) we apply torque to the nut,

(b) the nut turns,

(c) the bolt stretches,

(d) creating preload.'

The bolt is really a spring that stretches and creates preload on the joint.

We use the Power Screw equations to determine how torque results in preload. This can be approximated simply by:

$$T = KD_{crest}P$$

Where T is torque, D_{crest} is the bolt crest diameter, P is the preload, and K is a dimensionless constant. K = 0.20 for clean, dry threads and K = 0.15 for lubricated threads.

If you have enough info to use the Hamrock/Dornfeld equations, DO NOT use this.

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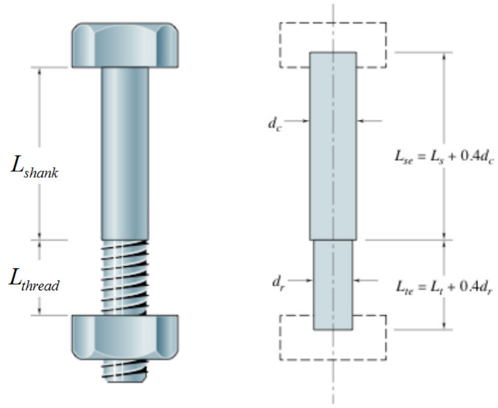
Bolt Stiffness



A bolt looks like two springs in series: one rod with the Crest diameter and one with the Root diameter.

Their lengths are increased to reflect the head and nut.

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$$\frac{1}{k_b} = \frac{4}{\pi E}\left(\frac{L_s + 0.4d_c}{d_c^2} + \frac{L_t + 0.4d_r}{d_r^2}\right)$$

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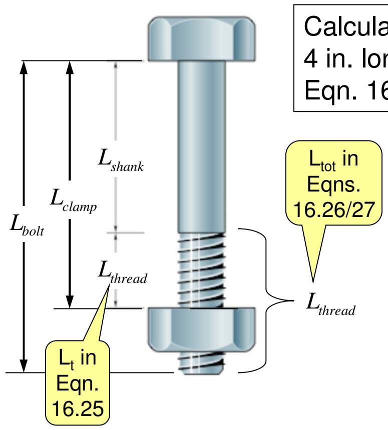
Bolt Stiffness Exercise

Calculate the stiffness of a 3/8-16 screw that is 4 in. long and clamps 3.5” of material. Use Eqn. 16.27 to determine shank length.



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Joint Stiffness

The material clamped by the bolt also acts like a spring – in compression.

Effectively, only the material in the red double conical area matters.

There are many methods to calculate this stiffness.

Compare these calculator stiffness results from tribology-abc.com with Hamrock's Example 16.6

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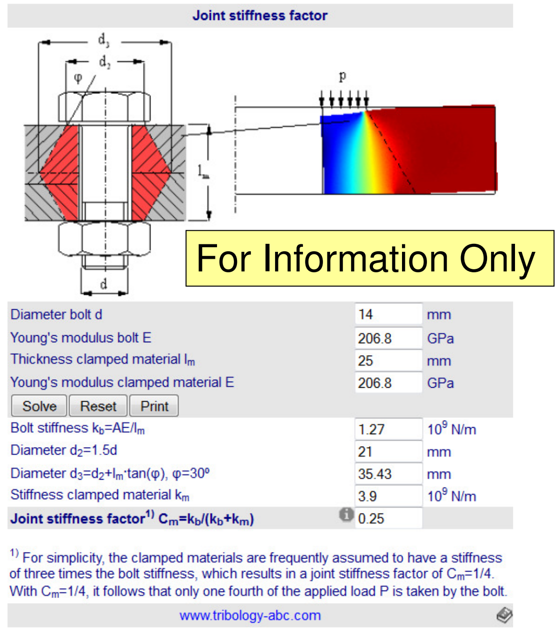

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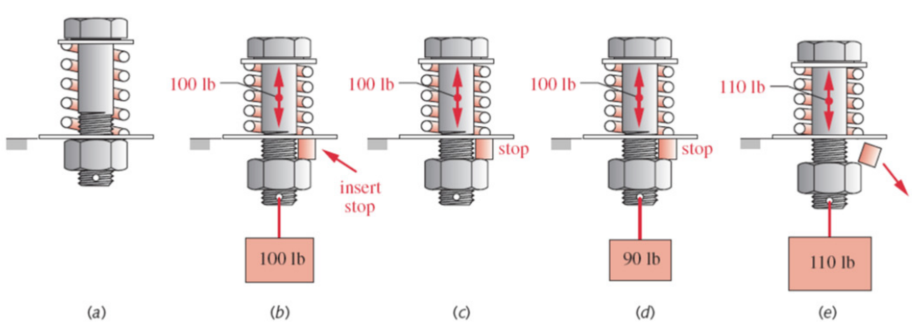
How Bolt Preload Works



Preload isolates the bolt from most of any external loads.

The joint stiffness factor, C_k, determines what fraction of external loads the bolt actually sees.

$$C_k = \frac{k_b}{k_b + k_j}$$

Hamrock
Eqn. 16.21

From Norton, Chap. 14

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Bolt Strength - Metric

Metric grade	Head marking	Crest diameter, d_c, mm	Ultimate tensile strength, S_u, MPa	Yield strength, S_y, MPa	Proof strength, S_p, MPa
4.6	4.6	M5 – M36	400	240	225
4.8	4.8	M1.6 – M16	420	340[a]	310
5.8	5.8	M5 – M24	520	415[a]	380
8.8	8.8	M17 – M36	830	660	600
9.8	9.8	M1.6 – M16	900	720[a]	650
10.9	10.9	M6 – M36	1040	940	830
12.9	12.9	M1.6 – M36	1220	1100	970

Table 16.7

For Metric grades, the first number x 100 = Sut in MPa. The fraction x Sut = Sy. Ex: grade 12.9 has Sut ≈1200 MPa and Sy ≈ 0.9x1200 = 1080 MPa.

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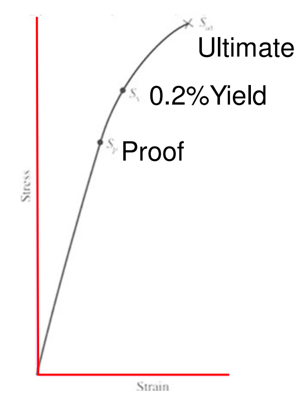
Bolt Loading

Generally, bolts are preloaded to:

- 75% of Proof Load for reused connections
- 90% of Proof Load for permanent connections

where Proof Load = Proof Strength x A_t.

The Proof Strength is approximately at the elastic limit for the material.



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Summary of Screws

1. Behavior
   - Torque → Axial Preload

$$T_{raise} = W\left[r_m \frac{\mu + \tan\alpha}{1 - \mu \tan\alpha} + \mu_c r_c \right]$$

$$\mu_{eff} = \frac{\mu}{\cos(\beta/2)}$$

2. Standards
   - Shape
     * Metric / UN
     * ACME Table 16.2
     * Other (Square)
   - Grade (Strength)
     * Inch (UN) Table 16.6
     * Metric Table 16.7
   - Sizes
     * Inch (UN) Table 16.8
     * Metric Table 16.9

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Recommended Site:
BoltScience.com

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