MEEG3311 Machine Design

Lecture 7: Columns

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Column Buckling

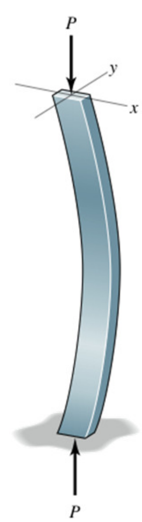
We have already discussed axially loaded bars. For a short bar, the stress = P/A, and the deflection is PL/AE. (Hamrock, §4.3)

If we load a long, slender bar, however, it will bend and buckle long before it will yield in compression.

The sudden nature of buckling makes it deserve special attention so it can be avoided.

This failure mode – instability – is different from the yield or fatigue failure modes.

Columns are classified by two means:
1. by their relative length (i.e., slenderness)
2. by whether or not the load is centered on them.



Hamrock
Section 9.3

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Concentrically Loaded Columns with Pinned Ends

You already learned this in Beer & Johnston.

The Euler column buckling formula [Eqn. 9.7]:

$$P_{crit} = \frac{\pi^2 EI}{l^2}$$

Notes:
- Swiss mathematician Leonhard Euler (Óil er) figured it out in ~1790.
- His name does <u>not</u> rhyme with Ferris Bueller's – it rhymes with "boiler".
- P_{crit} is independent of material strength, Sy.
- It depends on I and not on area, as P/A does.
- The derivation is simple and beautiful – see §9.3.1.

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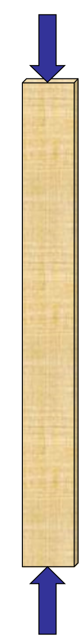
Yardstick Buckling

A typical yardstick is about 1/8" thick and 1 1/8" wide. What is the critical buckling load, assuming the ends are pinned?

Typical E for softwoods is 1.5x10^6 psi.

$$I = \frac{bh^3}{12}$$



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Radius of Gyration

In Chap. 4, the radius of gyration was defined from

$$I = Ar_g^2 \quad as$$

$$r_g = \sqrt{I/A} \qquad \text{Eqn. 4.14}$$

(It is similar in form to I = mr² for mass moment of inertia.)

Then can rewrite $$P_{crit} = \frac{\pi^2 EA}{(l/r_g)^2}$$

Now that we have Area, can define the critical stress:

$$\sigma_{cr} = \frac{P_{crit}}{A} = \frac{\pi^2 E}{(l/r_g)^2}$$

- This is an elastic stress, because it is < Sy
- l/r_g is called the Slenderness Ratio
- It depends only on geometry and Young's Modulus, not strength or heat treatment.

Hamrock
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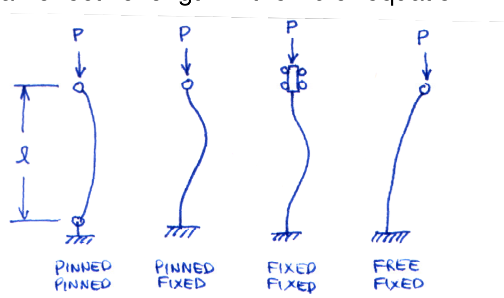
End Conditions

If the ends of the column are something other than pinned-pinned, must use an effective length in the Euler equation.



Effective Length:

Theoretical	L	0.7L	0.5L	2L
AISC Recommended	L	0.8L	0.65L	2.1L

Use These

- For other than Pinned-Pinned, replace the actual beam length, L, by effective column length, Le.

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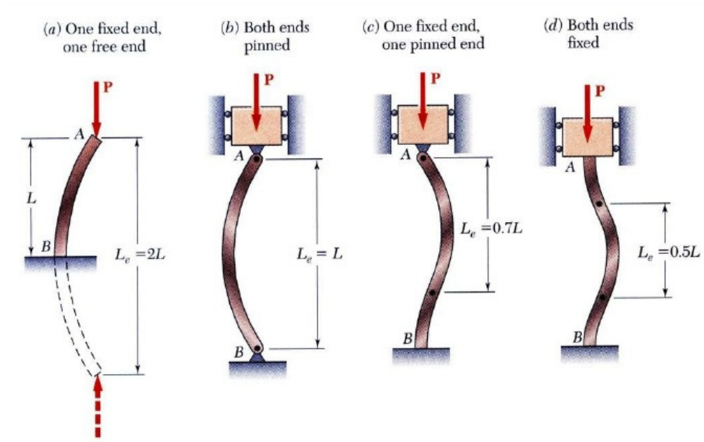
End Conditions



Another view of the Theoretical effective lengths of columns based on end conditions. (From Beer & Johnston 3rdEd, Fig. 10.18)

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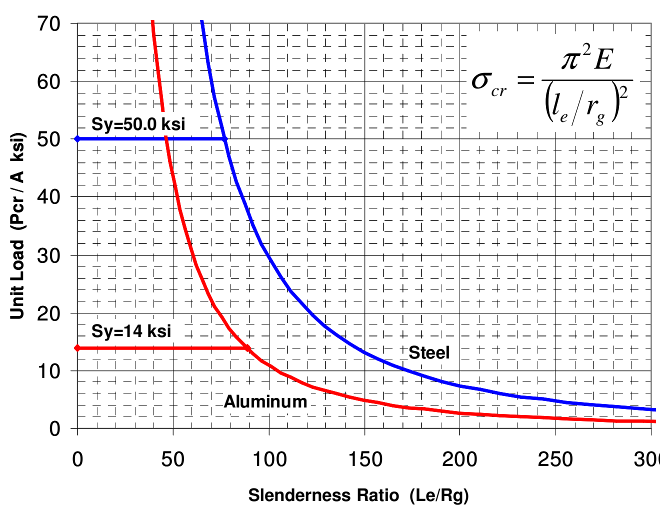
Critical Stress vs Slenderness

The curves only depend on Modulus; Upper limit is Sy.



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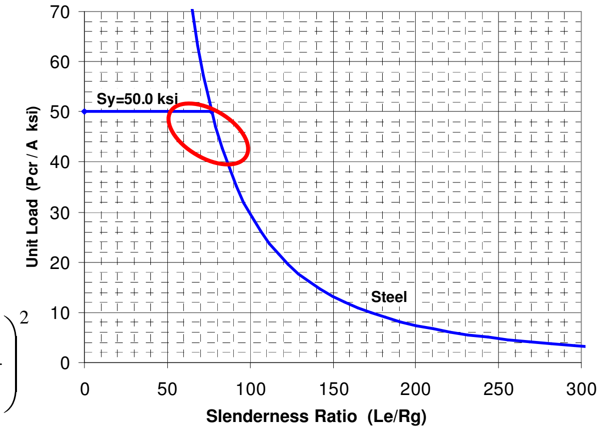
Transition to Yielding

Up near this corner, it was discovered that buckling failures did occur below the Euler/Yield lines.

Based on measured results around 1900, J.B. Johnson developed a parabolic transition formula for "Intermediate" length columns. [Eqn. 9.16]

$$\sigma_{cr_J} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{l_e}{r_g}\right)^2$$



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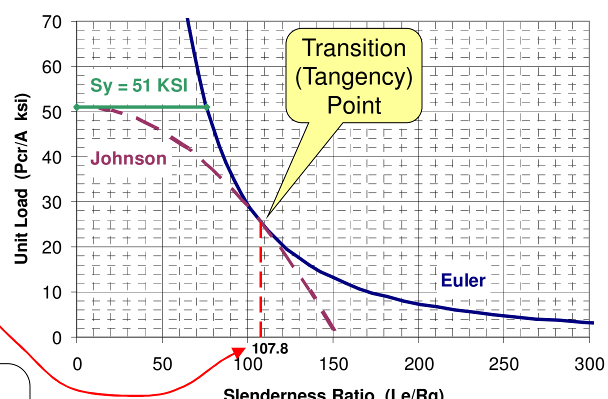
Euler-Johnson Equations

- Above the Transition Slenderness ratio, use Euler.
- Below the Transition Slenderness ratio, use Johnson.

Transition Slenderness Ratio:

$$\left(\frac{l_e}{r_g}\right)_{trans} = \sqrt{\frac{2\pi^2 E}{S_y}}$$

[Eqn. 9.18]



Johnson

$$\sigma_{cr_J} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{l_e}{r_g}\right)^2$$

Euler: $\sigma_{cr} = \dfrac{\pi^2 E}{\left(l_e/r_g\right)^2}$

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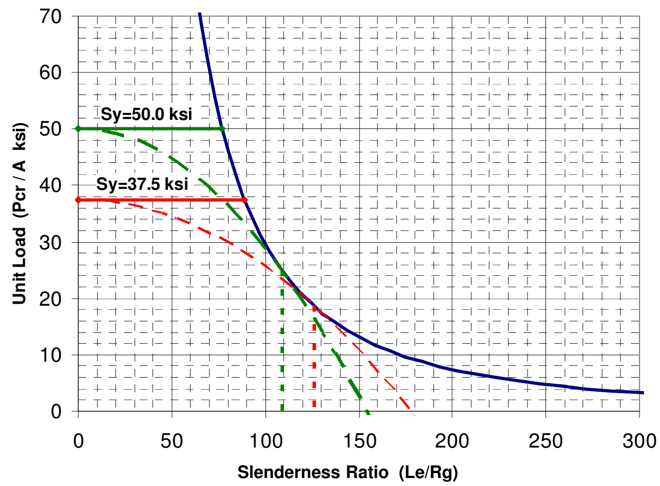
Euler-Johnson Equations

• As the Yield Strength changes, so does the transition point, to keep the two curves tangent.

Transition Slenderness Ratio:

$$\left(\frac{l_e}{r_g}\right)_{trans} = \sqrt{\frac{2\pi^2 E}{S_y}}$$

Note: It hits the Euler curve at about Sy/2.



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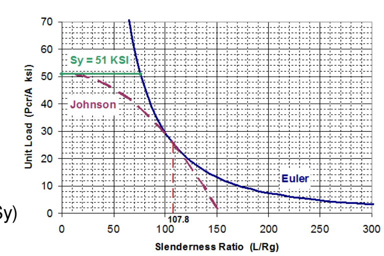
Columns Procedure



A. You always need:
1. The Material Modulus of Elasticity and Yield Strength
2. The Length
3. The Cross Section
4. The End Conditions

B. Find out if your Column is short or long. First compute the Critical or Transition Slenderness Ratio = Sqrt(2 Pi^2 E / Sy)

C. Compute the Slenderness Ratio of your column. Calculate the Area Moment of Inertia, I, and the Area, A. Then the Radius of Gyration, Rg = Sqrt(I/A).
Next, calculate the Effective Length, based on the End Conditions, Le = L * Coefficient based on end conditions.
Finally, your Slenderness Ratio = Le/Rg.

D. If Le/Rg > Transition Slenderness Ratio, use the Euler Equation to compute the buckling load or stress.
If Le/Rg < Transition Slenderness Ratio, use the Johnson Equation to compute the buckling stress. Buckling load = Buckling stress * Area.

E. The Factor of Safety = Buckling Load / Actual Load, or Buckling Stress / Actual Stress.
FOS is NOT Sy/anything.

Euler: $\sigma_{cr} = \dfrac{\pi^2 E}{\left(l_e/r_g\right)^2}$

Johnson: $\sigma_{cr_J} = S_y - \dfrac{S_y^2}{4\pi^2 E}\left(\dfrac{l_e}{r_g}\right)^2$

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Euler? or Johnson?

You have a 1” round bar of 1040 steel, annealed. Assuming it is loaded with its ends pinned-pinned:

A. How long would it be to be right at the Johnson-Euler transition point?

B. What load could it take before buckling?

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What if the column is loaded off-center?

The eccentricity causes a moment that contributes to bending the beam and aiding buckling. This will also happen if the column is initially crooked (bent).

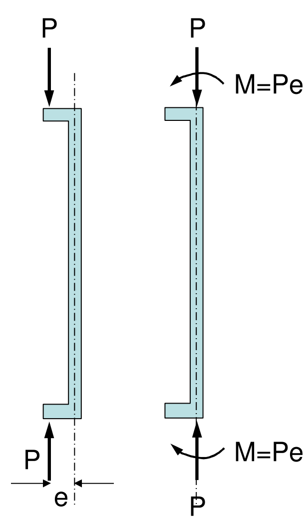
This case is described by the Secant Equation [Eqn. 9.32]:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_g^2}\sec\left(\frac{l_e}{2r_g}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\sec = \frac{1}{\cos}$$

or, rearranging

$$\frac{P}{A} = \frac{\sigma_{max}}{1 + \frac{ec}{r_g^2}\sec\left(\frac{l_e}{2r_g}\sqrt{\frac{P}{EA}}\right)}$$



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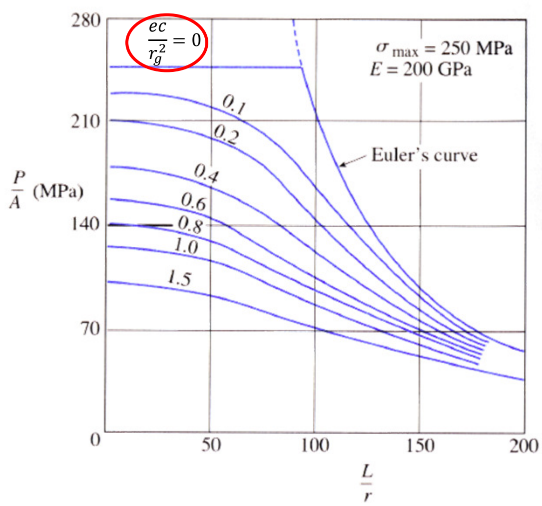
Secant Equation

The term ec/r_g^2 is called the Eccentricity Ratio, and c is the distance from neutral axis to surface (like Mc/I).

This is a messy equation because P/A is a function of itself. It must be solved iteratively, typically starting with the Johnson P_{cr}.

$$\frac{P}{A} = \frac{\sigma_{max}}{1 + \frac{ec}{r_g^2}\sec\left(\frac{l_e}{2r_g}\sqrt{\frac{P}{EA}}\right)}$$



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