MEEG3311 Machine Design

Lecture 6: Fluctuating Fatigue and the Goodman Diagram; Impact

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19Oct2023

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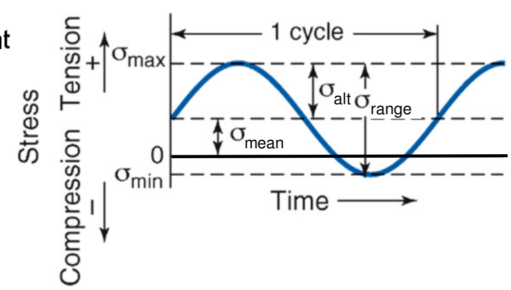
Fluctuating Fatigue

So far we have discussed loading that alternately went from tension to compression with the extremes equal and opposite.

Now we look at the more general case where there could be a mean value.

This is called Fluctuating Fatigue and is characterized by both a mean and an alternating component.



$$\sigma_{mean} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = Average\ or\ Steady\ Stress$$

$$\sigma_{alt} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = Stress\ Amplitude$$

Hamrock
Section 7.2

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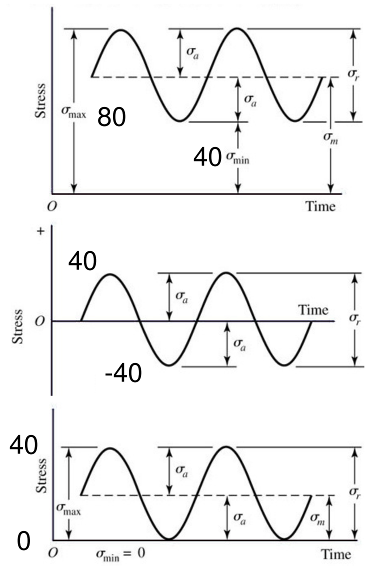
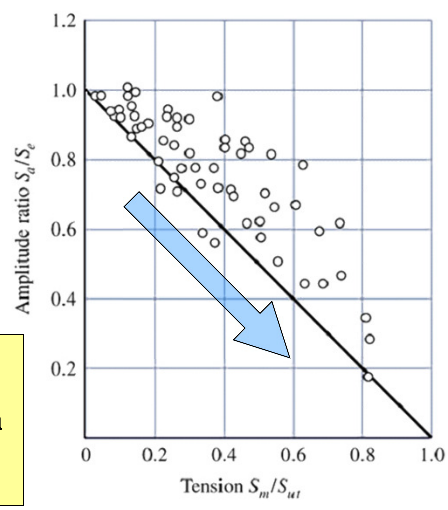
$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_{alt} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

σ_{mean}	σ_{alt}	Fluctuating or Fully Reversing?



Effect of Mean Stress

This axis is the fraction that σ_{alt} is of the Endurance Strength, Se



Circles represent failures

As the Mean stress increases, the material breaks at a lower Alternating stress amplitude.

This axis is the fraction that σ_{mean} is of the Ultimate Strength, Sut

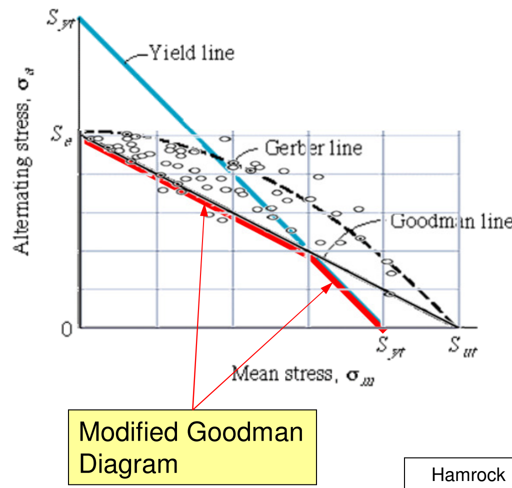
Fluctuating Fatigue Diagrams

Several ways are available to characterize the Fluctuating Fatigue behavior. Two common ones are:

- Goodman line and
- Gerber line.

Both approximate the material behavior. We will use the Goodman line because it is simpler and conservative.

The Modified Goodman Diagram is the red line and is the Goodman line truncated by the Yield line.



Hamrock Section 7.10

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Drawing the Modified Goodman Diagram

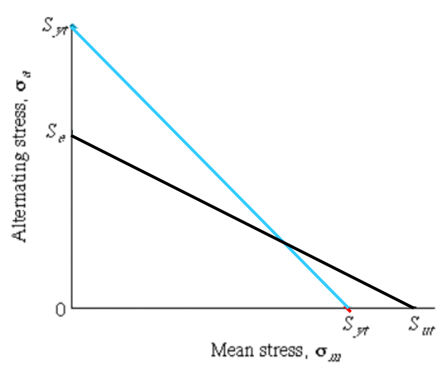
The diagram is based on material properties Sut, Sy, and Se.

As with the S-N curve, Se should be derated to reflect your part:

$S_e = k_f\, k_s\, k_r\, k_t\, k_m\, S_e'$

(My Part) (Test Specimen)

For torsional (shear) loading, use
Ssy = 0.577 Sy,
Sus = 0.67 Sut, and
Se for Torsion



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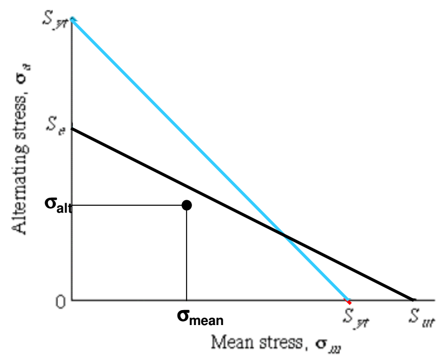
Drawing the Modified Goodman Diagram

Then plot your alternating and mean stress.

If your point is below the Mod Goodman line, the part should have unlimited life.



Note: This is very different from the "complete" Modified Goodman Diagram that Hamrock details on P. 178 – 179.

We will not use that version – it is pretty confusing.

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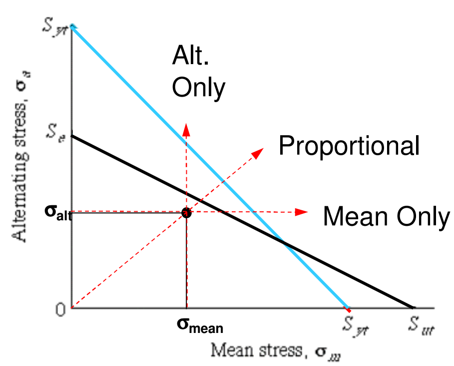
Factors of Safety

The Factor of Safety depends on how the stresses behave. They might:

1. Grow proportionately
2. Only grow in mean
3. Only grow in alternating

How they behave depends on the actual hardware and loading involved.



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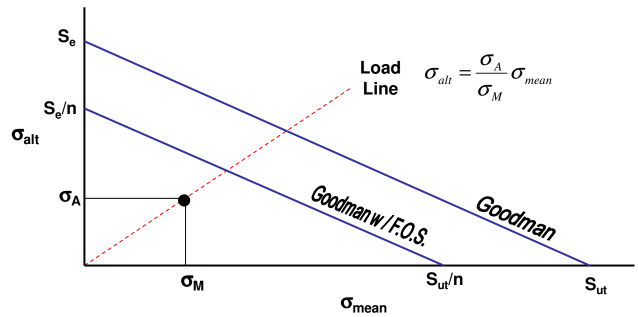
If σ_{alt} and σ_{mean} Increase Proportionately

To make the Goodman Line w/F.O.S. go through (σ_M, σ_A) :

$$\sigma_A = S_e\left(\frac{1}{n} - \frac{\sigma_M}{S_{ut}}\right)$$

$$\frac{\sigma_A}{S_e} = \frac{1}{n} - \frac{\sigma_M}{S_{ut}}$$

$$\boxed{\frac{1}{n} = \frac{\sigma_A}{S_e} + \frac{\sigma_M}{S_{ut}}}$$



Equations of the Goodman Line:

$$\sigma_{alt} = -\frac{S_e}{S_{ut}}\sigma_{mean} + S_e = S_e\left(-\frac{\sigma_{mean}}{S_{ut}} + 1\right) \quad or \quad \sigma_{alt} = S_e\left(1 - \frac{\sigma_{mean}}{S_{ut}}\right)$$

$$For\ a\ Factor\ of\ Safety\ of\ n: \quad \sigma_{alt} = S_e\left(\frac{1}{n} - \frac{\sigma_{mean}}{S_{ut}}\right)$$

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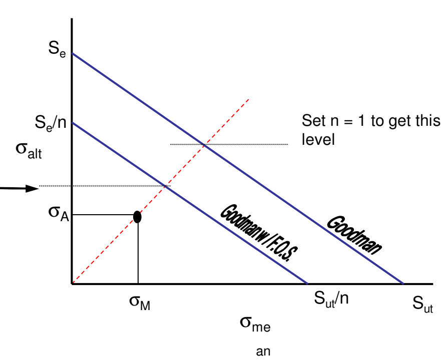
If σ_{alt} and σ_{mean} Increase Proportionately

$$\sigma_{alt_{Lim}} = \frac{S_e}{n}\frac{1}{\left(1 + \frac{\sigma_M}{\sigma_A}\frac{S_e}{S_{ut}}\right)}$$

Equation to find alternating stress when operating point hits the FOS line or Goodman line.



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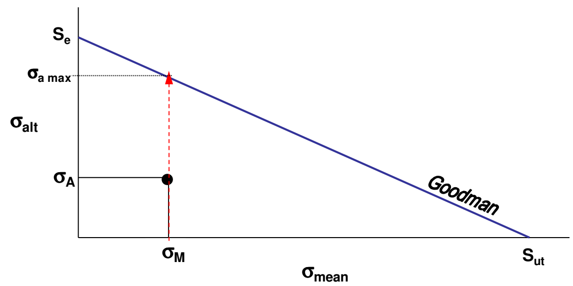
If Only σ_{alt} Increases

Equation of the Goodman Line:

$$\sigma_{alt} = S_e(1 - \frac{\sigma_{mean}}{S_{ut}})$$

$$\sigma_{a\max} = S_e(1 - \frac{\sigma_M}{S_{ut}})$$

Equation to find alternating stress when operating point hits the Goodman line.



$$n = \frac{\sigma_{a\max}}{\sigma_A}$$

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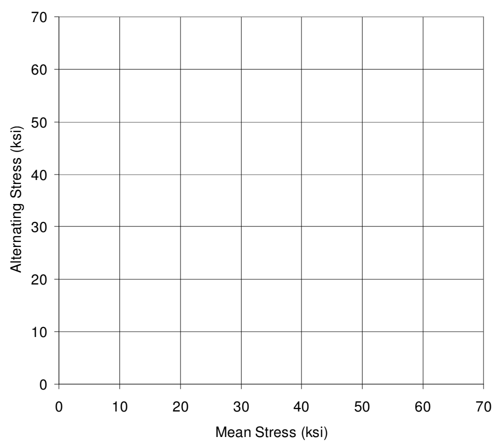
Fatigue Exercise

Given a bar of steel with these properties

Yield	40	ksi
UltimateTensile	65	ksi
Endurance	30	ksi

On a Goodman Diagram, predict fatigue for these loadings:

	Min	Max	Mean	Alt	
A.	0	36			ksi
B.	-27	37			ksi
C.			14	32	ksi



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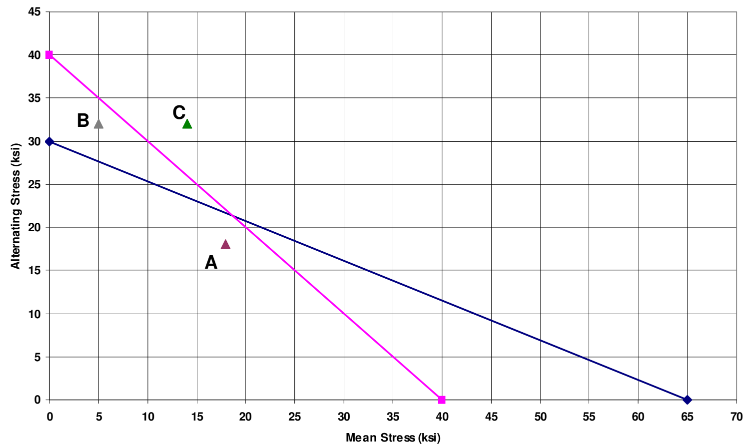
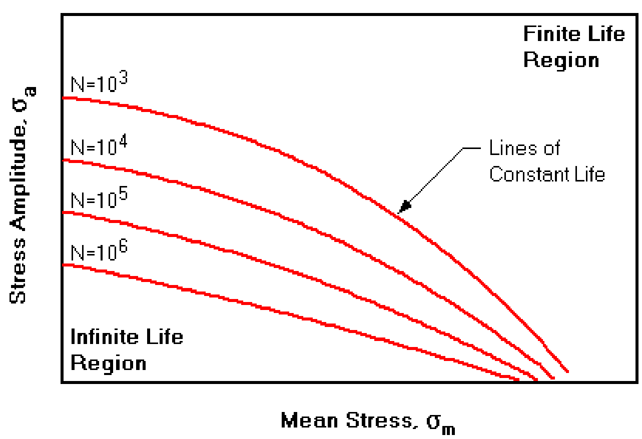
Fatigue Exercise

Yield	40	ksi
UltimateTensile	65	ksi
Endurance	30	ksi

	Min	Max	Mean	Alt
A.	0	36	18	18 ksi
B.	-27	37	5	32 ksi
C.	-18	46	14	32 ksi

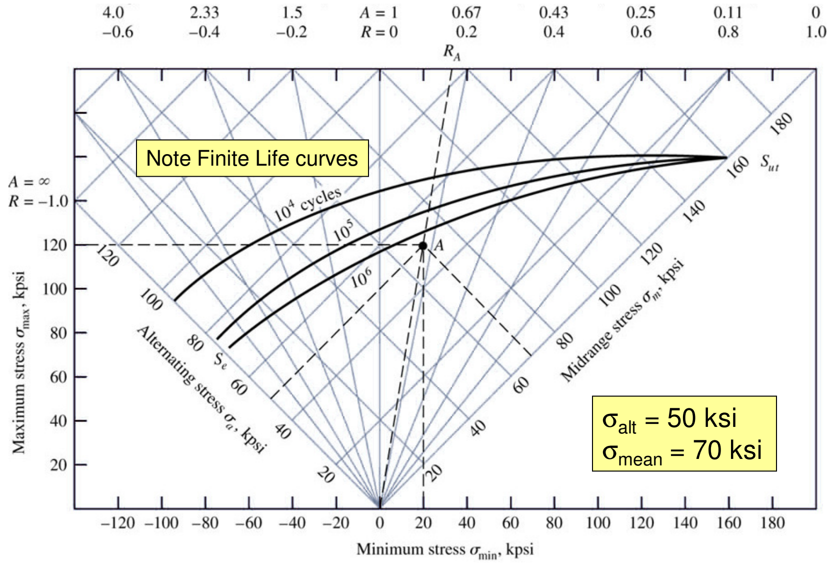


Fatigue Diagram for Finite Life



From EngRasp ETBX

Another Type of Fatigue Diagram



σ_{alt} = 50 ksi
σ_{mean} = 70 ksi

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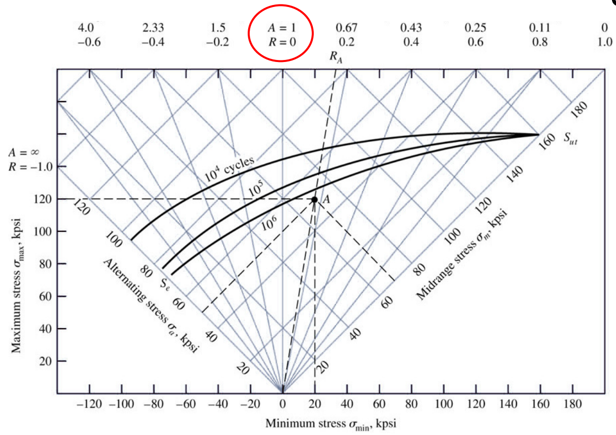
Stress Ratios for This Diagram



σ_{alt} = 50 ksi
σ_{mean} = 70 ksi

$$\textit{Stress ratio } R = \frac{\sigma_{min}}{\sigma_{max}}$$

$$\textit{Amplitude ratio } A = \frac{\sigma_{alt}}{\sigma_{mean}}$$

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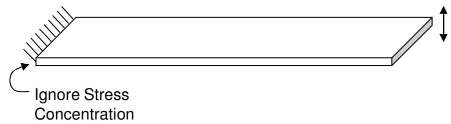
More Cantilever Beam



Details

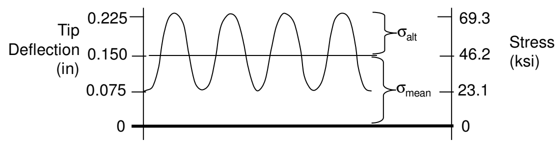
12 Gauge (0.1094" thick)
0.75 in. wide
4 in. long
High Strength Steel, with S_{ut} = 245 ksi
Machined finish
Room Temperature

$S_e = k_f k_s k_r S'_e$ = (0.63)(1)(0.87)(100) = 54.8 ksi

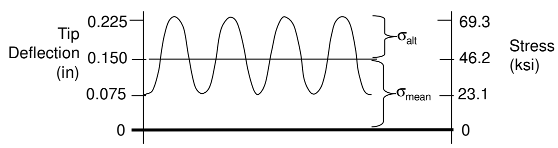
CASE 2: Tip is flexed between 0.075 in and 0.225 in. What is life for 95% survival?

By proportioning, the force now fluctuates between 8.631 lb and 3 x 8.631 = 25.893 lb.

Stresses go from +23.1 ksi to +69.3 ksi.

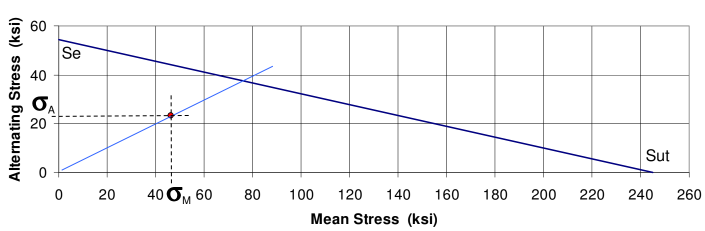


Cantilever Beam, contd.



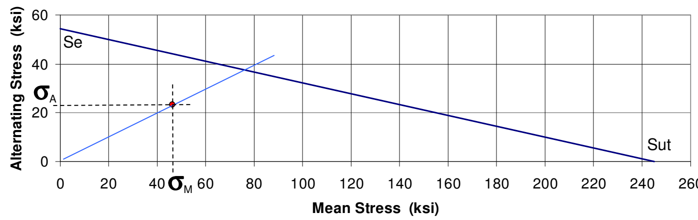
$$\sigma_{mean} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{69.3 + 23.1}{2} = 46.2\,ksi$$

$$\sigma_{alt} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{69.3 - 23.1}{2} = 23.1\,ksi$$



Cantilever Beam FOS

$\sigma_{mean} = 46.2\,ksi$
$\sigma_{alt} = 23.1\,ksi$



What is the Factor of Safety?

A. If both alternating and mean stresses increase proportionately:

$$\frac{1}{n} = \frac{\sigma_A}{S_e} + \frac{\sigma_M}{S_{ut}} = \frac{23.1}{54.8} + \frac{46.2}{245} = 0.422 + 0.189 = 0.611 \qquad n = \frac{1}{0.611} = 1.64$$

B. If only alternating stress increases:

$$\sigma_{a\max} = S_e(1 - \frac{\sigma_M}{S_{ut}}) = 54.8(1 - \frac{46.2}{245}) = 54.8(1 - 0.189) = 44.4\,ksi$$

$$n = \frac{\sigma_{a\max}}{\sigma_A} = \frac{44.4}{23.1} = 1.92$$

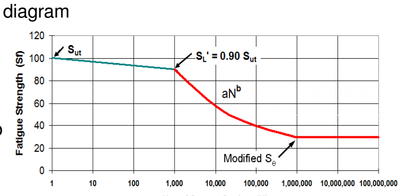
General Fatigue Analysis Procedure

A. You always need:
1. The material – Sut & maybe Sy
2. The type of Loading
   - Axial; Bending; or Shear

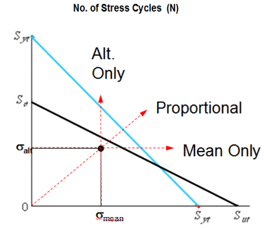
   } Use these to get Se'
3. The Max and Min Stress → Get σ_{mean} & σ_{alt}

B. If you have Reliability, Surface, or Size information, then you must adjust Se' to represent your part. Se = Kf * Ks * Kr * Se'.

C. If σ_{mean} = 0, then it's fully reversing and we use an S/N diagram
1. If σ_{alt} < Se, life is infinite and you're done.
2. Otherwise, calculate the 1000 Cycle value SL, and draw the S/N plot.
3. Calculate a = SL^2/Se, and b = -1/3 Log10 (SL / Se)
4. Then you can calculate S = aN^b or N = (σ_{alt}/ a)^1/b



D. If σ_{mean} ≠ 0, then it's fluctuating and we use a Goodman diagram
1. Draw the Goodman line between Se and Sut.
2. Add the Yield Line between Sy and Sy.
3. Plot the operating point (σ_{mean}, σ_{alt}).
4. Depending on how the stress might increase, calculate the FOS.



Impact

The energy of the falling block is transferred to stored energy in the spring:

$$W\,(h+\delta_{\max}) = \frac{1}{2}(k\,\delta_{\max})\,\delta_{\max}$$

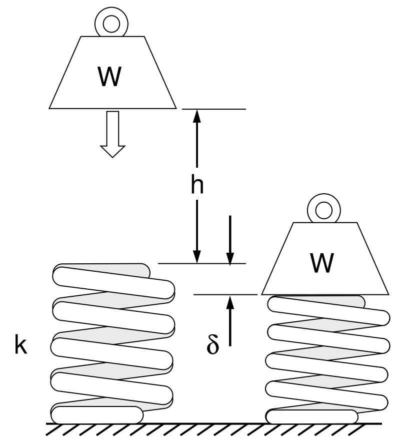
Static displacement of Weight <u>gently placed</u> on Spring of stiffness, k

$$\delta_{st} = \frac{W}{k}$$

Impact factor is

$$I_m = \frac{\delta_{\max}}{\delta_{static}} = \frac{P_{\max}}{W} = 1+\sqrt{1+\frac{2h}{\delta_{st}}}$$

Effect is dependent on Spring Stiffness. A soft spring means large static deflection, which means smaller Impact factor.



Impact significantly increases forces!

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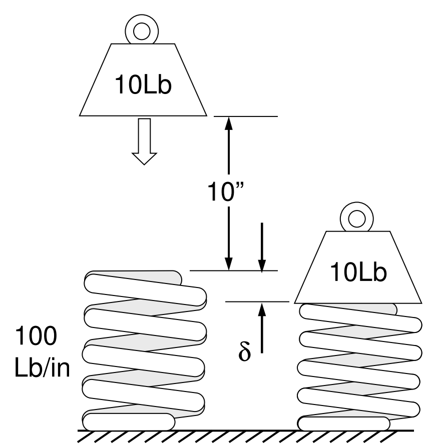
Impact

For this situation, what is:

1. The static deflection
2. The Impact factor
3. The max deflection
4. The max force



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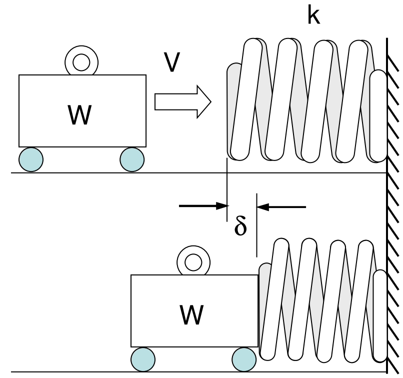
Impact

For the case of a weight sliding horizontally with a velocity, V, and hitting the spring

$$\delta_{max} = \sqrt{\frac{WV^2}{gk}}$$

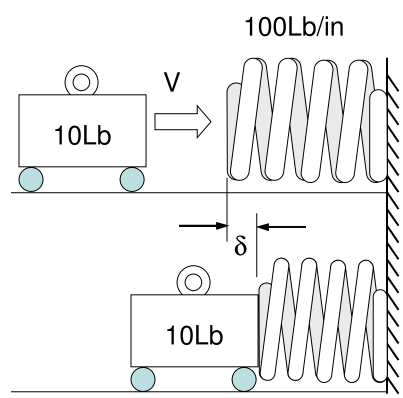
where g is the gravitational constant, 386 in/s^2 or 9.8 m/s^2. Recognizing that W/k = δ_{st}

$$\delta_{max} = \sqrt{\frac{\delta_{st} V^2}{g}}$$



Impact

For this setup, what velocity gives the same max force as the falling weight just did?

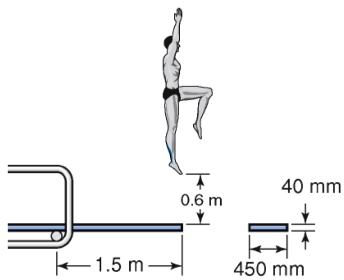


Impact

Read Hamrock's Example 7.11 of a diver landing on a diving board.

Note that the spring here is a beam, whose stiffness is calculated as Force/Deflection.

Also note that he deflects the end of the board 92mm [3.62"], and sees a max force of 13.5kN [3035lb]!



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