MEEG3311 Machine Design

Lecture 5: Fully-Reversing
Fatigue and the S-N Diagram

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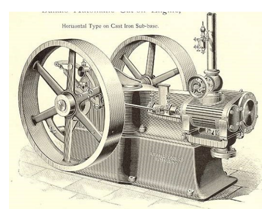
Discovering Fatigue

With the Industrial Revolution came the steam engine (~1760-1775) and then the locomotive (~1830). This began an age of dynamic loading that had never been seen before.



Even though equipment was designed with stresses below Yield, failures were still occurring. This was very publicly visible in the failure of bridges and of train wheel axles.

Hamrock
Section 7.2

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The Versailles Train Crash

One of the worst rail disasters of the 19th century occurred in May 1842 near Versailles, France. Following celebrations at the Palace of Versailles, a train returning to Paris crashed after the leading locomotive broke an axle. The carriages behind piled into the wrecked engines and caught fire. At least 55 passengers were trapped in the carriages and died.

From Wikipedia.

Examination of several broken axles from British railway vehicles showed that they had failed by brittle cracking across their diameters (fatigue). The problem of broken axles was widespread on all railways at the time and continued to occur for many years before engineers developed better axle designs, mostly resulting from improved testing of axles.

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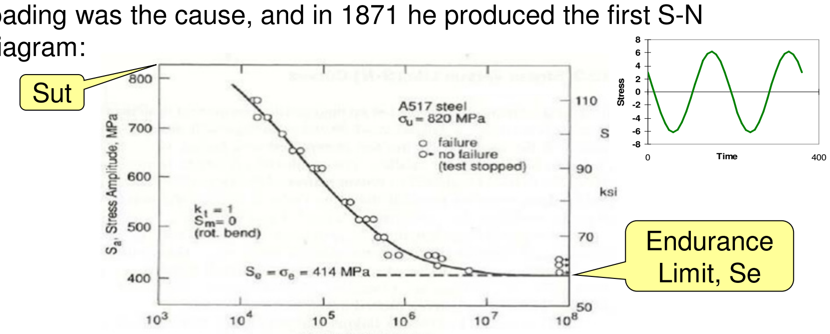
The S-N Diagram

A German engineer, August Wöhler, discovered that reversed loading was the cause, and in 1871 he produced the first S-N diagram:



The diagram shows the relationship between the stress amplitude of reversing loading and the number of cycles to failure.

Note that some materials have an Endurance Limit – a stress level below which the material can take unlimited cycles.

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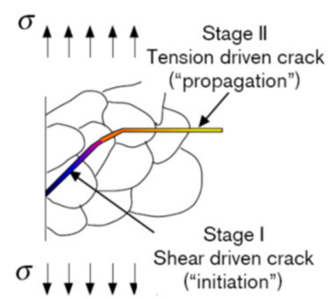
What's up with fatigue?

Stage I: Initiation with Small Cracks
- **Shear** driven
- Interact with **microstructure**
- Mostly analyzed by **continuum mechanics** approaches

Stage II: Propagation with Large Cracks
- **Tension** driven
- Fairly **insensitive to microstructure**
- Mostly analyzed by **fracture mechanics** models



From Anders Ekberg, Chalmers U.

Microcracks act as localized stress concentrations to exceed Sy and support local plastic yielding. As the crack grows, the section is reduced, stresses increase, and propagation accelerates until the part fractures – usually rapidly and unexpectedly.

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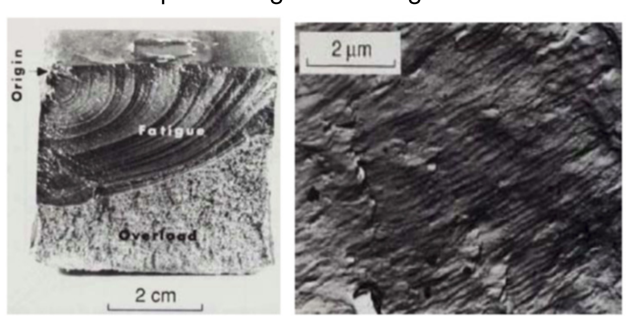
Crack Growth and Failure

The progression of fatigue cracking is sometimes visible as "beach marks" with each line representing the crack growth due to one load cycle:



From Majid Mirzaei, TMU.

The left photo shows that the crack grew and the remaining section reduced until P/A exceeded yield and the remainder of the part broke by yielding.

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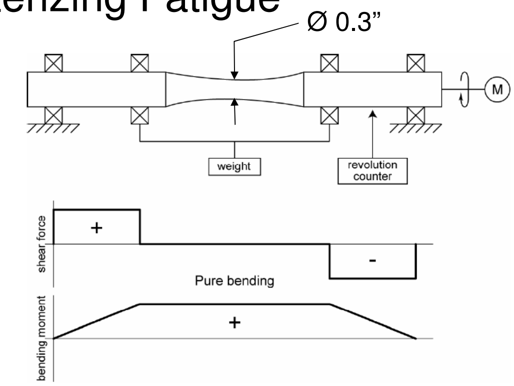
Characterizing Fatigue

Fatigue is very complex, so has been largely driven by test data from rotating beam tests.



Although moment is uniform, the narrowing specimen has max bending stress at the middle.

- Standard R.R.Moore test specimen is 0.3 in. diameter at thinnest section
- Specimen is polished
- Each revolution gives one fully reversing cycle
- At 3600RPM = 60 Hz, get 216,000 cycles per hour, 5.2 million cycles per day

Hamrock Section 7.5

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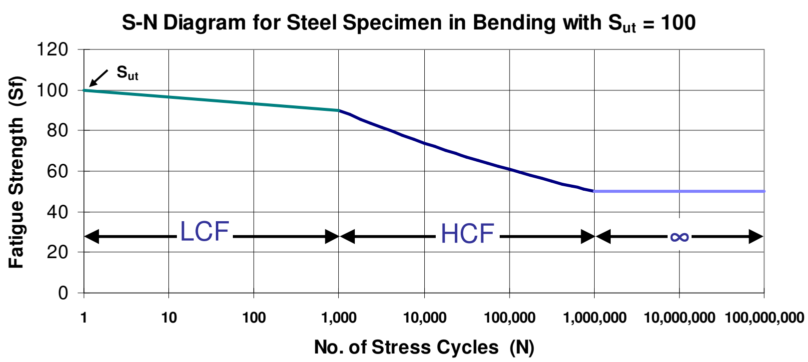
The S-N Diagram

The S-N diagram has three regions for materials with Endurance Limits: Low Cycle, High Cycle, & Infinite Life



The Low Cycle Fatigue (LCF) region is from 1 to 1000 cycles.
The High Cycle Fatigue (HCF) region is from 1000 to 1 million cycles.
The Infinite Life region is above 1 million cycles.

Some consider the Infinite Life region to begin at 10 million cycles.

Hamrock Section 7.6

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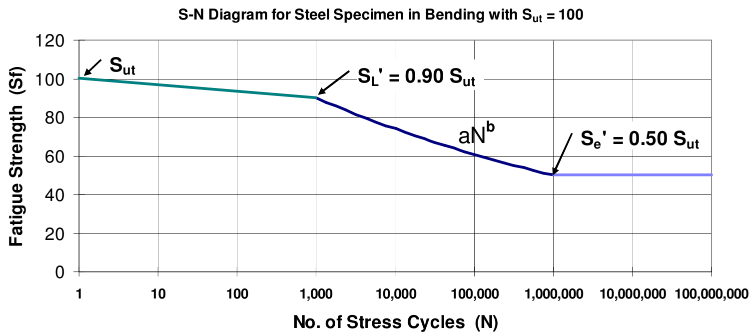
Constructing our S-N Diagram

1. Start with Sut at 1 cycle (Because it can endure 1 cycle to Sut)
2. Plot the LCF value SL' at 1000 cycles
3. Plot the Endurance Strength Se' at 1 million cycles
4. Draw the first and third segments as straight lines
5. Connect SL' and Se' with a curve = aN^b



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SL' and Se' Depend on the Type of Loading

Find the fatigue strength at the 1000 cycle Low Cycle Fatigue point, SL'

For steel, use:

Bending:	SL' = 0.90 Sut	
Axial:	SL' = 0.75 Sut	(Eqn. 7.7)
Torsion:	SL' = 0.72 Sut	

Find the "raw" endurance strength, Se'

For steel, use (from Eqn. 7.6):

Bending:	Se' = 0.50 Sut	up to a max Se' of 100 ksi (690 Mpa)
Axial:	Se' = 0.45 Sut	up to a max Se' of 90 ksi (621 Mpa)
Torsion:	Se' = 0.29 Sut	up to a max Se' of 58 ksi (400 Mpa)

This is Se' for a polished 0.3" diameter bar.

This is because Se' tops out when Sut ~200 ksi (1380 MPa). See Fig. 7.8

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Plotting the 1000 to 1 million Curve

Connect (1000, SL') and (10^6, Se) with the curve

I use this.

$$S_f = aN^b \; or \; S_f = 10^C N^b$$

Hamrock uses this.

where

$$a = \frac{(S_L^{'})^2}{S_e{'}}, \quad C = \log_{10}\frac{(S_L^{'})^2}{S_e{'}}, \; and \;\; b = -\tfrac{1}{3}\log_{10}\left(\frac{S_L^{'}}{S_e{'}}\right)$$

and N = Number of cycles.

If you have a or C and b , and want to know the life for an alternating stress σ_{alt} between Se' and SL', compute

$$N = \left(\frac{\sigma_{alt}}{a}\right)^{1/b} \; or \; \left(\frac{\sigma_{alt}}{10^C}\right)^{1/b}$$

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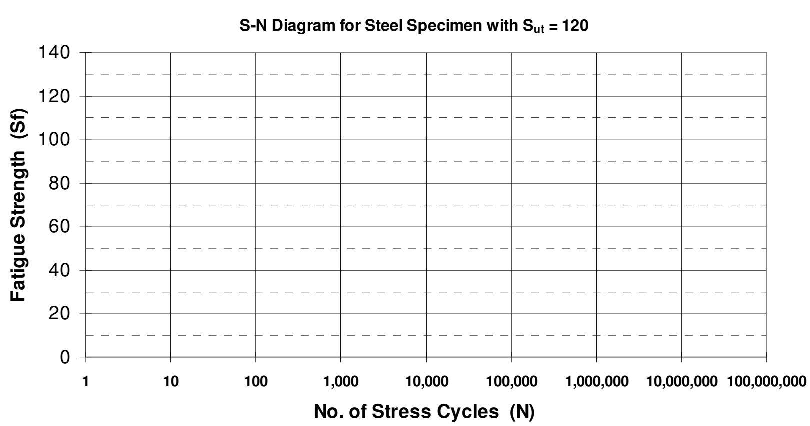
S-N Exercise

Plot the S-N diagram for a steel with Sut = 120ksi under fully reversing Axial loading. Find a & b, and the Sf at 100,000 cycles.



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Reality Sets In

If your actual product doesn't happen to be 0.3" diameter polished bars, then you must make some modifications to the Endurance Limit to make this apply to your part.

The modification (Marin) factors are:

Main 3:
- k_f Surface effect – if not polished
- k_s Size effect – if not $\leq$ 0.3" diameter
- k_r Reliability effect – if other than 50% survival

- k_t Temperature effect – if not at Room Temperature
- k_m Miscellaneous – Mat'l processing, Residual stress, Coatings, Corrosion

Then $S_e = k_f\, k_s\, k_r\, k_t\, k_m\, S_e'$

(My Part) (Test Specimen)

Another factor is K_f, the fatigue stress concentration factor.

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Surface Finish Factor

Equation 7.19 gives

$$k_f = eS_{ut}^{f}$$

where Sut is the material's Ultimate Tensile Strength, and e & f are coefficients defined in Table 7.3, shown here.

Manufacturing process	Factor e		Exponent f
	MPa	ksi	
Grinding	1.58	1.34	-0.085
Machining or cold drawing	4.51	2.70	-0.265
Hot rolling	57.7	14.4	-0.718
As forged	272.0	39.9	-0.995

Note 1: e is NOT the "exponential".
Note 2: Sut value is entered as MPa or ksi, NOT Pa or psi. In other words, if Sut = 100ksi, you use the value 100.

Exercise: What is k_f for AISI 1080 steel that has a machined surface?

Why do we need to correct for surface finish?

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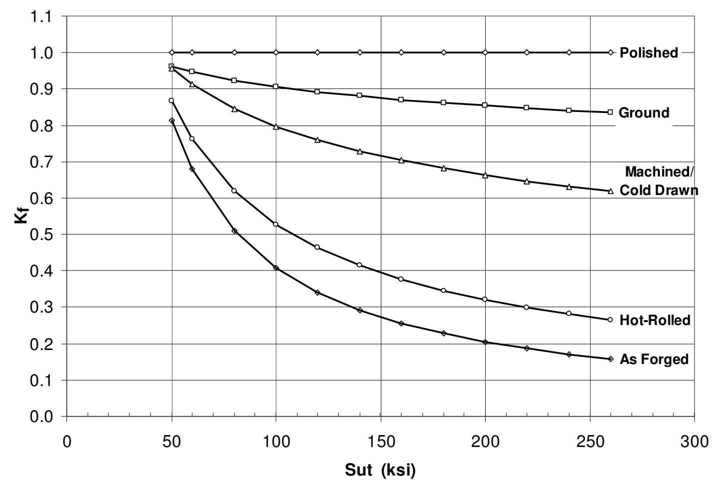
Surface Finish Factor Plot

Note that Hamrock's Fig. 7.11a is distorted, especially for the Ground and As Forged curves. It should look like this:



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Size Factor

Equation 7.20 gives:

$$k_s = 0.869 d^{-0.112}$$ For d in inches from 0.3 to 10"

$$k_s = 1$$ For d ≤ 0.3" or ≤ 8mm

$$k_s = 1.248 d^{-0.112}$$ For d in mm from 8 to 250mm

for Bending or Torsional Loading.

If the loading is Axial, k_s = 1 for all sizes.

Exercise: What is k_s for a 1 inch diameter steel bar?

Why do we need to correct for part size?

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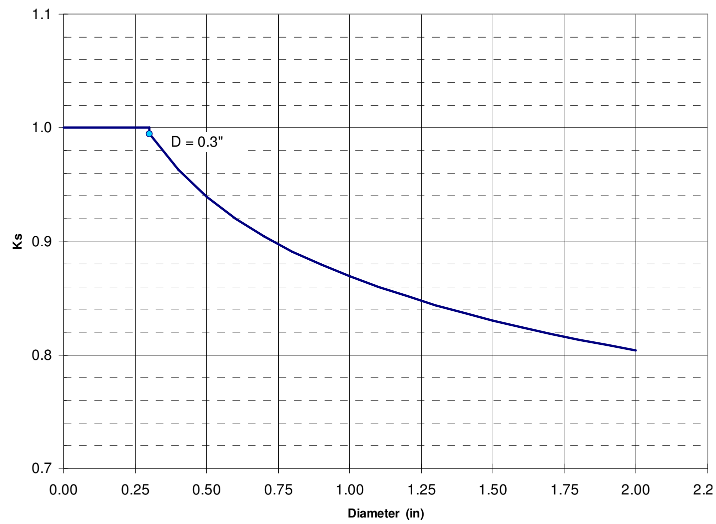
Size Factor

Here is what the Size Factor looks like:



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Reliability Factor

A standard Endurance Strength value is based on 50% survival.

If you think your customers would like a better durability than that, you need to correct for it.

Probability of Survival, %	Reliability Factor, k_r
50	1.00
90	0.90
95	0.87
99	0.82
99.9	0.75
99.99	0.70

Exercise: What is k_r for a product where you only want 1 in 10,000 to fail?

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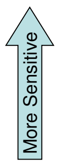
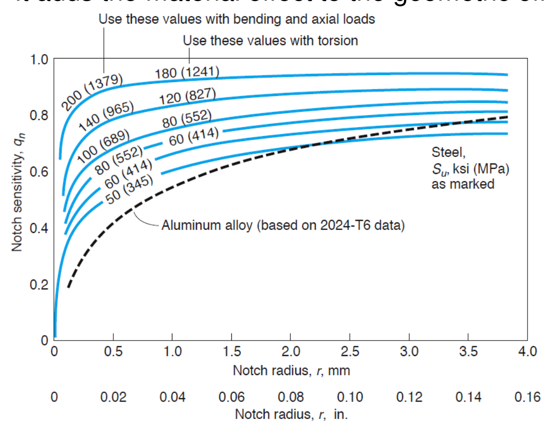
Stress Concentration with Notch Sensitivity

For fatigue, stress concentration is a function of both geometry AND the material and type of loading. That's where Notch Sensitivity comes in. It adds the material effect to the geometric effect.

$K_f = Kc$



$K_f = 1$



Info Only

$K_f = 1+(Kc-1)q$, so $q=0$ means $K_f=1$; $q=1$ means $K_f=Kc$

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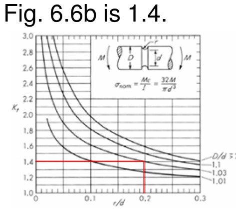
Stress Concentration with Notch Sensitivity

Stress concentration for fatigue is a three-step process:
1. Calculate the Kc as you would for a static case.
2. Use a notch sensitivity table for your material, loading type, and notch radius.
3. Compute $K_f = 1+(Kc-1)q$

Info Only

Then you can either divide Se by K_f or multiply your actual alternating stress σ_{alt} by K_f.

Exercise: What is K_f for an AISI 4130 Annealed Steel shaft loaded in bending with a 1mm notch radius, if the Kc read from Fig. 6.6b is 1.4.



Kf is an "awareness" topic and will not be in homework or numerically on an exam.

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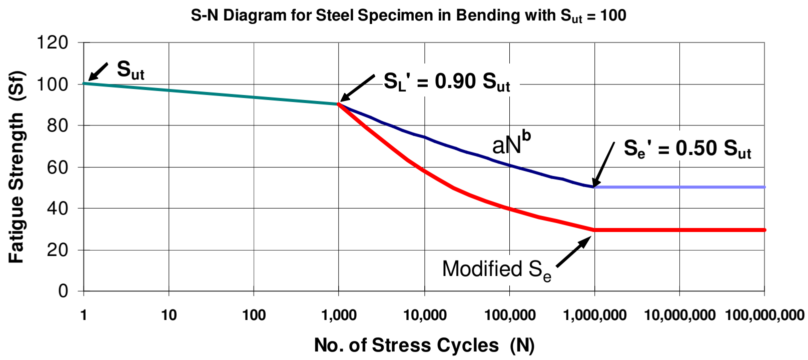
Summary

The effect of all of this modification is to correct the Endurance Limit to properly represent your part. The corrected S-N diagram looks like this:

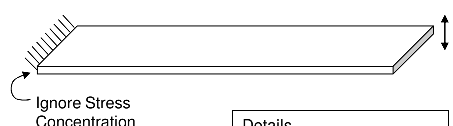


The red lines show the changes from the test specimen values to ones that represent your part.

FATIGUE EXAMPLE

Cantilever Beam

CASE 1: Tip is flexed ±0.075 in. What is life for 95% survival?



<u>Details</u>
12 Gauge (0.1094" thick)
0.75 in. wide
4 in. long
High Strength Steel, with S_{ut} = 245 ksi
Machined finish
Room Temperature

$$y_{max} = \frac{Fl^3}{3EI} = \frac{Fl^3 12}{3Ebh^3}$$

$$0.075 = \frac{F(4)^3(12)}{(3)(30\times10^6)(0.75)(0.1094)^3}$$

$$F = 8.631 lb$$

$$Stiffness\ k = \frac{F}{\delta} = \frac{8.631}{0.075} = 115.1 lb/in$$

$$M = Fl = (8.631)(4) = 34.52\ lb.in.$$

$$\sigma = \frac{Mc}{I} = \frac{(34.52)(0.1094/2)(12)}{(0.75)(0.1094)^3} = \pm 23{,}076\ psi$$

Since S_{ut} = 245 ksi > 200 ksi, we set S_e' not equal to S_{ut}/2 = 122.5 ksi, but limit it to the maximum of 100 ksi.

FATIGUE EXAMPLE, cont'd

Surface Factor

From Table 7.3: e = 2.70 ksi, f = -0.265

$$k_f = 2.7(245)^{-0.265} = (2.7)(0.233) = 0.628$$

Note: The "e" here is not the exponential, but a variable from Table 7.3. It just happens to be almost equal to e = 2.7183 in this example.

Size Factor

Area = (0.75)(0.1094) = 0.08205 in^2

Area loaded > 95% stress is 5% of Area = (0.05)(0.08205) = 0.0041 in^2

Equivalent diameter from Eqn. 7.21:

$$d = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{0.0041}{0.0766}} = \sqrt{0.05356} = 0.2314\, in.$$

Because this diameter is less than 0.3 in, we don't use $k_s = 0.869 d^{-0.112}$, but just set k_s = 1.

Reliability

From Table 7.4, k_r = 0.87

Derated Endurance Strength

$S_e = k_f k_s k_r S'_e$ = (0.63)(1)(0.87)(100) = 54.8 ksi

σ_{alt} = 23.1 ksi < 54.8 ksi Endurance Strength, ∴ Life is ∞ .

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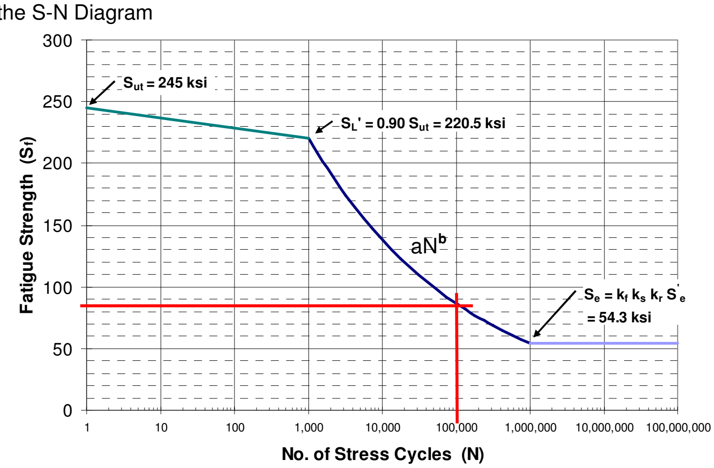
FATIGUE EXAMPLE, cont'd

Question: How big could the stress concentration at the attachment be and still have 100,000 cycles of life?

Draw the S-N Diagram



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FATIGUE EXAMPLE, cont'd

Calculate the stress amplitude for a life of 100,000 cycles.

$$a = \frac{S_L^2}{S_e} = \frac{220.5^2}{54.8} = 887.2\,ksi$$

$$b = -\frac{1}{3}\log_{10}\left(\frac{S_L}{S_e}\right) = -\frac{1}{3}\log_{10}\left(\frac{220.5}{54.8}\right) = -\frac{1}{3}\log_{10}(4.024) = -\frac{0.605}{3} = -0.202$$

$$S_f = aN^b = 887.2(100{,}000)^{-0.202} = (887.2)(0.098)$$

$$S_f = 87.16\,ksi$$

So the stress concentration would have to be $\frac{87.16}{23.1} = 3.77$.

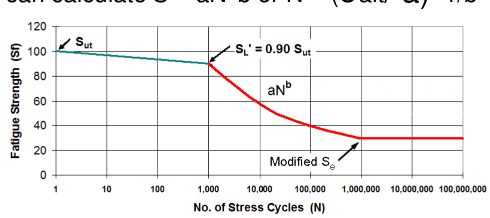
Fully Reversing Fatigue Procedure

A. You always need:
1. The material – Sut
2. The type of Loading
   - Axial; Bending; or Shear

} Use these to get Se'

B. If you have Reliability, Surface, or Size information, then you must adjust Se' to represent your part. Se = Kf * Ks * Kr * Se'.

C. If the stress is fully reversing, we use an S/N diagram
1. If σ_{alt} < Se, life is infinite and you're done.
2. Otherwise, calculate the 1000 Cycle value SL, and draw the S/N plot.
3. Calculate a = SL^2/Se, and b = -1/3 Log10 (SL / Se)
4. Then you can calculate S = aN^b or N = (σalt/ a)^1/b

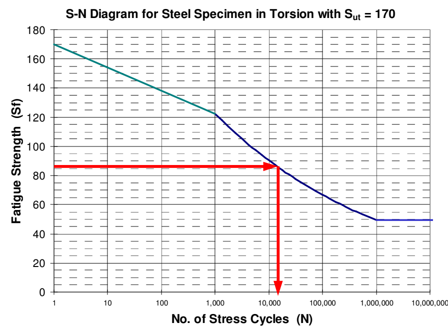


Cumulative Fatigue Damage: Miner's Rule

- In practice, the fatigue loading on structures is rarely constant amplitude.
- To estimate the effect of operating at a variety of stress levels, you can use the linear damage rule, aka Miner's rule.
- You take the number of cycles at each stress level and divide that by the life if only run at that stress level to get a damage fraction.
- You add up all of the damage fractions and see if the total exceeds one.

Example:
A stress amplitude of 87ksi has a life of 13,300 cycles.

Running 5,000 cycles at 87ksi is a damage fraction of 5/13.3 = 0.38



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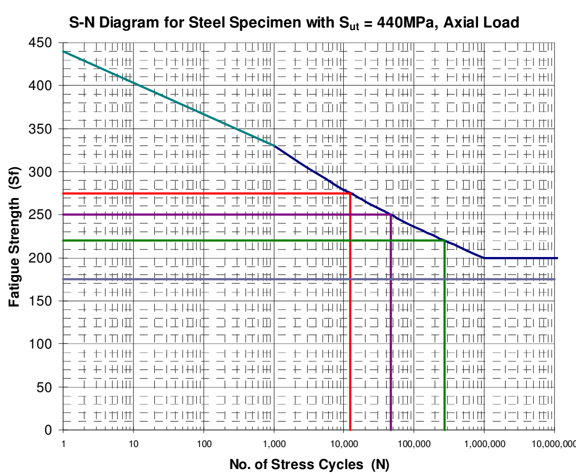
Example 7.7: Cumulative Damage

Loading	20% @ 175MPa	30% @ 220MPa	40% @ 250MPa	10% @ 275MPa
Life @ Load	∞	268,564	46,048	12,367



$$\frac{0.2L}{\infty}+\frac{0.3L}{268{,}564}+\frac{0.4L}{46{,}048}+\frac{0.1L}{12{,}367}=1$$

$$\frac{0.2}{\infty}+\frac{0.3}{268{,}564}+\frac{0.4}{46{,}048}+\frac{0.1}{12{,}367}=\frac{1}{L}$$

$$0+1.117\text{E-}06+8.687\text{E-}06+8.086\text{E-}06=\frac{1}{\text{L}}$$

$$1.78899\text{E-}05=\frac{1}{L}$$

$$L=55{,}897 \text{ Cycles}$$

Mat'l: Steel with Axial Loading
Sut=440MPa.
SL'=75%Sut =330MPa.
Se'=45%Sut = 200MPa

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Avoiding Fatigue Failure

A checklist of good practices.

1. Control stress risers like holes, sharp corners, grooves, threads, keyways, and stamped markings.
2. Control surface finish, including scratches in critical areas.
3. Avoid corrosion and exposure to embrittling gases.
4. Be very careful with welds – spot welds and fillet welds.
5. Higher temperatures usually reduce fatigue strength.
6. Manage residual stresses from fabrication operations, including welding.
7. Make critical areas inspectable for cracks.
8. Design in stress margin.
9. Measure real load environment and test parts to determine actual fatigue life.
10. Design in redundant load paths.

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Avoiding Fatigue Failure

One more thing – don't put square windows in airplanes…

Metal fatigue became apparent to aircraft engineers in 1954 after three de Havilland Comet passenger jets had broken up in mid-air and crashed within a single year.

The sharp corners around the plane's window openings acted as initiation sites for cracks. The skin of the aircraft was also too thin, and cracks from manufacturing stresses were present at the corners. All aircraft windows were immediately redesigned with rounded corners.

One last thing – this lecture applies to Ferrous alloys and to Titanium. It does <u>not</u> apply to Aluminum alloys.

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