# MEEG3311 Machine Design

Lecture 4: Stress Concentrations; Static Failure

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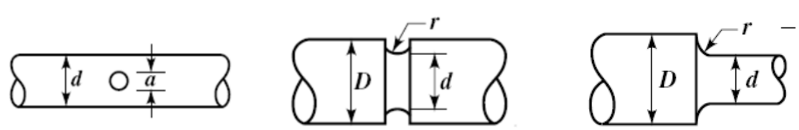
## Stress Concentration

We saw that in a curved beam, the stress was distorted from the uniform distribution of a straight beam, and higher at the inside radius.

A similar distortion happens near holes, notches, and changes in section:




The effect varies with the loading type: tension, bending, or torsion.

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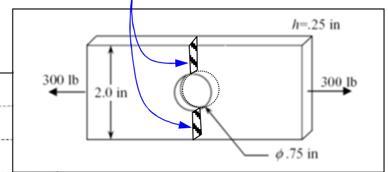
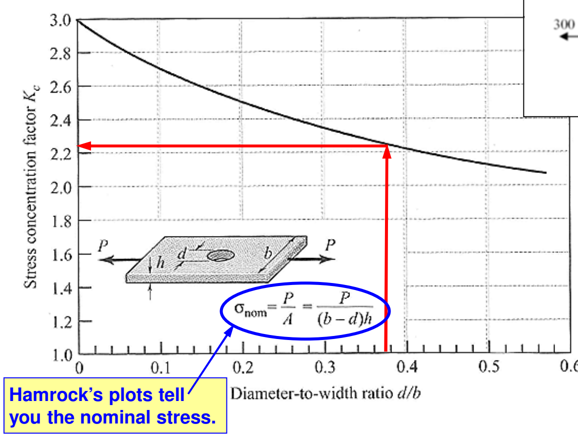
# Stress Concentration

"Stress Concentration Factors" relate the maximum stress to the average stress in the minimum cross section of the part.




$$d/b = \frac{0.75}{2.0} = 0.375$$

$$\sigma_{nom} = \frac{300}{(2.0-0.75)0.25} = \frac{300}{0.3125} = 960\,psi$$

$$\sigma_{\max} = K_c \times \sigma_{nom} = 2.25 \times 960 = 2160\,psi$$

Hamrock Fig. 6.2a

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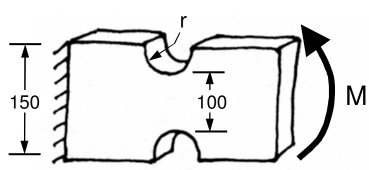
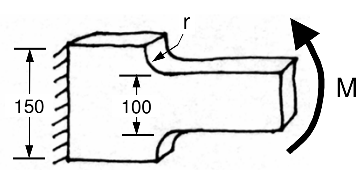
# Which has the bigger $K_c$?




| | |
|---|---|
| $Use\ Fig.:$ ________ | $Use\ Fig.:$ ________ |
| $\frac{H}{h} =$ | $\frac{H}{h} =$ |
| $r =$ | $r =$ |
| $\frac{r}{h} =$ | $\frac{r}{h} =$ |
| $K_c =$ | $K_c =$ |

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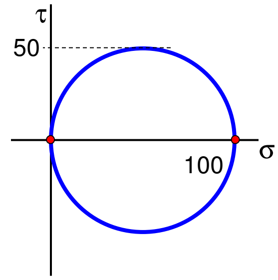
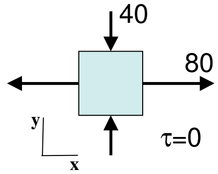
# Predicting Failure

We are given some material that is tensile tested and yields at $S_y$=100ksi.

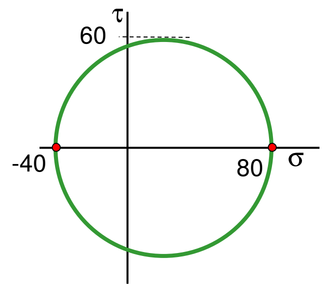
It has a Mohr's circle that looks like this:




We want to use this material in an application where it sees this loading:




It has a Mohr's circle that looks like this:




The Max Principal stress is less than 100ksi, but will it yield?

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# Failure Theory

Failure Theory addresses how to translate a real, multi-axial state of stress into something that can be compared with a simple uniaxial (tensile) test result.

For <u>ductile</u> materials, there are two prevailing theories:

1. Max Shear Stress Theory (MSST) or Tresca Theory
2. Distortion Energy Theory (DET) or von Mises criterion.

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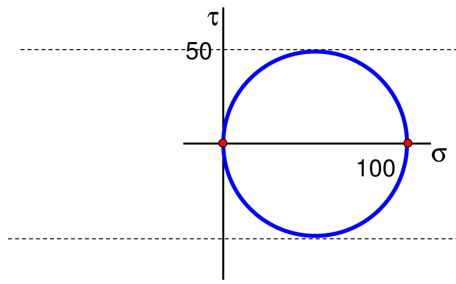
# Max Shear Stress

MSST says "Yield occurs whenever the maximum shear stress in an element exceeds the max shear at yield in a tensile test"

In other words, the Mohr's circle lies within the shear bounds (upper and lower) of the tensile test.

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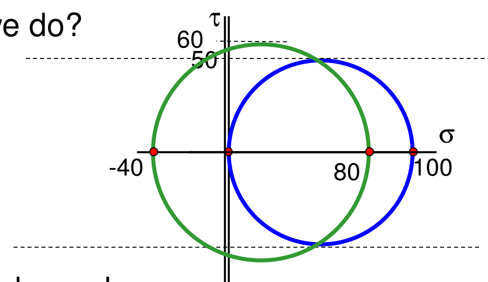
# Max Shear Stress

How did we do?




Doesn't look good.

MSST predicts yielding when $\sigma_1 - \sigma_3 \geq Sy$. (~Eqn. 6.8)

$\sigma_1$ is Max principal and $\sigma_3$ is Min principal.

In our case, $\sigma_1 - \sigma_3 = [80 - (-40)] = 120 > Sy$ of 100.

Another way to say is that $\tau_{max} \leq Sy / 2$

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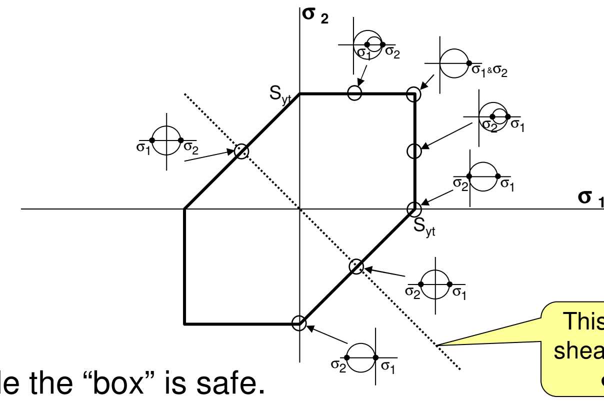
## Max Shear Stress

Here is a representation of MSST in terms of limits to Max and Min principal stresses:



Inside the "box" is safe.

This is the pure shear line, where $\sigma_1 = -\sigma_2$.

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## Distortion Energy

DET observes that materials can take enormous hydrostatic pressures (e.g., rocks deep in the earth that see pressures well above their compressive strength) and not fracture – suggesting that it must be distortion that causes failure.

DET essentially computes the total strain energy and subtracts the volume change energy to get the distortion energy.

In terms of principal stresses:

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$$

where $\sigma_e$ is the von Mises equivalent stress.

Note that for hydrostatic loading, $\sigma_1=\sigma_2=\sigma_3$, and $\sigma_e=0$!

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## Distortion Energy

In the more common case of biaxial principal stresses ($\sigma_3$=0), this reduces to:

$$\sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

In our example, this is:

$$\begin{aligned}\sigma_e &= \sqrt{80^2 - (80)(-40) + (-40)^2} \\ &= \sqrt{6400 + 3200 + 1600} \\ &= \sqrt{11{,}200} = 105.8 ksi\end{aligned}$$

This exceeds Sy = 100ksi, so DET also predicts yielding.

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## Distortion Energy - Simplified

If you have only direct biaxial stress ($\sigma_z$=0), you can calculate DET directly from $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ without getting the principal stresses:

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

And, if you have only bending and torsion ($\sigma_y$=0), this reduces to:

$$\sigma_e = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

(Hamrock buries this in Ex. 7.4 on page 170.)

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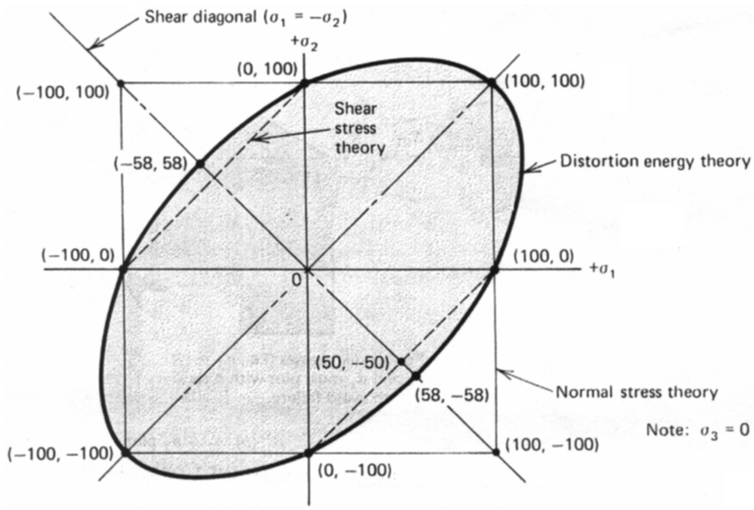
# The Failure Theories, overlaid




MSST is a little more conservative than DET.

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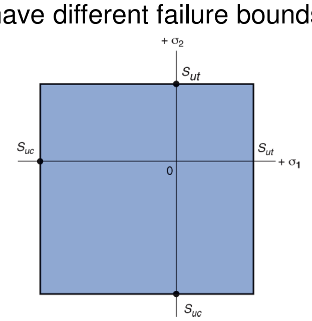
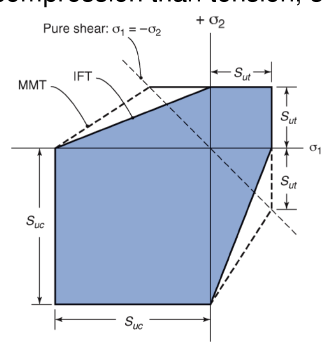
# Brittle Theories

Brittle materials can take more compression than tension, so have different failure bounds.




Max Normal Stress Theory




Internal Friction Theory & Modified Mohr Theory

The Brittle theories use Ultimate Strengths because they fracture rather than yield, and both Compressive and Tensile strengths because they are different.

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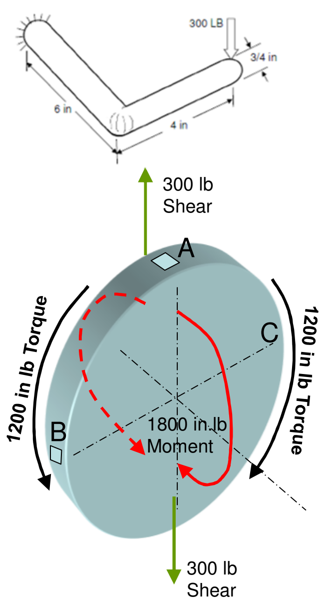
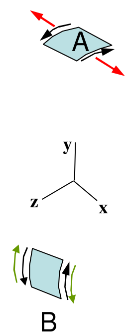



## Remember the Crank Arm ?

At "A": Bending M = 1800 in.lb.
Torsion T = 1200 in.lb.




$$\sigma_x = \frac{Mc}{I} = \frac{(1800)(0.375)}{\pi(0.75)^4/64} = \frac{675}{0.01553} = 43{,}460\ psi$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{(1200)(0.375)}{\pi(0.75)^4/32} = \frac{450}{0.03106} = 14{,}487\ psi$$

$$\tau_{BENDING} = \frac{4V}{3A} = \frac{(4)(300)}{3\pi(0.75)^2/4} = \frac{1200}{1.325} = 905\ psi$$

$$\tau_{TOTAL} = \tau_{xz} - \tau_{BENDING} = 13{,}582\ psi$$

$$\rightarrow Pure\ Shear$$

At "B": Torsion T = 1200 in.lb.
Shear V = 300 lb

Note that at location "C", the two shears would ADD to 15,392 psi.

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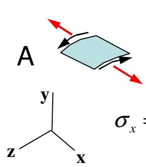
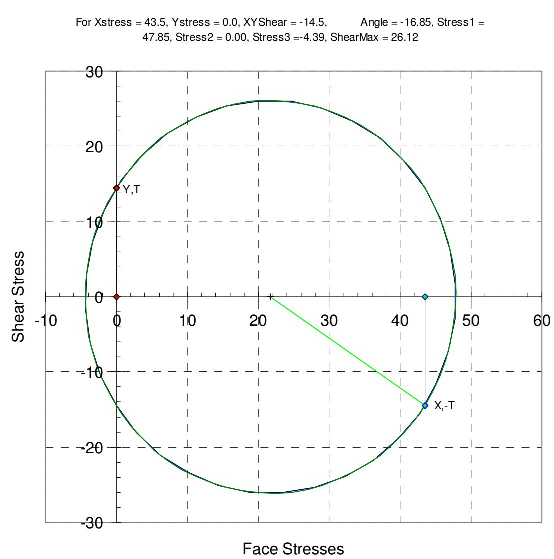
## Stresses in a Crank Arm

The stresses at A occur <u>at the same point</u>, so can use Mohr.




$$\sigma_x = \frac{Mc}{I} = 43{,}460\ psi$$

$$\tau_{xz} = \frac{Tr}{J} = 14{,}487\ psi$$




The Mohr circle gives:
$\sigma_1$ = 47.846 ksi
$\sigma_2$ = 0.000 ksi
$\sigma_3$ = -4.386 ksi
$\tau_{Max}$ = 26.116 ksi

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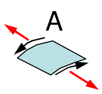
## What are the DET stresses at "A" on the Crank Arm?

At "A": Bending M = 1800 in.lb.
Torsion T = 1200 in.lb.




$$\sigma_x = \frac{Mc}{I} = 43{,}460\ psi$$
$$\tau_{xz} = \frac{Tr}{J} = 14{,}487\ psi$$

The Mohr circle for "A" gave:
$\sigma_1$ = 47.846 ksi
$\sigma_2$ = 0.000 ksi
$\sigma_3$ = -4.386 ksi
$\tau_{Max}$ = 26.116 ksi

Using the Principal stresses:

$$\sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2} = \sqrt{47.8^2 - (47.8)(-4.39) + (-4.39)^2}$$
$$= \sqrt{2285 + 210 + 19} = \sqrt{2518} = 50.18 ksi$$

Using the Direct stresses:

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} = \sqrt{43.46^2 + 0 - 0 + 3(14.487)^2}$$
$$= \sqrt{1888.8 + 629.6} = \sqrt{2518.4} = 50.18 ksi$$

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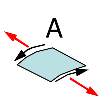
## What are the MSST stresses at "A" on the Crank Arm?

At "A": Bending M = 1800 in.lb.
Torsion T = 1200 in.lb.




$$\sigma_x = \frac{Mc}{I} = 43{,}460\ psi$$
$$\tau_{xz} = \frac{Tr}{J} = 14{,}487\ psi$$

The Mohr circle for "A" gave:
$\sigma_1$ = 47.846 ksi
$\sigma_2$ = 0.000 ksi
$\sigma_3$ = -4.386 ksi
$\tau_{Max}$ = 26.116 ksi

Using the Principal stresses (our only option with MSST):

$$MSST = \sigma_1 - \sigma_3 = 47.85 - (-4.39) = 52.24 ksi$$

This number is slightly higher than the von Mises stress of 50.2 – consistent with the MSST being more conservative.

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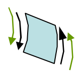
## What are the MSST & DET stresses at "C" on the Crank Arm?

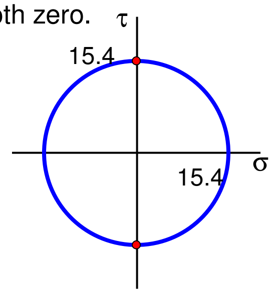
At "C": Torsion T = 1200 in.lb.
Shear V = 300 lb



C

$$\tau_{BENDING} = \frac{4V}{3A} = 905\ psi$$
$$\tau_{TOTAL} = \tau_{xz} + \tau_{BENDING} = 15{,}392\ psi$$
$$\rightarrow Pure\ Shear$$

"C" sees pure shear, with $\sigma_x$ & $\sigma_y$ both zero.




So $\sigma_1$ = 15.4 & $\sigma_3$ = -15.4

$$MSST = \sigma_1 - \sigma_3$$
$$= 15.4 - (-15.4) = 30.8ksi$$

DET using the Principal stresses:

$$\sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2}$$
$$= \sqrt{15.4^2 - (15.4)(-15.4) + (-15.4)^2}$$
$$= \sqrt{3(15.4)^2} = \sqrt{711.5} = 26.7ksi$$

DET using the Direct stresses:

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$
$$= \sqrt{0^2 + 0^2 - 0 + 3(15.4)^2}$$
$$= \sqrt{3(15.4)^2} = \sqrt{711.5} = 26.7ksi$$

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## Factors of Safety

The Factor of Safety, $n_s$, for a component is the ratio of the stress allowed by the component's material, divided by the maximum stress predicted (or measured). We usually use the yield strength, Sy, as the allowable stress for ductile materials. See Hamrock's Appendix A for Sy.

$$n_s = \frac{Sy}{\sigma_{\max}}$$

Using the Failure Theories, we will generally compute $n_s$ either by:

$$n_s = \frac{Sy}{\sigma_e}$$

For DET (von Mises)

or

$$n_s = \frac{Sy}{(\sigma_1 - \sigma_3)}$$
$$or$$
$$n_s = \frac{0.5Sy}{\tau_{\max}}$$

For MSST

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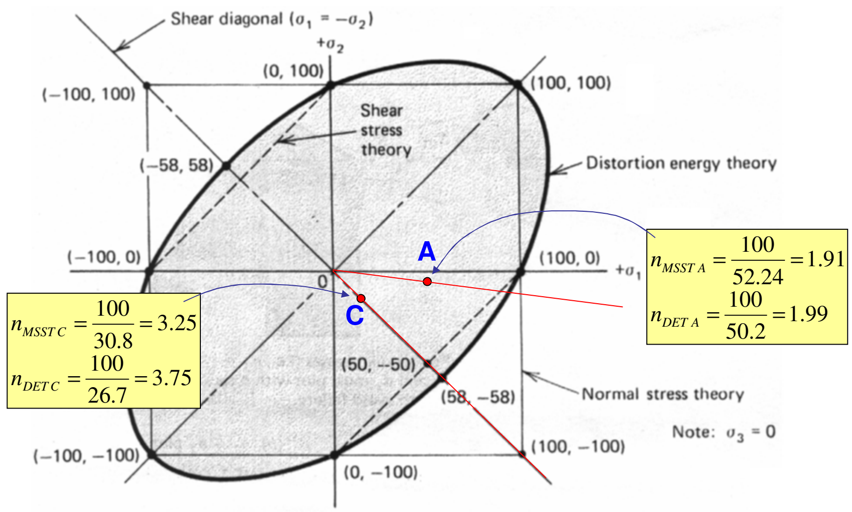
# Crank Arm Safety Factors




$$n_{MSST\,A} = \frac{100}{52.24} = 1.91$$

$$n_{DET\,A} = \frac{100}{50.2} = 1.99$$

$$n_{MSST\,C} = \frac{100}{30.8} = 3.25$$

$$n_{DET\,C} = \frac{100}{26.7} = 3.75$$

This assumes a material with Sy = 100ksi.
Can you find a material with Sy = 100ksi?

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# Static Safety Factors

A bar of AISI 1040 Steel sees a bending and twisting load, resulting in a bending stress of 15 ksi and a shear stress of 6.5 ksi at its most highly stressed location.

Find the MSS and the DET factors of safety against yielding.

Calculate both FoS's using both Direct stresses and Principal stresses.

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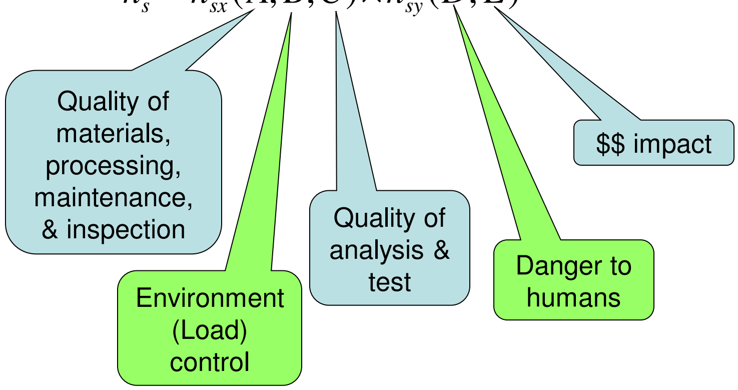
# Selecting Factors of Safety

Hamrock gives guidelines for selecting Safety Factors in Section 1.4.1.

$$n_s = n_{sx}(A, B, C) \times n_{sy}(D, E)$$




$1.1 \leq n_{sx} \leq 3.95$; $1.0 \leq n_{sy} \leq 1.6$; so $1.1 \leq n_s \leq 6.32$

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