MEEG3311 Machine Design

Lecture 2: Materials; Stress & Strain; Power Transmission

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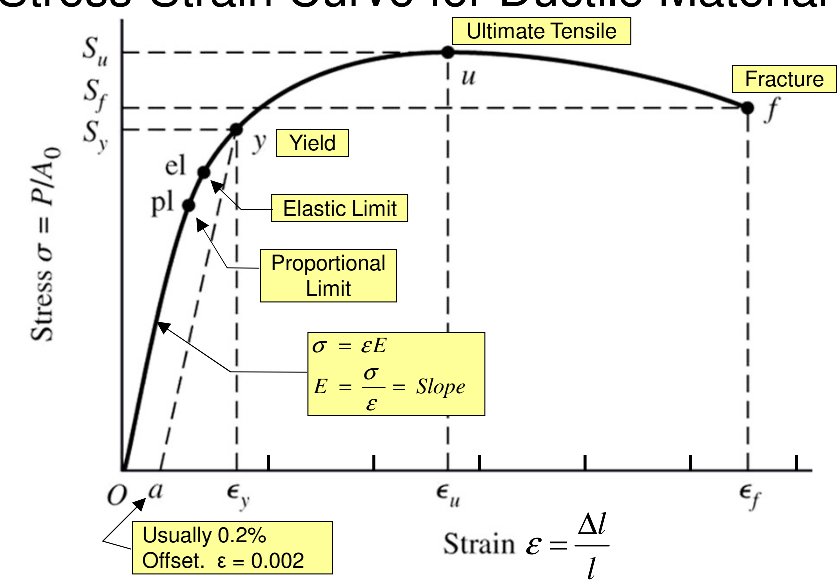
Stress-Strain Curve for Ductile Material



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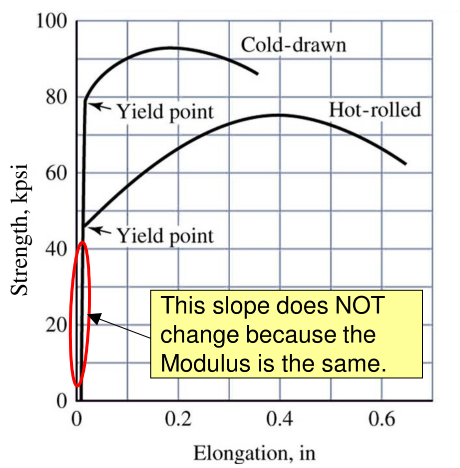
Effects of Hardness; Brittleness



Hardening by Heat Treating or Cold Working increases the Yield Strength of materials.

Terminology:
Strength = Material Property;
Stress = Applied Loading

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Manufacturing/Metalworking

- Metal fabrication, where sheets and bars are bent and formed, obviously depends on going beyond yield into the plastic forming range.
- It is common for highly-formed metals to require annealing to reduce their yield strengths for further forming.

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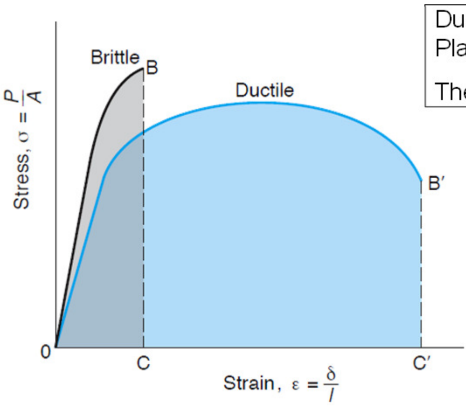
Brittle versus Ductile



Ductility is related to the amount of Plastic Deformation (Strain) at fracture.

The strain at fracture = % Elongation

$$\%El = \frac{l_{fracture} - l_0}{l_0} \times 100\%$$

Ductile if %El $\geq$ 5%
Brittle if %El < 5%

Hamrock
Fig. 3.7

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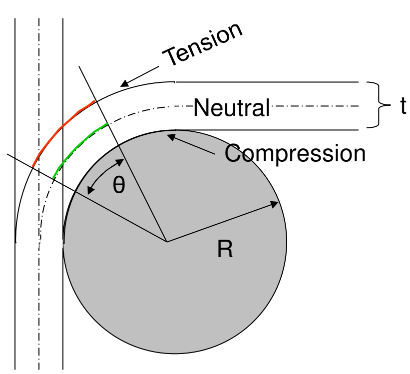
How to Crack Sheet Metal

- Bend it in a real tight radius. Why?



For any wrap angle θ, the circumferential wrap is r θ.

The Neutral length is (R+t/2)θ, and the length in Tension is (R+t)θ.

The strain is:

$$\frac{\Delta l}{l} = \frac{(Tension - Neutral)}{Neutral} =$$
$$\frac{(R+t)\theta - (R+t/2)\theta}{(R+t/2)\theta} = \frac{t/2}{(R+t/2)}$$
$$= \frac{t}{(2R+t)} = \frac{t}{D+t}$$

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How to Crack Sheet Metal

- Example: Aluminum Dogbone

Material is 6061-T6 Aluminum

Inner Bend Radius = 0.102”
Thickness = 0.101”

Calculate the strain. → $\varepsilon = \frac{t}{2R+t}$

Should it have cracked? ______
What do you think the stress was? ____________

From MatWeb.com:

Aluminum 6061-T6; 6061-T651

Mechanical Properties	Metric	English	Comments
Ultimate Tensile Strength	310 MPa	45.0 ksi	AA; Typical
Tensile Yield Strength	276 MPa	40.0 ksi	AA; Typical
Elongation at Break	12.0 %	12.0 %	AA; Typical; 1/16 in. (1.6 mm) Thickness
	17.0 %	17.0 %	AA; Typical; 1/2 in. (12.7 mm) Diameter
Modulus of Elasticity	68.9 GPa	10000 ksi	AA; Typical; Average of tension and compression. Compression modulus is about 2% greater than tensile modulus.
Tensile Yield Strength	276 MPa	40.0 ksi	AA; Typical
Elongation at Break	12.0 %	12.0 %	AA; Typical; 1/16 in. (1.6 mm) Thickness
	17.0 %	17.0 %	AA; Typical; 1/2 in. (12.7 mm) Diameter
Modulus of Elasticity	68.9 GPa	10000 ksi	AA; Typical; Average of tension and compression. Compression modulus is about 2% greater than tensile modulus.

See Example 3.1, Hamrock

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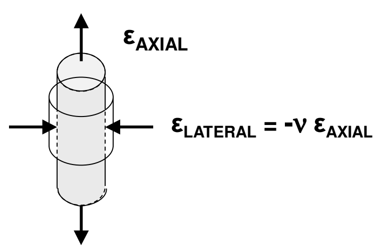
What’s nu?

Poisson’s Ratio
ν (nu)



Question: Is there a resulting lateral stress? _______
So, could you say there is strain without stress? ______
Can you think of another way to get strain without stress? ______

Poisson’s ratio is around 0.3 for most metals.
Lowest is ≈ 0.2 for Cast Iron; Highest is ≈ 0.44 for Lead.

Shear modulus (“stiffness in twisting”) is $G = \frac{E}{2(1+\nu)}$

Because most metals have a ν of about 0.3,
this means that for most metals,
G ≈ what percent of E? __________

Hamrock
Eqn. 3.7

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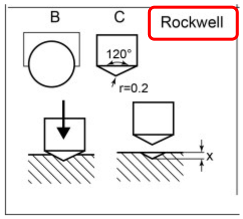
Hardness Tests

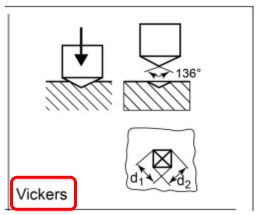


Sphere or Rounded-Tip Cone

$$HR = N - \frac{d}{s}$$

d = depth

N and s are scale factors

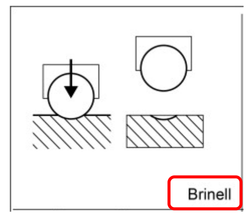


Diamond Pyramid

$$HV \approx 0.1891 \frac{F}{d^2}$$

d = Average Diagonal, mm

F = Load, N



Sphere

$$\mathrm{HBW} = 0.102 \frac{2F}{\pi D \left(D - \sqrt{D^2 - d^2}\right)}$$

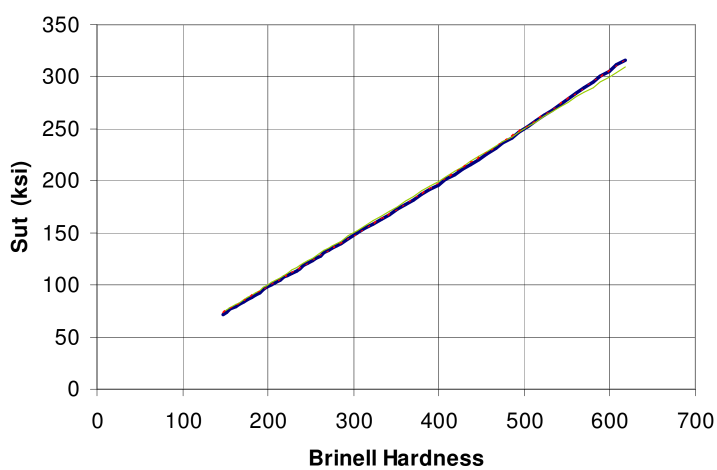
d = Indent Diameter, mm

D = Sphere Diameter, mm

F = Load, N

https://www.sciencedirect.com/topics/engineering/rockwell-test

UTS vs. Hardness



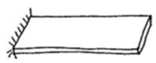
Ultimate Tensile Strength correlates very well with Brinell hardness.

Ashby Charts

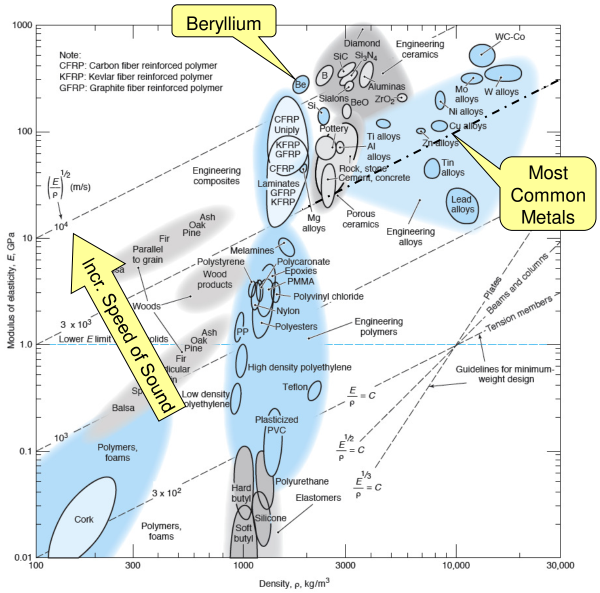
Look them over.

Understand what they are and how they represent differences among materials.

My favorite is Fig. 3.19, comparing Modulus of Elasticity and Density.



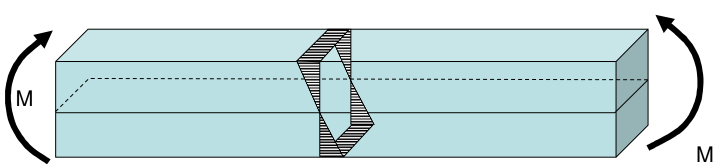
The natural frequency of a cantilever beam is nearly the same for steel or aluminum, but for Beryllium it is almost 3x higher!



What is $\sqrt{E/\rho}$ for Aluminum ______ and for Steel _____? What are the units? _____

Stresses in Straight Beams

- A uniform beam in pure bending



The maximum stress is $\sigma_{\max} = \dfrac{Mc}{I}$

where I is the Area Moment of Inertia about the Centroid, and c is the distance from the Neutral Axis, or Centroid.

We need to know these properties of the beam cross section to be able to calculate bending stress, one of the most common large stresses on structures.

Hamrock Chapter 4:
Area Moment of Inertia

Definitions:

- Centroid of an Area $$\bar{x} = \frac{\int_A x\,dA}{A}$$
- Area Moment of Inertia $$I_x = \int_A y^2\,dA$$
- Parallel Axis Theorem $$I_{x'} = I_x + Ad^2$$ CENTROID

This is NOT same as J = Polar Moment of Inertia, or as Mass Moment of Inertia, I_m, (which has units Lb.In.Sec^2, and sets how fast a torque can rotationally accelerate a device.)

Hamrock Section 4.2

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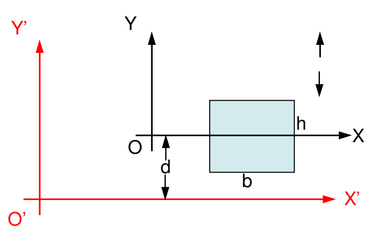
Parallel Axis Theorem



$Ix' = Ix + Ad^2 = bh^3/12 + bhd^2$

where:

- Ix' = moment of inertia about an axis that IS NOT through the center
- Ix = moment of inertia about an axis that is through the centroid = $bh^3/12$
- d = distance between the x axes
- A = area of the cross section = bh

Note: the axes must be parallel

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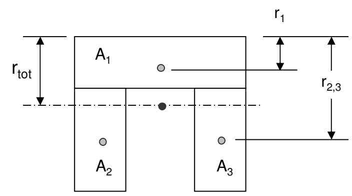
Moment of Inertia Procedure



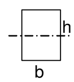
1. Find neutral axis (centroid) of total area.
   Why? Because we will use **I** to calculate bending stress about the neutral axis.

   ($A_1 + A_2 + A_3$) $r_{tot} = A_1 r_1 + A_2 r_2 + A_3 r_3$

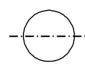
   Solve for r_{tot} .

- Cut-outs have negative areas.
- Pick a convenient axis to measure r's from.
- If you pick an outer edge, you will always know which direction r_{tot} is!

2. Compute Moment of Inertia about the centroid of each sub-piece.

For example: 

$I_{CG} = \frac{bh^3}{12}$



$I_{CG} = \frac{\pi D^4}{64}$

3. Use Parallel Axis Theorem to translate to about the neutral axis

$$I_X = I_{CG} + Ad^2$$

where "d" is the distance from the sub-piece's centroid to the total area centroid, and "A" is the area of the sub-piece.

4. Add up the translated moments of inertia of all the pieces, subtracting moments if they are cut-outs.

Hamrock
Section 4.2.3

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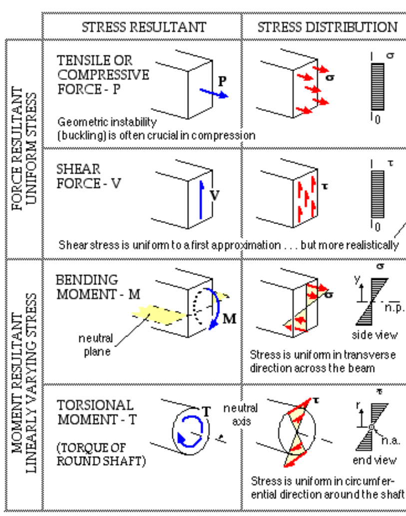
Elementary Load Building Blocks



STRESS	
$\sigma = \frac{P}{A}$	Hamrock Eqn. 4.22
$\tau = \frac{P}{A}$	
$\sigma = \frac{My}{I}$	Eqn. 4.46
$\tau = \frac{Tr}{J}$	Eqn. 4.32

From DAWright at U of Western Australia

The practical unit of stress is neither 10^6 N/m^2 nor 10^6 Pa, but **MPa** (equivalent to N/mm^2). Stress conversion factor: 6.895 kPa per lbf/in^2

stress in member (at distance from neutral axis, if linear) = stress resultant / property of member's cross-sectional geometry = material property (elastic modulus) × measure of deformation (strain)

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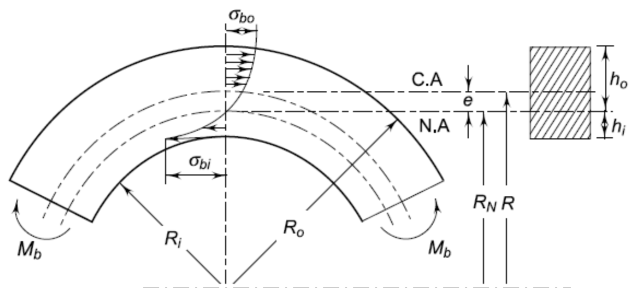
Stresses in Curved Beams

Bending Stresses in a curved beam are not linearly distributed across the beam section, but have a hyperbolic distribution that is higher at the inside surface than for a straight beam.



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Stresses in Curved Beams

1) Draw a very good picture.
   - Show the applied Force, F
   - Show ri, ro, Area

2) Calculate the <u>centroidal</u> radius, R, based on the cross-section shape. (Hamrock § 4.5.3)

Rectangular: $R = \dfrac{r_i + r_o}{2}$ Circular: $R = r_i + r_c$

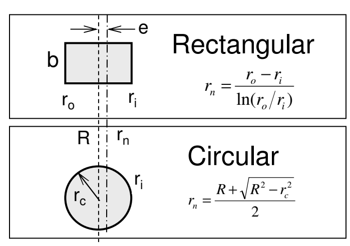
3) Compute the <u>neutral</u> radius, r_n, based on the section shape. (Hamrock § 4.5.3)

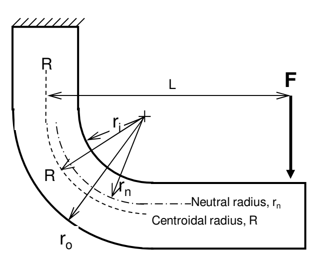
Rectangular:

$$r_n = \frac{r_o - r_i}{\ln(r_o/r_i)}$$

Circular:

$$r_n = \frac{R + \sqrt{R^2 - r_c^2}}{2}$$





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Stresses in Curved Beams (2)

4) Compute the eccentricity, $e = R - r_n$

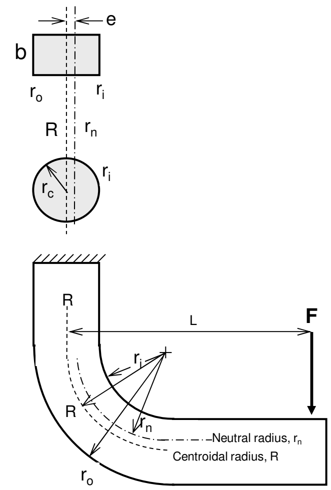
5) Compute the moment about the centroidal radius, R.
Here M = F x L, not F x R, because the force is not through the center of curvature.

6) Calculate the distances from the neutral axis to the inner and outer surfaces:
$c_i = r_n - r_i$ and $c_o = r_o - r_n$.

7) Calculate the stresses at the inside and outside surfaces:

$$\sigma_i = \frac{Mc_i}{Aer_i} \quad \text{and} \quad \sigma_o = -\frac{Mc_o}{Aer_o} ,$$

where A = the section area



Stresses in Curved Beams (3)

8) Add or subtract any P/A stresses in this section (using superposition).

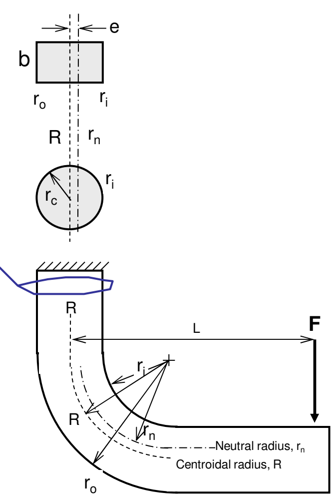
Rectangular:

$$\sigma = \frac{F}{(r_o - r_i)b}$$

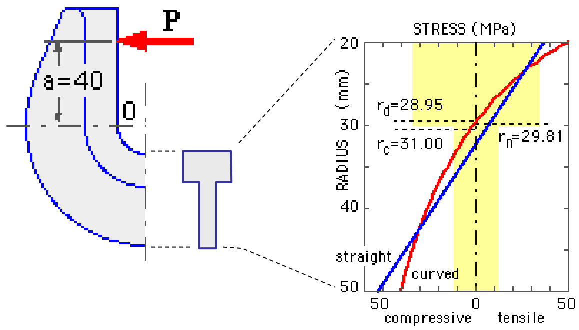
Circular:

$$\sigma = \frac{F}{\pi r_c^2}$$

9) As a check, compare the answer to MC / I for a straight beam with neutral axis on the centroid and see if it makes sense.



Stresses in Curved Beams (4)



From DAWright at U of Western Australia

The stress distribution:

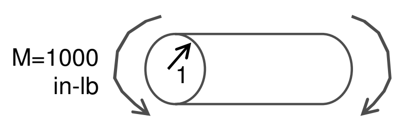
1) is hyperbolic, rather than linear for a straight beam,
2) passes through zero at the neutral axis rather than at the centroid,
3) has the highest stress at the inside radius.

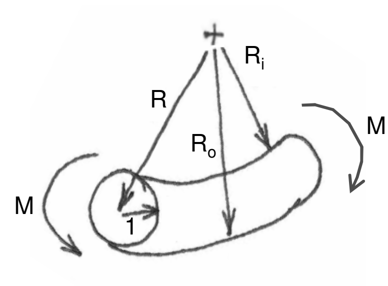
Stresses in Curved Beams

Calculate and compare the stresses on a 2” diameter bar with an applied bending moment of 1000 in-lb for these cases:

1) A straight bar

2) A curved bar with R = 3 in.



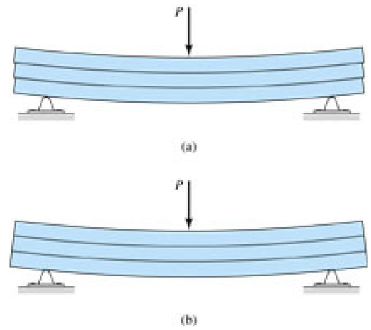


$$I = \frac{\pi}{4} r^4 \qquad r_n = \frac{R + \sqrt{R^2 - r_c^2}}{2} \qquad \sigma_i = \frac{Mc_i}{Aer_i} \qquad \sigma_o = -\frac{Mc_o}{Aer_o}$$

Transverse Shear

Hamrock Fig. 4.19 shows difference between behavior of boards not glued together (top) and boards glued together (bottom).

Solid beams behave like boards glued together, and the interior layers see shear stress along them.



The outside surfaces have nothing to constrain them, and therefore see no shear stress.

Hamrock
Section 4.6

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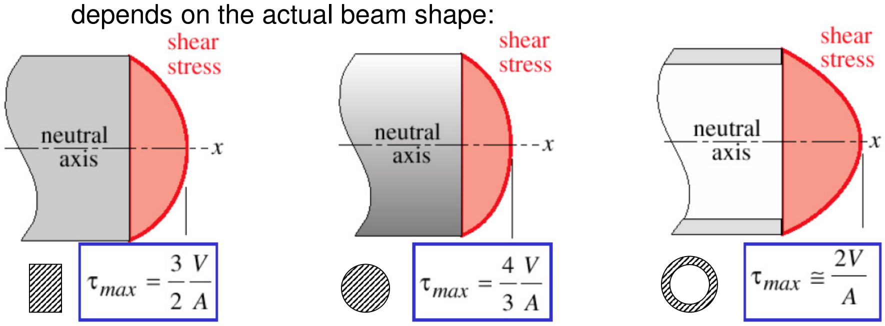
Transverse Shear

Even though we usually write transverse shear as τ = P/A, that is just the average shear. In reality, it has a distribution that depends on the actual beam shape:



(*a*) rectangular beam (*b*) solid round beam (*c*) hollow round beam

As Hamrock says below Eqn. 4.66: “In all cases, shear stress is zero on extreme fibers, is maximum on the neutral axis, and has a parabolic distribution through the thickness.”

From http://users.wpi.edu/~cfurlong/me3320/lect06-07/Lect06-07.pdf

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Notes on Power

Power is the rate at which work gets done, so has units of
(Force x Distance) / Time = Work / Time.
Example: 1 HP = 550 Lb x Ft / Sec

This can also be looked at as
Force x (Distance / Time) = Force x Velocity.

If a motor is involved, it can also be viewed as
Torque x Rotational Velocity = (Force x Distance) x (Radians / Time)

$$1\,HP = 550\frac{Ft \times Lb}{Sec}\left(\frac{12\,In}{Ft}\right)\left(\frac{60\,Sec}{\min}\right)\left(\frac{1\,\mathrm{Rev}}{2\pi\,Rad}\right) = 63025\,In \cdot Lb \cdot RPM$$

Note the orderly unit conversion.

In the SI system, it is simply
Power (Watts) = Tω = Torque (Nm) x Rotational Speed (Radians/Sec)

For conversion, 1 HP = 745.7 Watts

Hamrock Section 4.4.2

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Power Example

Horsepower: 1-1/2
Frame: 56
Shaft Diameter: 5/8"
Volts: 115 volt
Full load amps: 14.2
Phase: single
Enclosure: Open dripproof
No load speed: 3600 RPM
Reversible: yes
Service factor: 1.0
Weight: 29 lbs.

A. If this motor didn't slow down when delivering its full rated power (it does), what torque would it be delivering?
B. What would the maximum stress in the shaft be then?
C. What is the efficiency (mechanical power/electrical power) of the motor?

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Motor Calculations

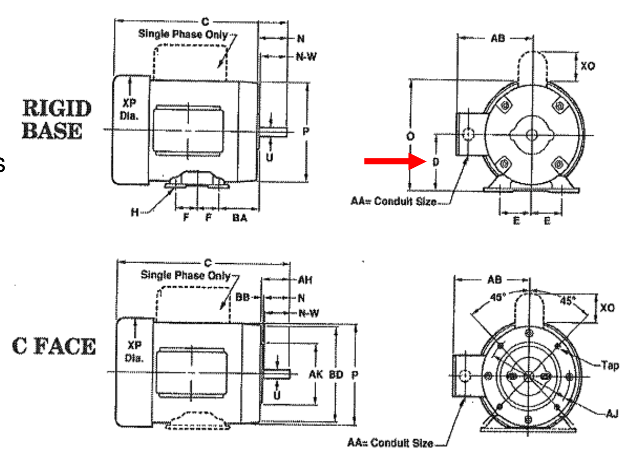
- Torque = Power / Speed
- $\tau = \frac{Tr}{J}, \; J = \frac{\pi}{2} r^4$
- Electrical power (W) = Volts x Amps

Motor Fun Facts

For a NEMA D Frame motor, the frame number is how many 1/16ths of an inch from the shaft center to base (the "D" dimension).

A size 56 frame is 56/16 = 3.5".

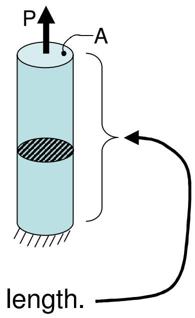
The standard controls mostly the motor mounting details, not the motor body dimensions



Simple Stress Distributions

- Axial load on a uniform bar

$$\sigma = P/A$$



Describe the maximum stress:

1. What position along the beam?
   Answer: Everywhere along its length.
2. What location in the cross section?
   Answer: It is uniform over the whole section.
3. What is the value?
   Answer: $\sigma = P/A$

Note that all material is equally stressed.
What about E?

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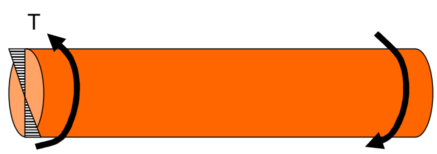
Simple Stress Distributions

- Torsional load (torque) on a uniform round bar

$$\tau_{max} = \frac{Tr}{J}$$



$$J = \frac{\pi}{2} r^4$$

Describe the maximum stress:

1. What position along the beam?
   Answer: Everywhere along its length.
2. What location in the cross section?
   Answer: It is maximum at the outer surface.
3. What is the value?
   Answer: $\tau = Tr/J$

Note that the material in the center of the bar (along its axis) isn't loaded very much.

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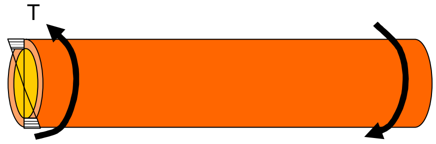
Simple Stress Distributions

- Torsional load (torque) on a uniform <u>tube</u>

$$\tau_{\max} = \frac{Tr_o}{J}$$



$$J = \frac{\pi}{2}(r_o^4 - r_i^4)$$

Describe the maximum stress:

1. What position along the beam?
   Answer: Everywhere along its length.
2. What location in the cross section?
   Answer: It is maximum at the outer surface.
3. What is the value?
   Answer: $\tau = Tr/J$

Removing the material in the center of the bar (along its axis) saves weight and $ without too much loss.

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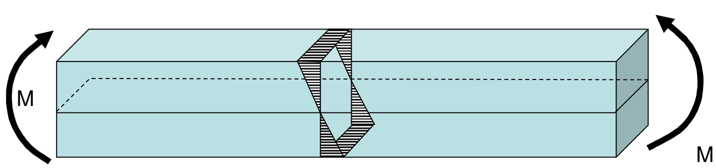
Simple Stress Distributions

- For a uniform beam in pure bending



Describe the maximum stress: $\sigma_{\max} = \frac{Mc}{I}$

1. What position along the beam?
   Answer: Everywhere along its length.
2. What location in the cross section?
   Answer: It is maximum at the top & bottom surfaces.
3. What is the value?
   Answer: $\sigma = Mc/I$

Note that the material in the center of the bar (along its axis) isn't loaded very much.

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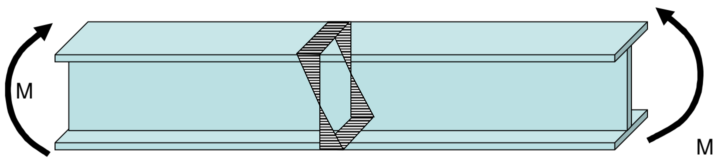
Simple Stress Distributions

- For an "I" beam in pure bending



Describe the maximum stress:

$$\sigma_{max} = \frac{Mc}{I}$$

1. What position along the beam?
   Answer: Everywhere along its length.
2. What location in the cross section?
   Answer: It is maximum at the top & bottom surfaces.
3. What is the value?
   Answer: $\sigma = Mc/I$

Removing the material in the center of the bar (along its axis) saves weight and $ without too much loss.

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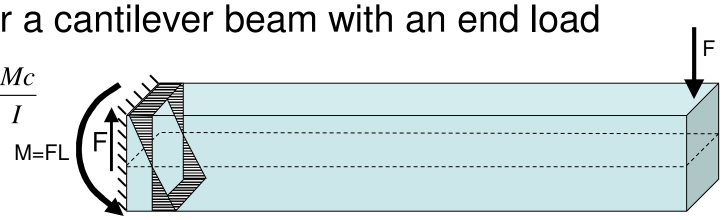
Non-Simple Stress Distributions

- For a cantilever beam with an end load

$$\sigma_{max} = \frac{Mc}{I}$$



Describe the maximum **bending** stress:

1. What position along the beam?
   Answer: At the left end of the beam.
2. What location in the cross section?
   Answer: It is maximum at the top and bottom faces.
3. What is the value?
   Answer: $\sigma = Mc/I$

What about the transverse shear?

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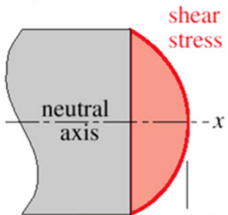
Non-Simple Stress Distributions

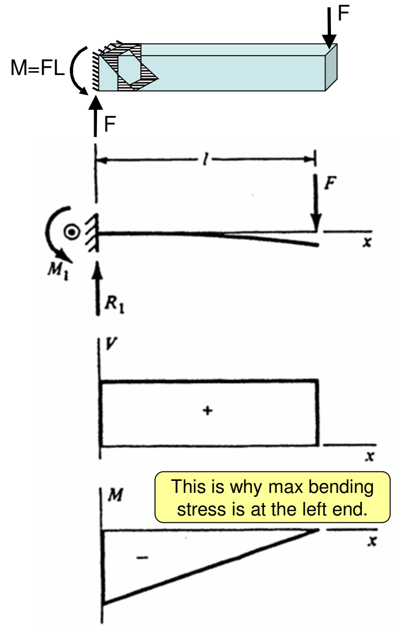
It is best to map out the Shear, V, and Moment, M, distribution when they vary along the beam.

The Transverse Shear, for this rectangular cross section beam, is parabolic, with a max of 1.5 x τ_{avg}:



$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

(a) rectangular beam



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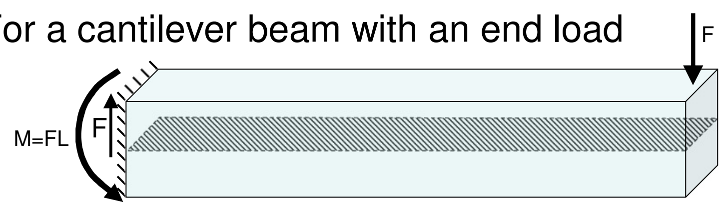
Non-Simple Stress Distributions

- For a cantilever beam with an end load



Describe the maximum **transverse shear** stress:

1. What position along the beam?
   Answer: Everywhere along its length.
2. What location in the cross section?
   Answer: It is maximum on the neutral axis (and zero at the top and bottom faces).
3. What is the value?
   Answer: τ = 1.5 V/A

How do I (or do I) combine the bending and the transverse shear?

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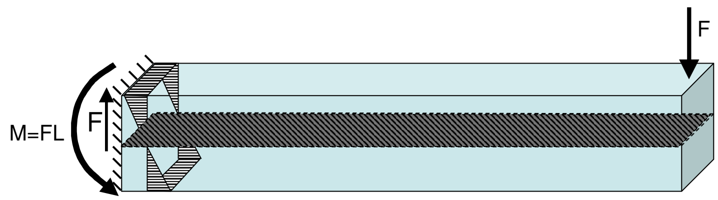
Dealing with Several Stresses

Considering:
1. Superposition
2. Mohr's circle
propose how to handle the bending plus shear stresses.



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Summary

Case	Type of Loading	Illustration	Stress Distribution	Stress Equations
1	Direct tension	F F	Uniform	$\sigma = \frac{F}{A}$ (9)
2	Bending	F_1 F_2 M x Bending moment diagram	$-\sigma$ $+\sigma$ Neutral plane	$\sigma = \pm\frac{M}{Z} = \pm\frac{My}{I}$ (11)
3	Bending	F_1 F_2 R_1 R_2 V F_1 F_2 x Shearing force diagram	τ Neutral plane	For beams of rectangular cross-section: $\tau = \frac{3V}{2A}$ (12) For beams of solid circular cross-section: $\tau = \frac{4V}{3A}$ (13) For wide flange and I beams (approximately): $\tau = \frac{V}{a}$ (14)
4	Direct shear	F F	Uniform	$\tau = \frac{F}{A}$ (15)
5	Torsion	T	τ τ	$\tau = \frac{T}{Zp} = \frac{Tc}{J}$ (16)

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Example Beam Loading

20 inch long beam with two 100lb loads

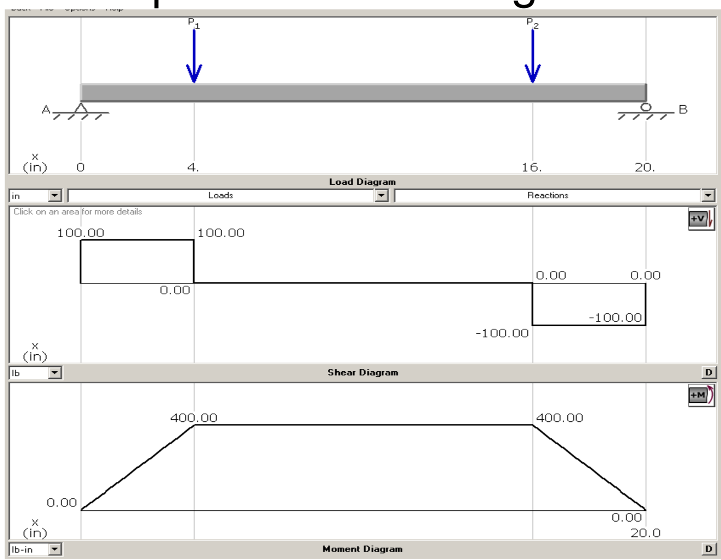
Beam cross section is 1 inch square.

Shear, V (Lb)

$\frac{dV}{dX} = -w$

Moment, M (In.Lb.)

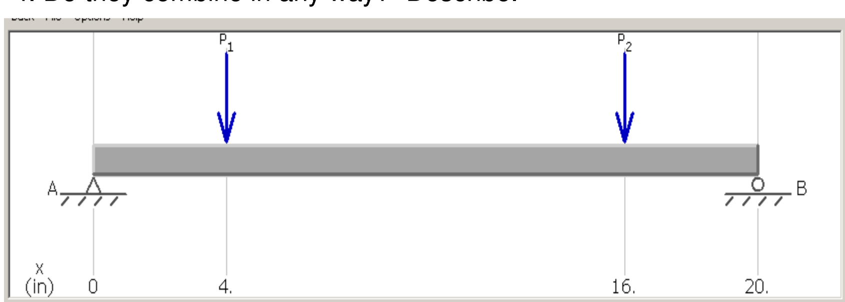
$\frac{dM}{dX} = V$



What stresses are where?

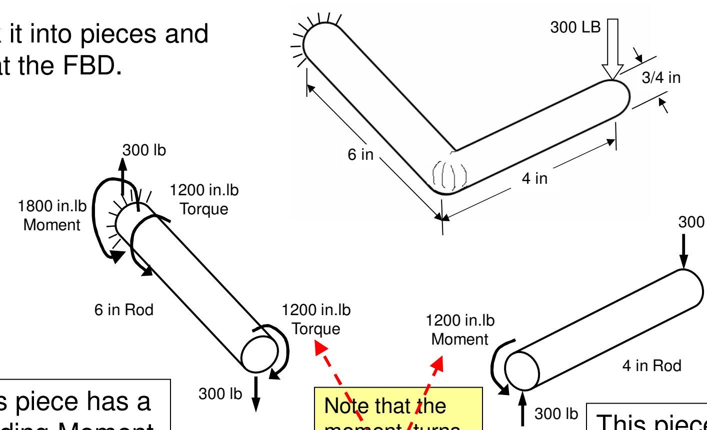
Calculate Amongst Yourselves

1. Show the location (along the beam) of the max stresses.
2. Show the position (in the cross section) of the max stresses.
3. Calculate the max stress values.
4. Do they combine in any way? Describe.



Stresses in a Crank Arm

Break it into pieces and look at the FBD.



This piece has a bending Moment and a twisting Torque.

Note that the moment turns into a torque where the arm bends the corner.

This piece has a bending Moment.

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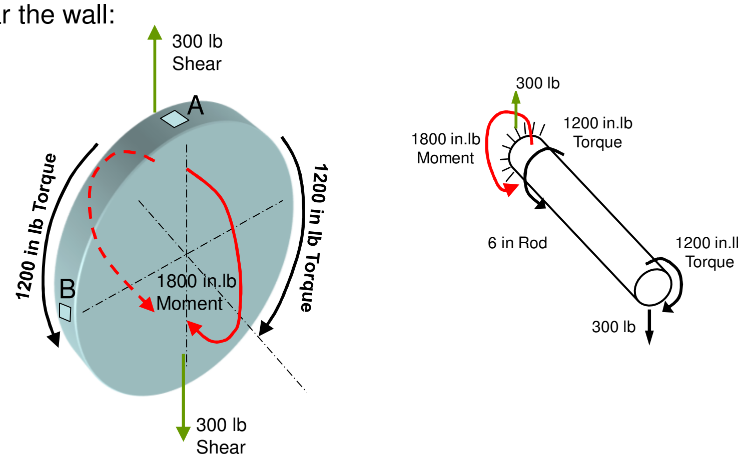
Stresses in a Crank Arm

We already know how to do a tip-loaded cantilever. Let's take a look at the piece that bends and twists. Let's slice off a thin section near the wall:



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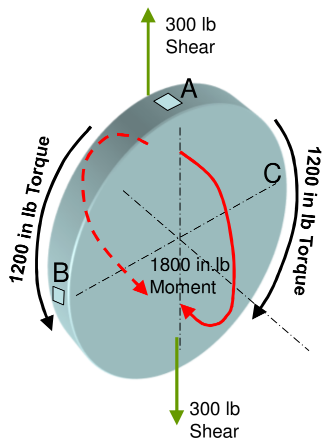
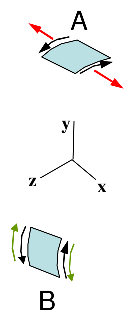
Stresses in a Crank Arm

Now let's get the stresses.





At "A": Bending M = 1800 in.lb.
Torsion T = 1200 in.lb.
[Shear V = 300 lb, but is at neutral axis, not on top surface.]

$$\sigma_x = \frac{Mc}{I} = \frac{(1800)(0.375)}{\pi(0.75)^4/64} = \frac{675}{0.01553} = 43{,}460\ psi$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{(1200)(0.375)}{\pi(0.75)^4/32} = \frac{450}{0.03106} = 14{,}487\ psi$$

$$\tau_{BENDING} = \frac{4V}{3A} = \frac{(4)(300)}{3\pi(0.75)^2/4} = \frac{1200}{1.325} = 905\ psi$$

$$\tau_{TOTAL} = \tau_{xz} - \tau_{BENDING} = 13{,}582\ psi$$

$\rightarrow Pure\ Shear$

At "B": Torsion T = 1200 in.lb.
Shear V = 300 lb
[Bending is at top and bottom only, not at neutral axis.]

Note that at location "C", the two shears would ADD to 15,392 psi.

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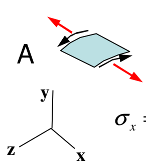
Stresses in a Crank Arm

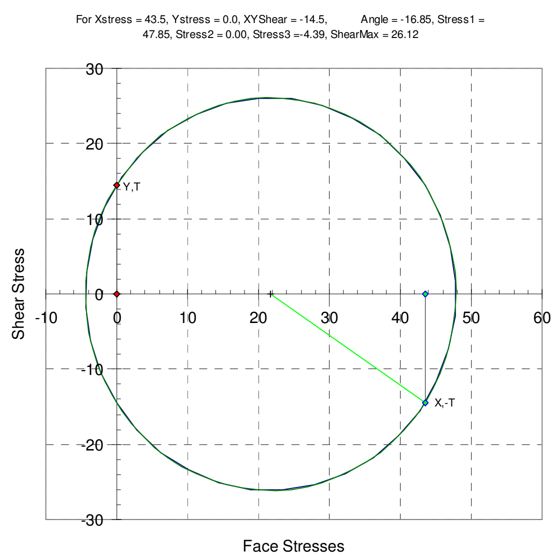
These stresses occur <u>at the same point</u>, so can use Mohr.



$$\sigma_x = \frac{Mc}{I} = 43{,}460\ psi$$

$$\tau_{xz} = \frac{Tr}{J} = 14{,}487\ psi$$

The Mohr circle gives:
σ_1 = 47.846 ksi
σ_2 = 0.000 ksi
σ_3 = -4.386 ksi
τ_{Max} = 26.116 ksi



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