ME311 Machine Design

Lecture 8: Cylinders

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Thin-Walled Cylinders

(You already covered this in Beer & Johnston.)

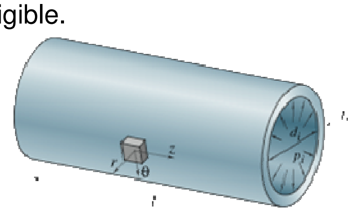
A pressurized cylinder is considered to be Thin-Walled if its wall thickness is less than 2.5% (1/40th) of its inside diameter.

Under these conditions:
1. We assume the stress distribution is uniform throughout the wall thickness – both in the hoop (circumferential) direction and in the longitudinal (axial) direction.
2. We assume that the radial stress is negligible.

Then:

$$Hoop \;\; \sigma_\theta = \frac{p_i r}{t} \quad and$$

$$Axial \;\; \sigma_z = \frac{p_i r}{2t} = \frac{\sigma_\theta}{2}$$



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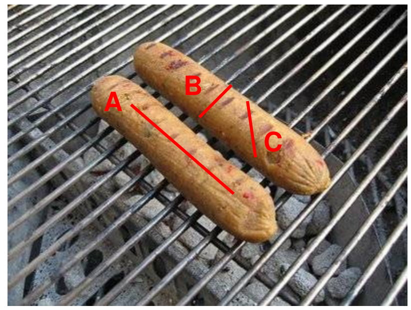
Thin-Walled Cylinder Quiz

Knowing all that you do about pressurized cylinders (i.e., that the hoop stress is twice the longitudinal stress), which direction would you predict that these pressurized cylinders will fracture?

A. Lengthwise due to hoop stress.

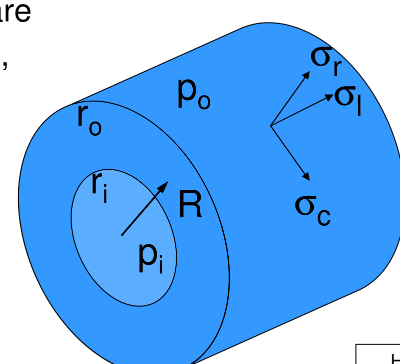
B. Crosswise due to axial stress.

C. On a 45° angle due to shear stress or torque.



Stresses in Thick-Walled Cylinders

- Thick-Walled cylinders have a wall thickness greater than 1/20th of their average radius.
- They are pressurized internally and/or externally.
- The principal stresses are circumferential (hoop) σ_c, radial σ_r, and longitudinal (axial) σ_l.



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Section 10.3.2

Circumferential & Radial Stresses

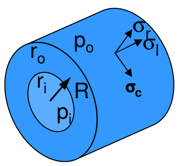
For the general case of both internal and external pressure, the circumferential and radial stresses at radius R in the wall are:

$$\sigma(R) = \frac{r_i^2 p_i - r_o^2 p_o \pm (p_i - p_o) r_i^2 r_o^2 / R^2}{r_o^2 - r_i^2}$$

Eqns 10.20/10.22

Where the ± is: + for circumferential, and - for radial stress.

For the special case of only internal pressure, $p_o = 0$, and the stresses at radius R are:



$$\sigma(R) = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

Eqns 10.23/10.24

The sign convention is the same.

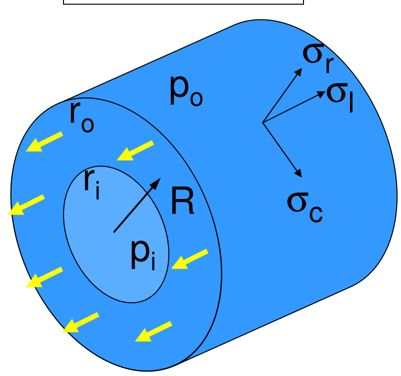
Longitudinal Stresses

The longitudinal stress is simply given by a Force/Area, where the Force is p_i times the circular inside area πr_i^2, and the Area is the annular area of the cylinder cross section, $\pi(r_o^2 - r_i^2)$, or:

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

Un-numbered Equation just below Eqn. 10.5

This is generally only considered for the case of internal pressurization ($p_o = 0$).

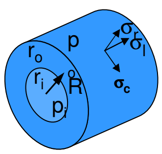


Stresses vs. Radius

First, the easy observation: Radial stresses at the inner and outer surfaces are equal to minus the pressurization.

- If a surface is <u>un</u>pressurized, the radial stress there is zero.
- If a surface is pressurized, the radial stress there = - p, because it is in compression.

Now let's look at an internally pressurized cylinder, and how the radial and circumferential stresses vary across the wall thickness at radius R.

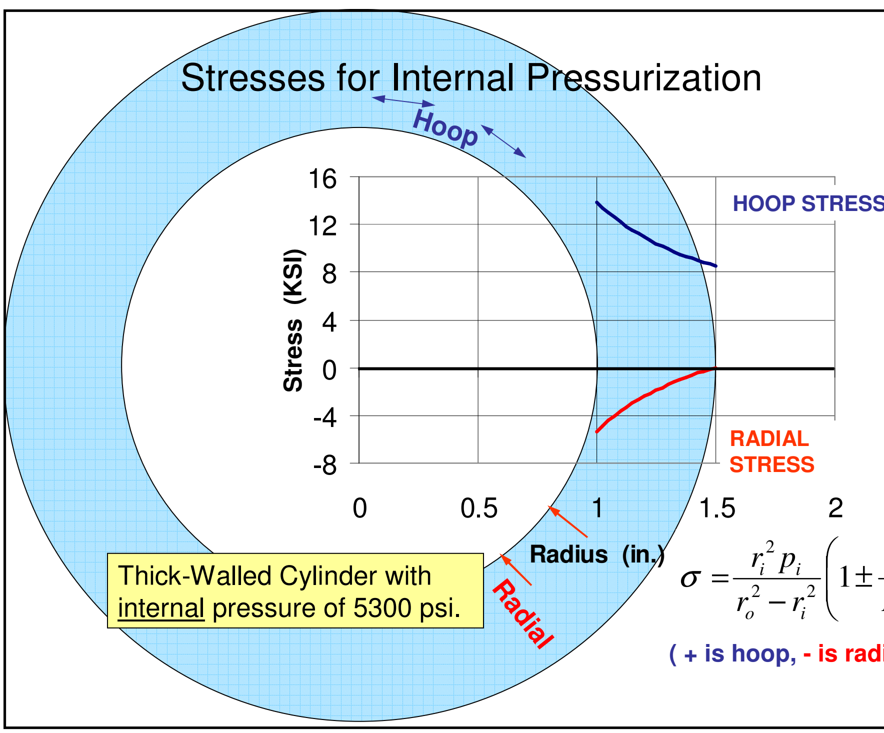


$$\sigma(R) = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

Eqns 10.23/10.24

(+ is circumferential, - is radial)

Stresses for Internal Pressurization



Thick-Walled Cylinder with <u>internal</u> pressure of 5300 psi.

$$\sigma = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

(+ is hoop, - is radial)

Stresses vs. Radius - Internal Pressure

Radial stress is as predicted:

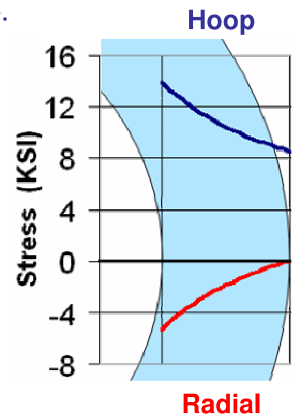
- -5300 psi at the inner, pressurized surface.
- 0 at the unpressurized outer surface.

Hoop stress is:

- Maximum at the inner surface, 13.9 ksi.
- Lower, but not zero, at the unpressurized outer surface, 8.5 ksi.
- Larger in magnitude than the radial stress

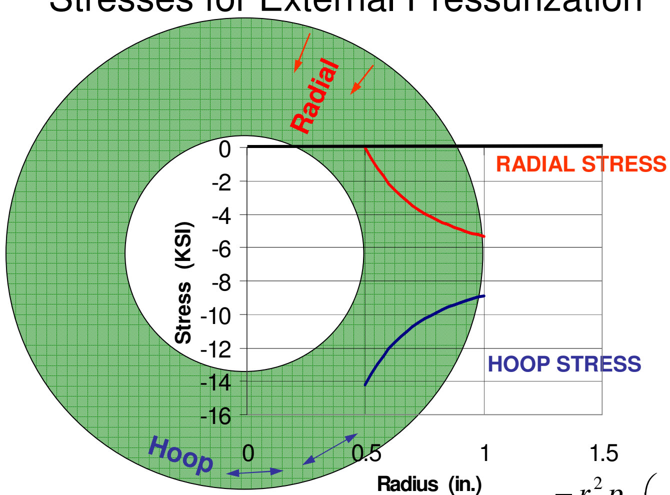
Longitudinal stress is (trust me):

- 4.3 ksi, considered as a uniform, average stress across the thickness of the wall.



Now let's look at an externally pressurized cylinder.

Stresses for External Pressurization



Thick-Walled Cylinder with external pressure of 5300 psi.

$$\sigma = \frac{-r_o^2 p_o}{r_o^2 - r_i^2}\left(1 \pm \frac{r_i^2}{R^2}\right)$$

(+ is hoop, - is radial)

Stresses vs. Radius - External Pressure

Radial stress is as predicted:

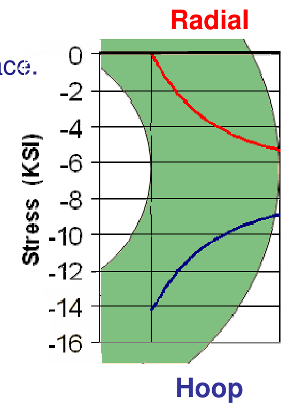
- 0 at the unpressurized inner surface.
- -5300 psi at the outer, pressurized surface.

Hoop stress is:

- Minimum at the outer surface, -8.9 ksi.
- Maximum at the (unpressurized) inner surface, -14.2 ksi.
- Larger than the radial stress

Longitudinal stress is:

- Not usually considered for external pressurization.

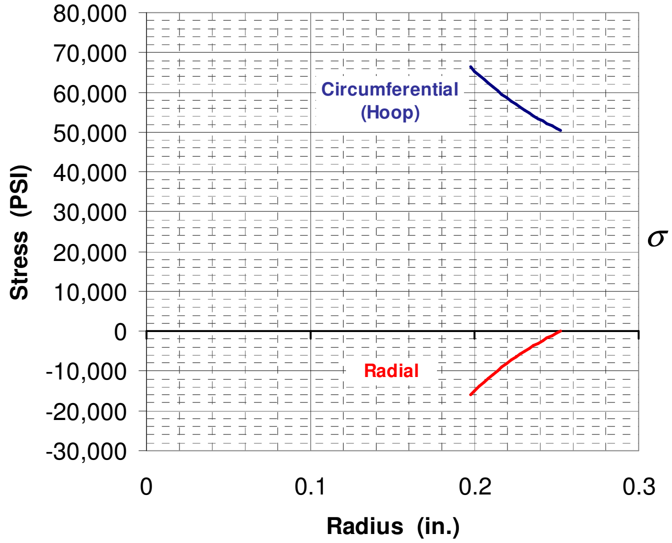


Burst Tubing Analysis

ID = 0.395”; OD = 0.505”; p = 16,000psi

- What was the hoop stress in the tube?
- Analyze it as both thin-wall and thick-wall. Which is it?

Stresses vs R for Tube

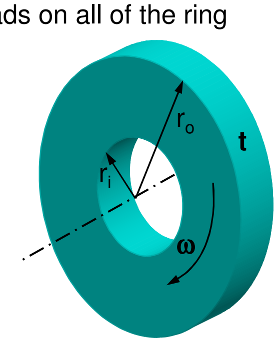


$$\sigma = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 \pm \frac{r_o^2}{R^2}\right)$$

Rotating Rings

Stresses (radial & tangential) are similar to those in thick-walled cylinders. The forces come from centrifugal loads on all of the ring particles instead of from the internal pressure.

Conditions:
1. $r_o \geq 10\,t$
2. t is constant



Eqn 10.35

$$\sigma_{hoop} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2\right)$$

where

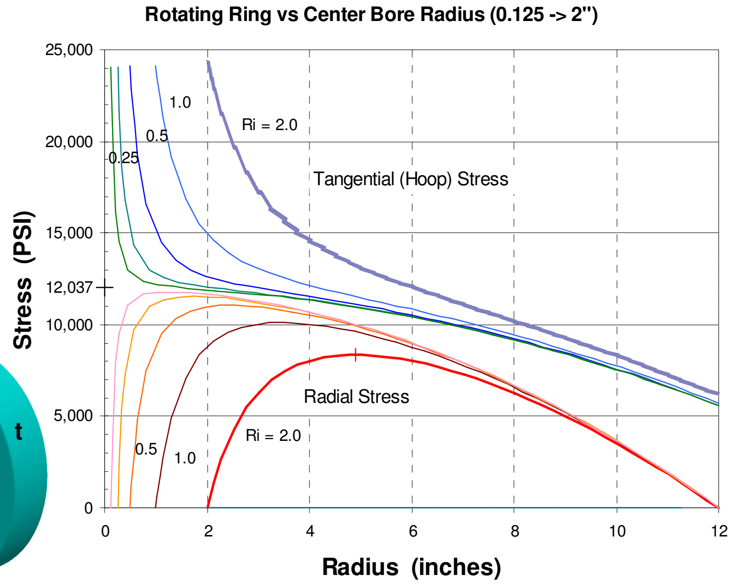
$$\rho = Mass\,Density = \frac{Lb/in^3}{386\,in/s^2}, \quad \omega = radians/\sec, \; \nu = Poisson's\;\; ratio$$

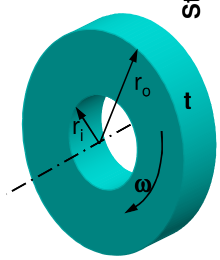
$$\sigma_{radial} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Eqn 10.36

Rotating Rings: Effect of Center Bore Radius on Stresses

Stresses for a 2 inch thick steel disk rotating at 5000 RPM.





At what radius is the peak radial stress?

Remember Differentiation?

$$\frac{d\sigma_{radial}}{dr} = \frac{d}{dr}\left[\rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)\right]$$

$$= \rho\omega^2\left(\frac{3+\nu}{8}\right)\frac{d}{dr}\left[\left(-\frac{r_i^2 r_o^2}{r^2} - r^2\right)\right] = 0 \ at\ peak$$

$$\frac{d}{dr}\left(-\frac{r_i^2 r_o^2}{r^2}\right) = \frac{d}{dr}\left(r^2\right)$$

$$\frac{d}{dr}\left(-\frac{r_i^2 r_o^2}{r^2}\right) = -\frac{-2r_i^2 r_o^2}{r^3} = \frac{d}{dr}\left(r^2\right) = 2r$$

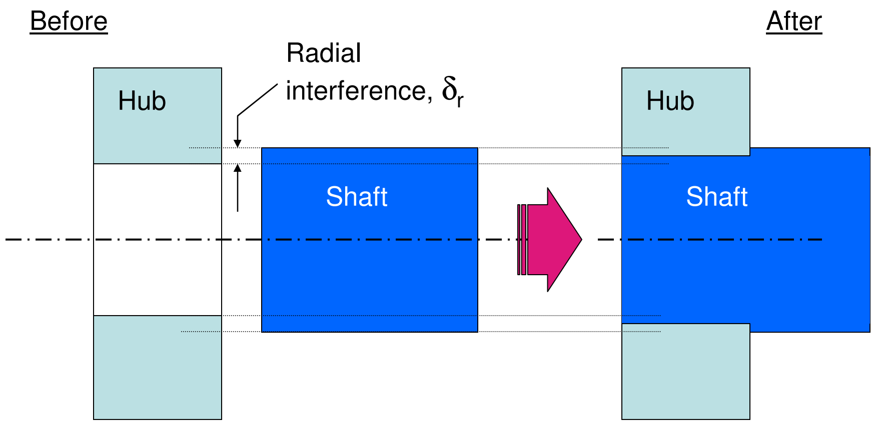
$$\boxed{\begin{array}{l}\frac{d}{dx}x^n = nx^{n-1} \\ \frac{d}{dr}r^{-2} = -2r^{-3}\end{array}}$$

$$r_i^2 r_o^2 = r^4$$

$$r = \sqrt{r_i r_o} \qquad \sqrt{(2)(12)} = \sqrt{24} = 4.90$$

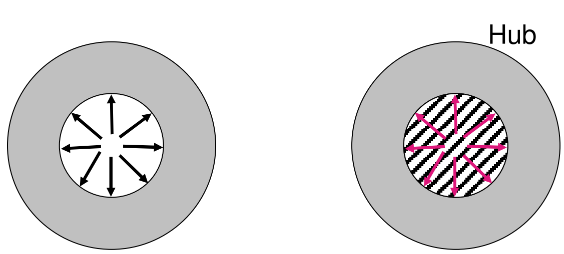
Press Fits

In a press fit, the shaft is compressed and the hub is expanded.



Press Fits

Press fits, or interference fits, are similar to pressurized cylinders in that the placement of an oversized shaft in an undersized hub results in a radial pressure at the interface.

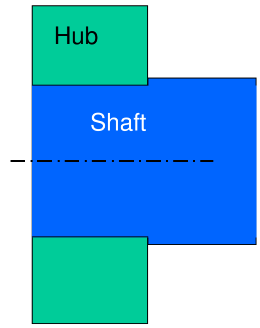


Characteristics of Press Fits

1) The shaft is compressed and the hub is expanded.

2) There are equal and opposite pressures at the mating surfaces.

3) The relative amount of compression and expansion depends on the stiffness (elasticity and geometry) of the two pieces.



4) The sum of the compression and the expansion equals the interference introduced.

5) The critical stress location is usually the <u>inner diameter of the hub</u>, where max tensile hoop stress occurs.

Analysis of Press Fits

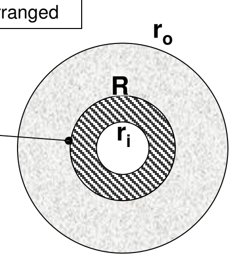
Start by finding the interface pressure.

If shaft and hub are of the same material:

$$p = \frac{E\delta_r}{R}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{2R^2\left(r_o^2 - r_i^2\right)}\right]$$

Eqn 10.52, rearranged

Where δ_r is the RADIAL interference for hub and shaft of the same material, with modulus of elasticity, E.



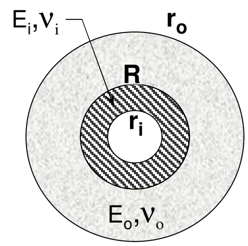
If the shaft is solid, $r_i = 0$ and

$$p = \frac{E\delta_r}{2R}\left[1 - \frac{R^2}{r_o^2}\right]$$

Eqn 10.53, rearranged

- $r_i \neq 0$ only if the shaft has a hole in it.
- R is where the shaft and hub contact.

Analysis of Press Fits



If the shaft and hub are of <u>different</u> materials

$$p = \frac{\delta_r}{\frac{R}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{R}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)}$$

Eqn 10.51, rearranged

$\nu_{i,o}$ = Poisson

Once we have the pressure, we can use the cylinder equations to compute the hoop stresses at the interface.

A) The ID of the hub is tensile:

$$\sigma_{o_c} = p\,\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

Eqn 10.45

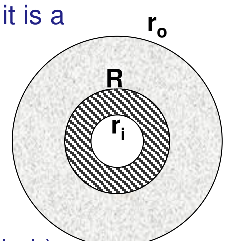
B) The OD of the shaft is compressive:

$$\sigma_{i_c} = -p\,\frac{R^2 + r_i^2}{R^2 - r_i^2}$$

Eqn 10.49

= -p if shaft is solid

Strain Analysis of Press Fits



The press fit has no axial pressure, so σ_l = 0, and it is a biaxial stress condition.

The circumferential strain

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu\,\sigma_r}{E}$$

Eqn 10.13

which equals the radial strain (because C = 2πr).

Because the radial change $\delta = R\,\varepsilon_r$, we get the <u>increase in Inner</u> <u>Radius</u> of the <u>outer member</u> (hub):

$$\delta_o = \frac{pR}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right)$$

Eqn 10.46

And the <u>decrease in Outer</u> <u>Radius</u> of the <u>inner member</u> (shaft):

$$\delta_i = -\frac{pR}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)$$

Eqn 10.50

These two deflections add up to the Radial Interference: $\left|\delta_i\right| + \left|\delta_o\right| = \delta_r$

Summary of Press Fits

1. Select amount of interference. Be careful about radial or diametral. Interference is really small – maybe 1 to 2 tenths of a percent of diameter.

2. Compute the pressure at the mating surface.

- If same materials, use Eqn. 10.52

$$p = \frac{E\delta_r}{R}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{2R^2\left(r_o^2 - r_i^2\right)}\right]$$

  - and if shaft is solid ($r_i = 0$): $p = \frac{E\delta_r}{2R}\left[1 - \frac{R^2}{r_o^2}\right]$

- If different materials, use Eqn. 10.51 (flipped)

$$p = \frac{\delta_r}{\frac{R}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{R}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)}$$

3. Compute the tensile hoop stress in the outer piece. Eqn. 10.45

$$\sigma_{o_t} = p\,\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

Press Fit Problem

A 1 inch diameter shaft is to be pressed into a 3 inch diameter hub with a radial interference of 0.001 inch. Both are AISI 1080 steel, Q&T 800°C. What is the resulting surface pressure and hoop stress in the hub?

Caution: Mind your radii and diameters.

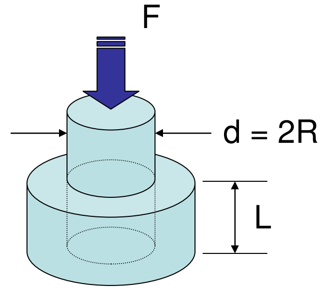
Press Fits: Force & Torque

The assembly force required will be

$$F_{max} = \pi d L p \mu$$

where p = the interface pressure

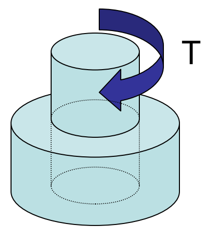
μ = the coefficient of friction



The torque capacity available is

$$T = FR = R\pi d L p \mu$$

where R = the interface radius, as before.



We know how to compute the interface pressure for these equations!

Hamrock Section 10.5.4

Press Fit Problem 2

For the 1 in. diameter steel shaft that we just calculated, if the friction coefficient is 0.15 and the hub is 1 in. thick, what are:

1. The force needed to press the parts together?
2. The maximum torque the joint could withstand?

Shrink Fits

If heating or cooling a part to achieve a shrink fit, the required radial interference is:

$$\Delta R = \boldsymbol{\delta_r} = R\alpha\Delta T$$

where R is the interface radius
α is the coefficient of thermal expansion
ΔT is the temperature change

To select an amount of interference see ANSI/ASME tables for class FN1 (light) to FN5 (Heavy-drive) fits.

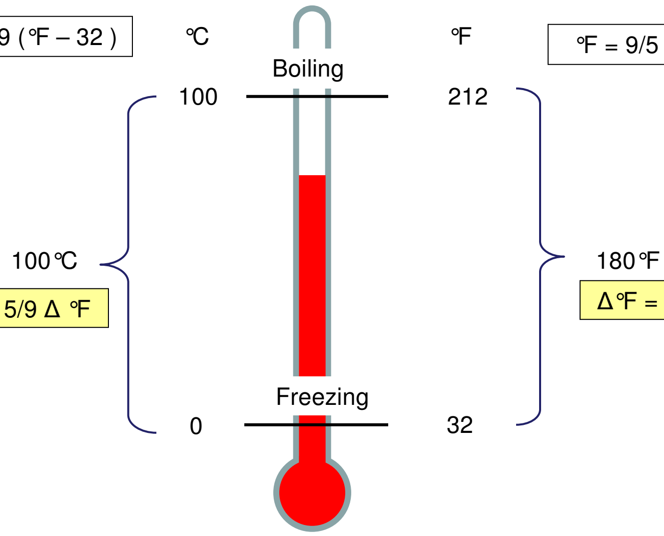
- They give interference in 0.001" on <u>diameter</u> for a range of diameters

Ex: FN4 for 0.95 to 1.19" diameter, interference = 1 to 2.3 mils on diameter.

Note: Conversion of <u>Temperature Change</u> is NOT the same as Converting Temperature

$°C = 5/9\ (°F - 32)$

$°F = 9/5\ °C + 32$



$\Delta °C = 5/9\ \Delta\ °F$

$\Delta °F = 9/5\ \Delta\ °C$

Press Fit Problem 3

For the 1 inch diameter steel shaft that we just calculated, how many degrees F would the hub need to heated to be able to assemble the parts without forcing? If we chose to cool the shaft instead, how many degrees C would the shaft need to be cooled to do the same thing?