ME311 Machine Design

Lecture 5: Fully-Reversing Fatigue and the S-N Diagram

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Discovering Fatigue

With the Industrial Revolution came the steam engine (~1760-1775) and then the locomotive (~1830). This began an age of dynamic loading that had never been seen before.

Even though equipment was designed with stresses below Yield, failures were still occurring. This was very publicly visible in the failure of bridges and of train wheel axles.
The Versailles Train Crash

One of the worst rail disasters of the 19th century occurred in May 1842 near Versailles, France. Following celebrations at the Palace of Versailles, a train returning to Paris crashed after the leading locomotive broke an axle. The carriages behind piled into the wrecked engines and caught fire. At least 55 passengers were trapped in the carriages and died.

Examination of several broken axles from British railway vehicles showed that they had failed by brittle cracking across their diameters (fatigue). The problem of broken axles was widespread on all railways at the time, and continued to occur for many years before engineers developed better axle designs, mostly resulting from improved testing of axles.
The S-N Diagram

A German engineer, August Wöhler, discovered that reversed loading was the cause, and in 1871 he produced the first S-N diagram:

The diagram shows the relationship between the stress amplitude of reversing loading and the number of cycles to failure.

Note that some materials have an Endurance Limit – a stress level below which the material can take unlimited cycles.
What’s up with fatigue?

Stage I: Initiation with Small Cracks
- **Shear** driven
- Interact with **microstructure**
- Mostly analyzed by **continuum mechanics** approaches

Stage II: Propagation with Large Cracks
- **Tension** driven
- Fairly **insensitive to microstructure**
- Mostly analyzed by **fracture mechanics** models

Microcracks act as localized stress concentrations to exceed Sy and support local plastic yielding. As the crack grows, the section is reduced, stresses increase, and propagation accelerates until the part fractures – usually rapidly and unexpectedly.

From Anders Ekberg, Chalmers U.
Crack Growth and Failure

The progression of fatigue cracking is sometimes visible as “beach marks” with each line representing the crack growth due to one load cycle:

The left photo shows that the crack grew and the remaining section reduced until P/A exceeded yield and the remainder of the part broke by yielding.

From Majid Mirzaei, TMU.
Characterizing Fatigue

Fatigue is very complex, so has been largely driven by test data from rotating beam tests.

Although moment is uniform, the narrowing specimen has max bending stress at the middle.

- Standard R.R. Moore test specimen is 0.3 in. diameter at thinnest section
- Specimen is polished
- Each revolution gives one fully reversing cycle
- At 3600RPM = 60 Hz, get 216,000 cycles per hour, 5.2 million cycles per day

Hamrock
Section 7.5
The S-N Diagram

The S-N diagram has three regions for materials with Endurance Limits: Low Cycle, High Cycle, & Infinite Life

S-N Diagram for Steel Specimen in Bending with $S_{ut} = 100$

The Low Cycle Fatigue (LCF) region is from 1 to 1000 cycles.
The High Cycle Fatigue (HCF) region is from 1000 to 1 million cycles.
The Infinite Life region is above 1 million cycles.

Some consider the Infinite Life region to begin at 10 million cycles.

Hamrock
Section 7.6
Constructing our S-N Diagram

1. Start with Sut at 1 cycle  (Because it can endure 1 cycle to Sut)
2. Plot the LCF value SL’ at 1000 cycles
3. Plot the Endurance Strength Se’ at 1 million cycles
4. Draw the first and third segments as straight lines
5. Connect SL’ and Se’ with a curve = aN^b
SL’ and Se’ Depend on the Type of Loading

Find the fatigue strength at the 1000 cycle Low Cycle Fatigue point, SL’

For steel, use:

Bending: \( SL' = 0.90 \text{ Sut} \)
Axial: \( SL' = 0.75 \text{ Sut} \) \((\text{Eqn. 7.7})\)
Torsion: \( SL' = 0.72 \text{ Sut} \)

Find the “raw” endurance strength, Se’

For steel, use (from Eqn. 7.6):

Bending: \( Se' = 0.50 \text{ Sut} \) up to a max \( Se' \) of 100 ksi
Axial: \( Se' = 0.45 \text{ Sut} \) up to a max \( Se' \) of 90 ksi
Torsion: \( Se' = 0.29 \text{ Sut} \) up to a max \( Se' \) of 58 ksi

This is \( Se' \) for a polished 0.3” diameter bar.

This is because \( Se' \) tops out when Sut \(~200 \text{ ksi. See Fig. 7.8}\)
Plotting the 1000 to 1 million Curve

Connect (1000, SL’) and (10^6, Se) with the curve

I use this.

\[ S_f = aN^b \text{ or } S_f = 10^cN^b \]

where

\[ a = \frac{(S_L')^2}{S_e'} \quad C = \log_{10} \left( \frac{(S_L')^2}{S_e'} \right) \quad \text{and} \quad b = -\frac{1}{3} \log_{10} \left( \frac{S_L}{S_e'} \right) \]

and \( N = \text{Number of cycles.} \)

If you have \( a \) or \( c \) and \( b \), and want to know the life for an alternating stress \( \sigma_{alt} \) between Se’ and SL’, compute

\[ N = \left( \frac{\sigma_{alt}}{a} \right)^{1/b} \text{ or } \left( \frac{\sigma_{alt}}{10^c} \right)^{1/b} \]
S-N Exercise

Plot the S-N diagram for a steel with $S_{ut} = 120$ksi under fully reversing Axial loading. Find a & b, and the $S_f$ at 100,000 cycles.
Reality Sets In

If your actual product doesn’t happen to be 0.3” diameter polished bars, then you must make some modifications to the Endurance Limit to make this real.

The modification (Marin) factors are:

\[ \begin{align*}
  k_f & \quad \text{Surface effect – if not polished} \\
  k_s & \quad \text{Size effect – if not } \leq 0.3” \text{ diameter} \\
  k_r & \quad \text{Reliability effect – if other than 50% survival} \\
  k_t & \quad \text{Temperature effect – if not at Room Temperature} \\
  k_m & \quad \text{Miscellaneous – Mat’l processing, Residual stress, Coatings, Corrosion}
\end{align*} \]

Then \[ S_e = k_f k_s k_r k_t k_m S'_e \] (My Part) \[ (\text{Test Specimen}) \]

Another factor is \( K_f \), the fatigue stress concentration factor.
Surface Finish Factor

Equation 7.19 gives

\[ k_f = e S_{ut}^f \]

where \( S_{ut} \) is the material’s Ultimate Tensile Strength, and \( e \) & \( f \) are coefficients defined in Table 7.3.

Note 1: \( e \) is NOT the “exponential”.
Note 2: \( S_{ut} \) value is entered as MPa or ksi, NOT Pa or psi. In other words, if \( S_{ut} = 100 \text{ksi} \), you use the value 100.

Exercise: What is \( k_f \) for AISI 1080 steel that has a machined surface?

Why do we need to correct for surface finish?
Surface Finish Factor Plot

Note that Hamrock’s Fig. 7.11a is distorted, especially for the Ground and As Forged curves. It should look like this:
Size Factor

Equation 7.20 gives:

\[ k_s = 0.869d^{-0.112} \quad \text{For } d \text{ in inches from 0.3 to 10"} \]

\[ k_s = 1 \quad \text{For } d \leq 0.3" \text{ or } \leq 8\text{mm} \]

\[ k_s = 1.248d^{-0.112} \quad \text{For } d \text{ in mm from 8 to 250mm} \]

for Bending or Torsional Loading.

If the loading is Axial, \( k_s = 1 \) for all sizes.

Exercise: What is \( k_s \) for a 1 inch diameter steel bar?

Why do we need to correct for part size?
Size Factor

Here is what the Size Factor looks like:

Diameter (in) vs. Size Factor ($K_s$) graph. The graph shows a decreasing trend for $K_s$ as the diameter increases from 0.3" to 2".
Reliability Factor

A standard Endurance Strength value is based on 50% survival.

If you think your customers would like a better durability than that, you need to correct for it.

<table>
<thead>
<tr>
<th>Probability of Survival, %</th>
<th>Reliability Factor, ( k_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.00</td>
</tr>
<tr>
<td>90</td>
<td>0.90</td>
</tr>
<tr>
<td>95</td>
<td>0.87</td>
</tr>
<tr>
<td>99</td>
<td>0.82</td>
</tr>
<tr>
<td>99.9</td>
<td>0.75</td>
</tr>
<tr>
<td>99.99</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Exercise: What is \( k_r \) for a product where you only want 1 in 10,000 to fail?
Stress Concentration with Notch Sensitivity

For fatigue, stress concentration is a function of both geometry AND the material and type of loading. That’s where Notch Sensitivity comes in. It adds the material effect to the geometric effect.

\[ K_f = 1 + (K_c - 1)q \]

\( q = 0 \) means \( K_f = 1 \); \( q = 1 \) means \( K_f = K_c \)
Stress Concentration with Notch Sensitivity

Stress concentration for fatigue is a three-step process:
1. Calculate the Kc as you would for a static case.
2. Use a notch sensitivity table for your material, loading type, and notch radius.
3. Compute $K_f = 1 + (Kc-1)q$

Then you can either divide $S_e$ by $K_f$ or multiply your actual alternating stress $\sigma_{alt}$ by $K_f$.

Exercise: What is $K_f$ for a 446 Stainless Steel shaft loaded in bending with a 1mm notch radius, if the Kc read from Fig. 6.6b is 1.4.

$K_f$ is an “awareness” topic, and will not be in homework or numerically on an exam.
Summary

The effect of all of this modification is to correct the Endurance Limit to properly represent your part. The corrected S-N diagram looks like this:

The red lines show the changes from the test specimen values to ones that represent your part.
FATIGUE EXAMPLE

Cantilever Beam

CASE 1: Tip is flexed ±0.075 in.  What is life for 95% survival?

\[ y_{\text{max}} = \frac{F l^3}{3EI} = \frac{F l^3}{3Eb h^3} \]

\[ 0.075 = \frac{F (4)^3 (12)}{(3)(30 \times 10^6)(0.75)(0.1094)^3} \]

\[ F = 8.631 \text{lb} \]

Stiffness \( k = \frac{F}{\delta} = \frac{8.631}{0.075} = 115.1 \text{lb/in} \)

\[ M = Fl = (8.631)(4) = 34.52 \text{ lb/in.} \]

\[ \sigma = \frac{Mc}{I} = \frac{(34.52)(0.1094/2)(12)}{(0.75)(0.1094)^3} = \pm 23,076 \text{ psi} \]

Since \( S_{ut} = 245 \text{ ksi} > 200 \text{ ksi} \), we set \( S_e' \) not equal to \( S_{ut}/2 = 122.5 \text{ ksi} \), but limit it to the maximum of 100 ksi.

Details

12 Gauge (0.1094” thick)
0.75 in. wide
4 in. long
High Strength Steel, with
\( S_{ut} = 245 \text{ ksi} \)
Machined finish
Room Temperature
FATIGUE EXAMPLE, cont’d

Surface Factor (Figure 7.10a with Sut = 245 ksi) = 0.63
Check: Table 7.3: $e = 2.70$ ksi, $f = -0.265$
\[ k_f = 2.7(245)^{-0.265} = (2.7)(0.233) = 0.628 \]

Size Factor
Area = $(0.75)(0.1094) = 0.08205 \text{ in}^2$
Area loaded > 95% stress is 5% of Area = $(0.05)(0.08205) = 0.0041 \text{ in}^2$
Equivalent diameter from Eqn. 7.20:
\[ d = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{0.0041}{0.0766}} = \sqrt{0.05356} = 0.2314 \text{ in.} \]
Because this diameter is less than 0.3 in, we don't use $k_s = 0.869d^{-0.112}$, but just set $k_s = 1$.

Reliability
From Table 7.4, $k_r = 0.87$

Derated Endurance Strength
\[ S_e = k_f k_s k_r S'_e = (0.63)(1)(0.87)(100) = 54.8 \text{ ksi} \]
\[ \sigma_{alt} = 23.1 \text{ ksi} < 54.8 \text{ ksi Endurance Strength}, \therefore \text{Life is } \infty \text{.} \]
FATIGUE EXAMPLE, cont’d

Question: How big could the stress concentration at the attachment be and still have 100,000 cycles of life?

Draw the S-N Diagram

\[
S_{ul} = 245 \text{ ksi} \\
S_{L'} = 0.90 S_{ul} = 220.5 \text{ ksi} \\
S_e = k_l k_s k_f S_e' = 54.3 \text{ ksi}
\]
FATIGUE EXAMPLE, cont’d

Calculate the stress amplitude for a life of 100,000 cycles.

\[ a = \frac{S_L^2}{S_e} = \frac{220.5^2}{54.8} = 887.2 \text{ ksi} \]

\[ b = -\frac{1}{3} \log_{10} \left( \frac{S_L}{S_e} \right) = -\frac{1}{3} \log_{10} \left( \frac{220.5}{54.8} \right) = -\frac{1}{3} \log_{10}(4.024) = -\frac{0.605}{3} = -0.202 \]

\[ S_f = aN^b = 887.2(100,000)^{-0.202} = (887.2)(0.098) \]

\[ S_f = 87.16 \text{ ksi} \]

So the stress concentration would have to be \( \frac{87.16}{23.1} = 3.77 \).
Cumulative Fatigue Damage: Miner’s Rule

- In practice, the fatigue loading on structures is rarely constant amplitude.
- To estimate the effect of operating at a variety of stress levels, you can use the linear damage rule, aka Miner’s rule.
- You take the number of cycles at each stress level and divide that by the life if only run at that stress level to get a damage fraction.
- You add up all of the damage fractions and see if the total exceeds one.

Example:
A stress amplitude of 87ksi has a life of 13,300 cycles.

Running 5,000 cycles at 87ksi is a damage fraction of 5/13.3 = 0.38
Example 7.7: Cumulative Damage

Mat'l: Steel with Axial Loading
Sut=440MPa.
SL'=75%Sut =330MPa.
Se'=45%Sut = 200MPa

<table>
<thead>
<tr>
<th>Loading</th>
<th>20% @ 175MPa</th>
<th>30% @ 220MPa</th>
<th>40% @ 250MPa</th>
<th>10% @ 275MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life @ Load</td>
<td>∞</td>
<td>268,564</td>
<td>46,048</td>
<td>12,367</td>
</tr>
</tbody>
</table>

S-N Diagram for Steel Specimen with Sut = 440MPa, Axial Load

\[
\begin{align*}
0.2 \frac{L}{268,564} + 0.3 \frac{L}{46,048} + 0.4 \frac{L}{12,367} &= 1 \\
0.2 + 0.3 \frac{L}{268,564} + 0.4 \frac{L}{46,048} + 0.1 \frac{L}{12,367} &= \frac{1}{L} \\
0 + 1.117 \times 10^{-6} + 8.687 \times 10^{-6} + 8.086 \times 10^{-6} &= \frac{1}{L} \\
1.78899 \times 10^{-5} &= \frac{1}{L} \\
L &= 55,897 \text{ Cycles}
\end{align*}
\]
Avoiding Fatigue Failure

A checklist of good practices.

1. Control stress risers like holes, sharp corners, grooves, threads, keyways, and stamped markings.
2. Control surface finish, including scratches in critical areas.
3. Avoid corrosion and exposure to embrittling gases.
4. Be very careful with welds – spot welds and fillet welds.
5. Higher temperatures usually reduce fatigue strength.
6. Manage residual stresses from fabrication operations, including welding.
7. Make critical areas inspectable for cracks.
8. Design in stress margin.
9. Measure real load environment and test parts to determine actual fatigue life.
10. Design in redundant load paths.
Avoiding Fatigue Failure

One more thing – don’t put square windows in airplanes…

Metal fatigue became apparent to aircraft engineers in 1954 after three de Havilland Comet passenger jets had broken up in mid-air and crashed within a single year.

The sharp corners around the plane's window openings acted as initiation sites for cracks. The skin of the aircraft was also too thin, and cracks from manufacturing stresses were present at the corners. All aircraft windows were immediately redesigned with rounded corners.

One last thing – this lecture applies to Ferrous alloys and to Titanium. It does not apply to Aluminum alloys.