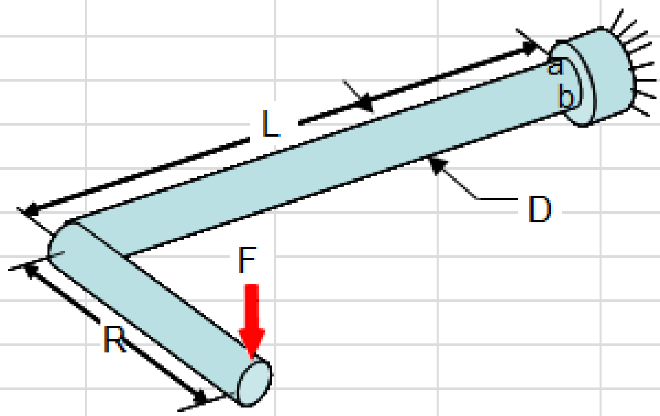
Bending and Torsional Stress

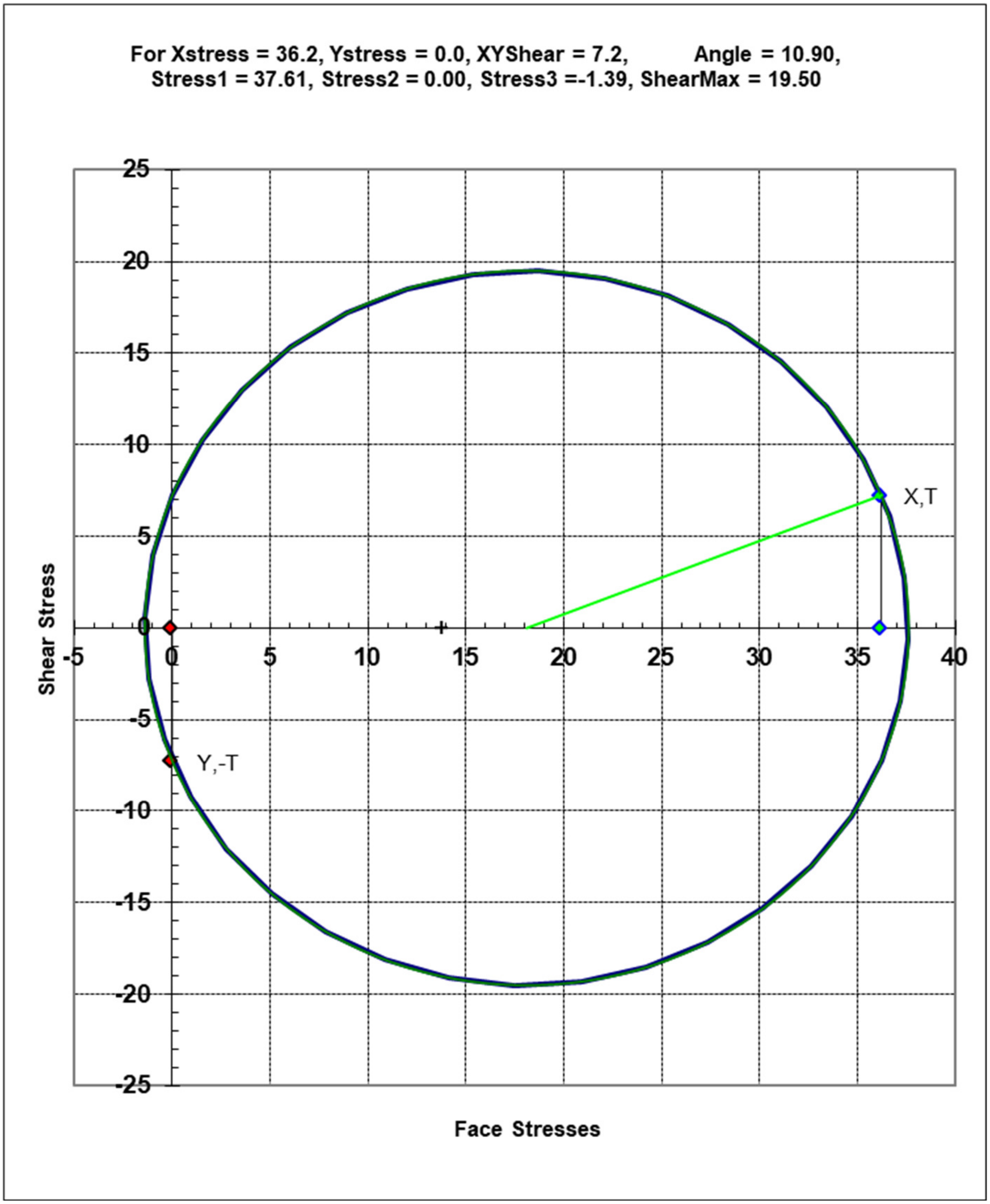
HW 3.1



R	6	in	Short Arm
L	15	in	Long Arm
D	0.75	in	Diameter
F	100	lb	Force
Sut	76000	psi	
Sy	42000	psi	
I	0.0155	in^4	=PI()*D^4/64
J	0.0311	in^4	=PI()*D^4/32
C_	0.375	in	=D/2
A	0.442	in^2	=PI()*D^2/4
Torsion	600	in lb	=F*R_
Moment	1500	in lb	=F*L
MaxBending	36,216.59	psi	=M*C_/I
MaxTorsion	7,243.32	psi	=T*(D/2)/J
AvgShear	226.4	psi	=F/A
TransverseShear	301.8	psi	=4*F/(3*A)
At "a":			
SigmaX	36,216.59	psi	MaxBending
SigmaY	0	psi	
TauXY	7,243.32	psi	MaxTorsion
At "b":			
SigmaX	0	psi	
SigmaY	0	psi	
TauXY	7545.12	psi	MaxTorsion + Transverse Shear

At "b", Sigma1 = Sigma2 = TauXY = 7545 psi

Mohr's Circle for "a"



Deflection of the Beam Tip at F

E	10,100,000	lb/in^2		
Poisson	0.33			
G	3,796,992	lb/in^2	=E/(2*(1+Poisson))	$G = \frac{E}{2(1+\nu)}$
Bending of Short Arm				
Ybs	0.0459	in	=F*R_^3/(3*E*I)	$y = \frac{Fl^3}{3EI}$
Bending of Long Arm				
Ybl	0.7172	in	=F*L^3/(3*E*I)	$y = \frac{Fl^3}{3EI}$
Twist of Long Arm				
ThetaL	0.0763	Rad	=T*L/(J*G)	$\theta = \frac{Tl}{JG}, \quad y = \theta R$
	4.37	°	=ThetaL*180/PI()	
Yts	0.4578	in	=R_*ThetaL	
Ytotal	1.221	in	=Ybs+Ybl+Yts	

HW 3.2

Base	b	0.04	m	
Height	h	0.1	m	
Length	L	2	m	
Modulus	E	207	GPa	
MomOfInertia	I	3.3333E-06	m^4	=b*h^3/12
Load	P	4000	N	
Location	a	1.2	m	
Moment	M	2500	Nm	

We use Superposition to determine the deflections first from the Force and then from the Moment and then we add them together at P and at the Tip.

$$y = \frac{Px^2}{6EI}(3a - x) \quad \text{for} \quad 0 < x < a$$

$$y = \frac{Pa^2}{6EI}(3x - a) \quad \text{for} \quad a < x < l$$

From Hamrock Appendix D.2

Deflections due to Force P

At P

XP	1.2	m	=a
YP-Force	-0.003339	m	=IF(XP<a,P*XP^2/(6*E*10^9*I)*(XP-3*a),P*a^2/(6*E*10^9*I)*(a-3*XP))
	-3.339	mm	

At Tip

Xtip	2	m	=L
Ytip-Force	-0.006678	m	=IF(Xtip<a,P*Xtip^2/(6*E*10^9*I)*(Xtip-3*a),P*a^2/(6*E*10^9*I)*(a-3*Xtip))
	-6.678	mm	

Deflections due to Moment M

At P

From Hamrock Appendix D.4

$$y = \frac{Mx^2}{2EI}$$

XP	1.2	m	=a
YP-Moment	0.002609	m	=M*XP^2/(2*E*10^9*I)
	2.609	mm	

At Tip

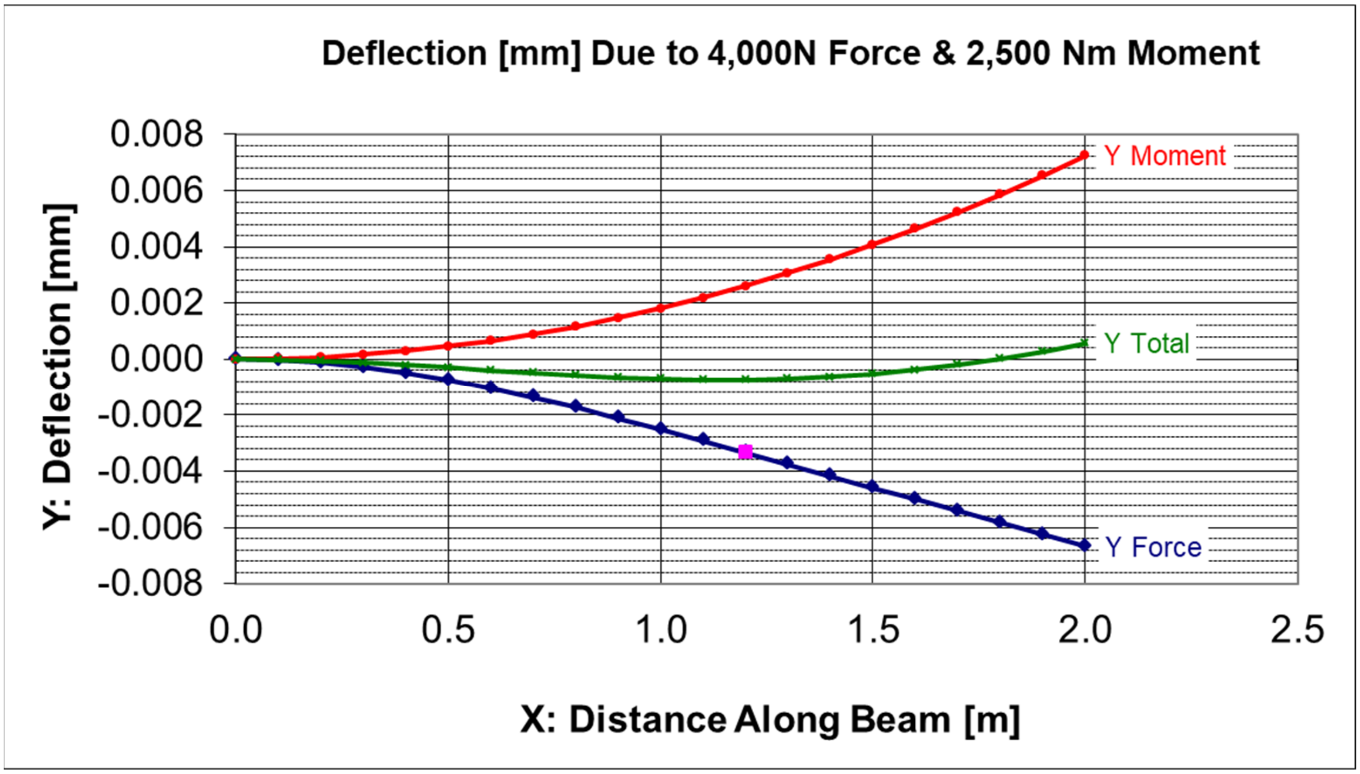
Xtip	2	m	=L
Ytip-Moment	0.007246	m	=M*Xtip^2/(2*E*10^9*I)
	7.246	mm	

X At P	y Force	y Moment	y Total	
1.2	-3.339	2.609	-0.730	mm

X At Tip	y Force	y Moment	y Total	
2	-6.678	7.246	0.568	mm

For Zero deflection at P, deflection from Moment must equal deflection from Force P, which is -3.339 mm
So we just proportion the Moment by the ratio of the deflections to reduce the deflection.
In other words, if 2,500 Nm caused 2.609mm deflection, how much moment causes 3.339mm deflection?
Deflection is directly proportional to force or moment, so we just proportion the Moment.

M target	3200	Nm	=M*YP_Force/-YP_Moment



		Deflection (mm)		
X		Y Force	Y Moment	Y Total
0.0		0.000	0.0000	0.000
0.1		-0.034	0.0181	-0.016
0.2		-0.131	0.0725	-0.059
0.3		-0.287	0.1630	-0.124
0.4		-0.495	0.2899	-0.205
0.5		-0.749	0.4529	-0.296
0.6		-1.043	0.6522	-0.391
0.7		-1.373	0.8877	-0.485
0.8		-1.731	1.1594	-0.572
0.9		-2.113	1.4674	-0.646
1.0		-2.512	1.8116	-0.700
1.1		-2.923	2.1920	-0.731
1.2		-3.339	2.6087	-0.730
1.3		-3.757	3.0616	-0.695
1.4		-4.174	3.5507	-0.623
1.5		-4.591	4.0761	-0.515
1.6		-5.009	4.6377	-0.371
1.7		-5.426	5.2355	-0.191
1.8		-5.843	5.8696	0.026
1.9		-6.261	6.5399	0.279
2.0		-6.678	7.2464	0.568