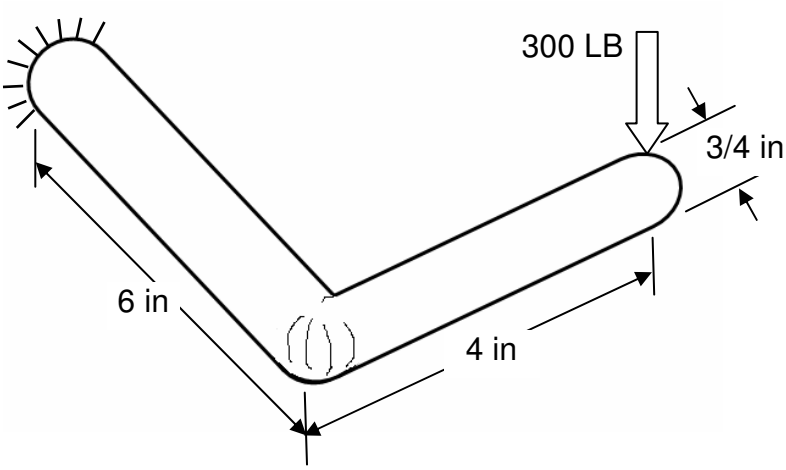
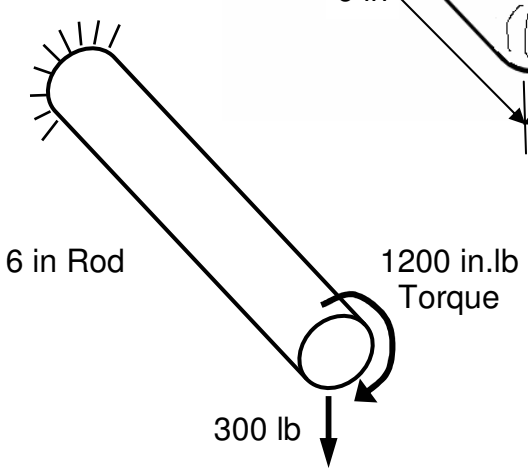
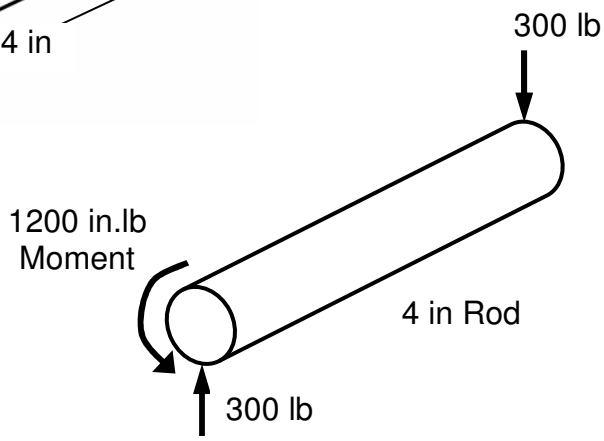
Piece-Wise Deflection of a Crank Arm



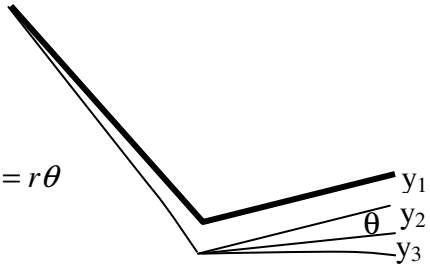




Deflection of the loaded end point is the sum of three deflections:

1. Bending of the 6" Rod due to the 300 lb load. $y_1 = \frac{Fl^3}{3EI}$

2. Twisting of the 6" Rod due to the 1200 in.lb torque, with rotation of the 4" Rod. $\theta = \frac{Tl}{JG}, \quad y_2 = r\theta$



3. Bending of the 4" Rod due to the 300 lb load. $y_3 = \frac{Fl^3}{3EI}$

$r = 0.375" \qquad I = \frac{\pi r^4}{4} = 0.01553\ in^4$

$J = \frac{\pi r^4}{2} = 2I = 0.03106\ in^4$

$E = 30\times10^6\ psi, \qquad G = 11.5\times10^6\ psi$

$$y_1 = \frac{(300)(6)^3}{3(30\times10^6)(0.01553)} = 0.0464\,in$$

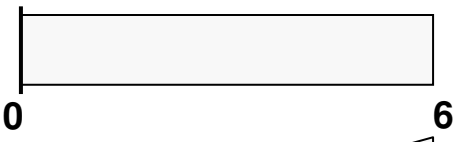
$$y_2 = r\theta = \frac{(4)(1200)(6)}{(0.03106)(11.5\times10^6)} = 0.0806\,in$$

$$y_3 = \frac{(300)(4)^3}{3(30\times10^6)(0.01553)} = 0.0137\,in$$

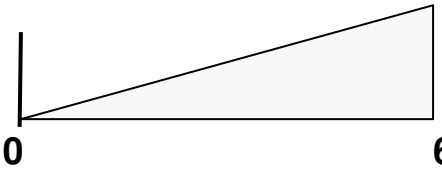
$$y_{total} = y_1 + y_2 + y_3 = 0.1407\,in$$

Crank Deflection Analysis by Castigliano

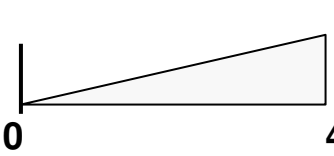
Torsion in 6" Rod



Bending in 6" Rod



Bending in 4" Rod



Strain Energy$_{TOTAL}$ = Bending (in 4") + Bending (in 6") + Torsion (in 6")

$$U = \int_{Tip}^{Elbow} \frac{M_4^2}{2EI}dx + \int_{Elbow}^{Base} \frac{M_6^2}{2EI}dx + \int_{Elbow}^{Base} \frac{T^2}{2GJ}dx$$

$$U = \frac{1}{2EI}\int_0^4 (Px)^2 dx + \frac{1}{2EI}\int_0^6 (Px)^2 dx + \frac{(4P)^2}{2GJ}\int_0^6 dx$$

$$U = \frac{P^2}{2EI}\int_0^4 x^2 dx + \frac{P^2}{2EI}\int_0^6 x^2 dx + \frac{16P^2}{2GJ}\int_0^6 dx$$

$$U = \frac{P^2}{2EI}\frac{1}{3}x^3\Big|_0^4 + \frac{P^2}{2EI}\frac{1}{3}x^3\Big|_0^6 + \frac{8P^2}{GJ}x\ \Big|_0^6$$

$$U = \frac{P^2}{6EI}4^3 + \frac{P^2}{6EI}6^3 + \frac{8P^2}{GJ}6 \ = \ \frac{64P^2}{6EI} + \frac{216P^2}{6EI} + \frac{48P^2}{GJ}$$

$$U = P^2\left(\frac{64}{6EI} + \frac{216}{6EI} + \frac{48}{GJ}\right) = P^2\left(\frac{10.67}{EI} + \frac{36}{EI} + \frac{48}{GJ}\right)$$

$$\delta_P = \frac{\partial U}{\partial P} = 2P\left(\frac{10.67}{EI} + \frac{36}{EI} + \frac{48}{GJ}\right)$$

$$\delta_P = 600\left(\frac{10.67+36}{(30\times10^6)(0.01553)} + \frac{48}{(11.5\times10^6)(0.03106)}\right)$$

$$\delta_P = 600(0.00002289 + 0.00007727 + 0.0001344)$$

$$\delta_P = (0.01374 + 0.04636 + 0.08064) = 0.1407\ in.$$

Finite Element Analysis of the Crank Arm with 300LB Tip Load