Solution of the Bean Can Cantilever Beam Design

A 1-g acceleration (gravity) makes the can's mass weigh 1lb, so a 10-g deceleration will make the force on the can be 10lb. Using the Impact equation (7.58)

$$I_m = \frac{P_{\max}}{W} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = \frac{10}{1} = 10$$

$$\sqrt{1 + \frac{2h}{\delta_{st}}} = 9 \ ; \quad 1 + \frac{2h}{\delta_{st}} = 9^2 = 81$$

$$\frac{2h}{\delta_{st}} = 80; \qquad \delta_{st} = \frac{2h}{80} = \frac{h}{40} = \frac{5}{40} = 0.125 inch$$

Rearranging Eqn. 7.55

$$k = \frac{W}{\delta_{st}} = \frac{1}{0.125} = 8 lb/in$$

Then we get the equation for force and displacement of the tip of a cantilever beam, either from Hamrock's Appendix D or from Example 7.10:

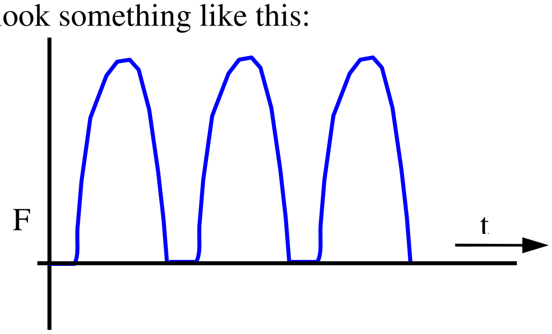
$\delta = \dfrac{Fl^3}{3EI}$ Then the stiffness of the cantilever beam $k = \dfrac{F}{\delta} = \dfrac{3EI}{l^3}$.

Because the area moment of inertia, I, for a rectangular beam is $bh^3/12$, we can write

$\dfrac{3Ebh^3}{12l^3} = 8; \quad or \dfrac{Ebh^3}{l^3} = 32.$ I'll flip this over, plug in 30 million psi for E, and write

$\dfrac{l^3}{bh^3} = \dfrac{30{,}000{,}000}{32} = 937{,}500 in^{-1}$.

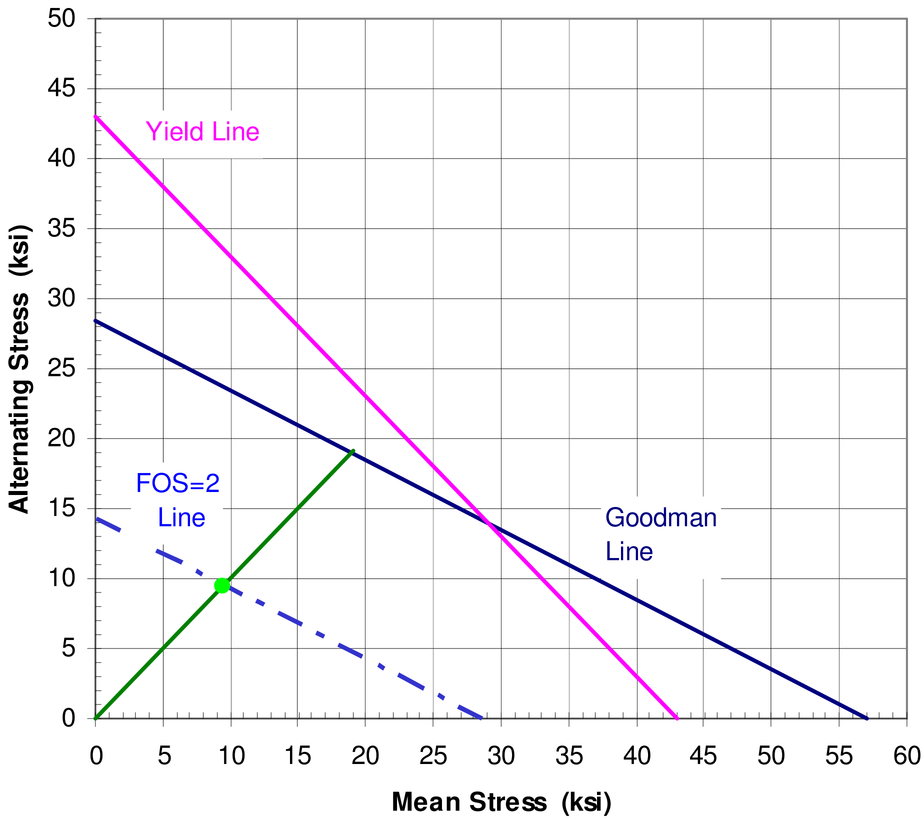
Now we have to figure out the maximum alternating stress to satisfy the infinite fatigue life. The cans hit the beam from only one side, so the deflection goes from zero when no can is there, to 1/8" x 10 = 1.25" when each can hits, and back to zero. The force might look something like this:



This is fluctuating fatigue, and we need to use a Goodman diagram. This is also a special case where $\sigma_{min} = 0$, so $\sigma_{alt} = \sigma_{mean} = \sigma_{max}/2$.

AISI 1020 steel Q&T 870°C has a Sut of 57ksi and an Sy of 43ksi. This is bending, so its endurance strength is Se' = 0.5 Sut = 28.5 ksi. We'll assume all of the modification factors are one.

Life is defined by the Goodman diagram shown here. Notice that our Load Line has a slope of 1, because $\sigma_{alt} = \sigma_{mean}$ for all values of σ_{max}. We want to find the Alternating Stress where the Load Line intersects the FOS = 2 line.



From Slide 10 of Lecture 6, we have an equation for this

$$\sigma_{alt_{Lim}} = \frac{S_e}{n}\frac{1}{\left(1+\frac{\sigma_M}{\sigma_A}\frac{S_e}{S_{ut}}\right)} = \frac{28.5}{2}\frac{1}{\left(1+(1)\frac{28.5}{57}\right)} = (14.25)\frac{1}{(1+0.5)} = \frac{14.25}{1.5} = 9.5\,ksi$$

Because $\sigma_{max} = \sigma_{alt}$ x 2, the max stress is 19 ksi. Because this is bending, we can write

$$\sigma_{max} = 19{,}000 = \frac{Mc}{I} = \frac{(F \times l)(h/2)}{bh^3/12} = \frac{10l}{bh^2/6} = \frac{60l}{bh^2}$$

$$or \qquad \frac{l}{bh^2} = 316.67$$

Now I'll go back and get the stiffness equation and regroup it to

$$\frac{l^3}{bh^3} = \left(\frac{l^2}{h}\right)\left(\frac{l}{bh^2}\right) = 937{,}500\,in^{-1}$$ and substitute in 316.67 for l/bh^2 to get

$$\frac{l^2}{h} = \frac{937{,}500}{316.67} = 2960.5$$

I can just pick a reasonable length for the beam, and compute the thickness.

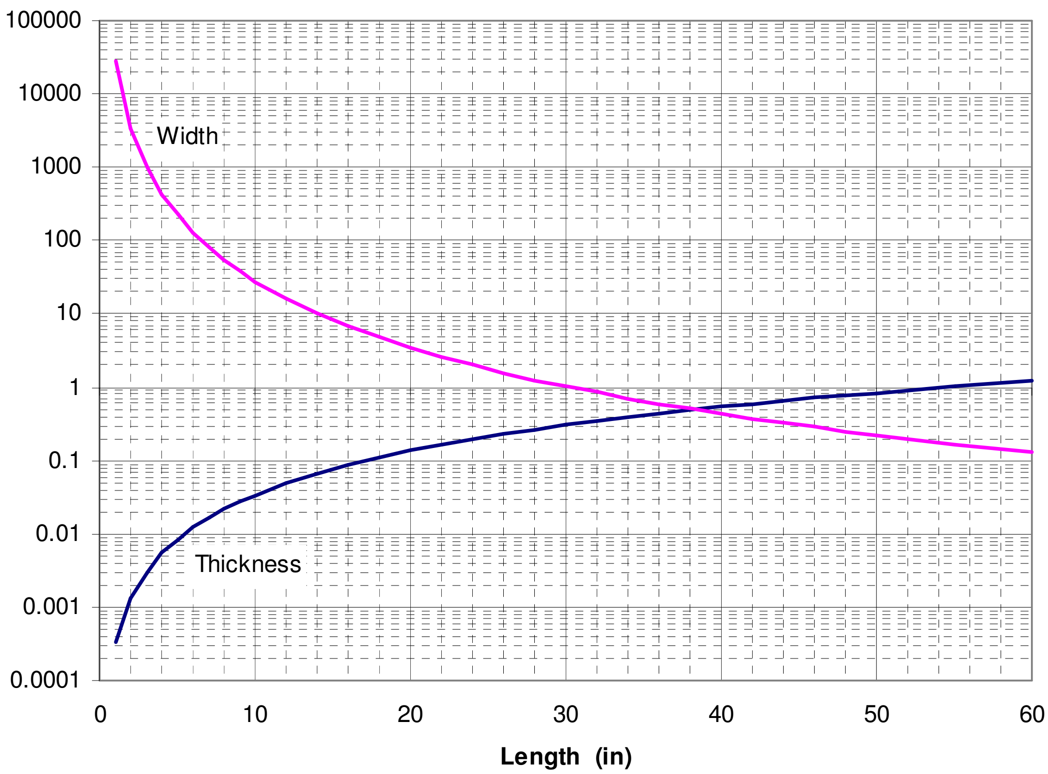
If I pick a length of 18", the thickness $h = \dfrac{l^2}{2960.5} = \dfrac{18^2}{2960.5} = \dfrac{324}{2960.5} = 0.1094"$.

I might go pick a standard thickness close to that, say 1/8" = 0.125", and recalculate the length $l^2 = 2960.5h = (2960.5)(0.125) = 370.06$, or $l = 19.237$ inches.

Then I can go back and get my stress equation and calculate the width.

$$\frac{l}{bh^2} = 316.67; \quad so\ b = \frac{l}{316.67h^2} = \frac{19.237}{(316.67)(0.125)^2} = \frac{19.237}{4.95} = 3.89\,inches.$$

If I go put this in Excel, I can see how width and thickness vary for a range of lengths. Short beams get really wide and thin; long beams get really narrow.



A very interesting thing (I think) is that the volume of all of these beams is exactly the same, 9.34903 cubic inches.