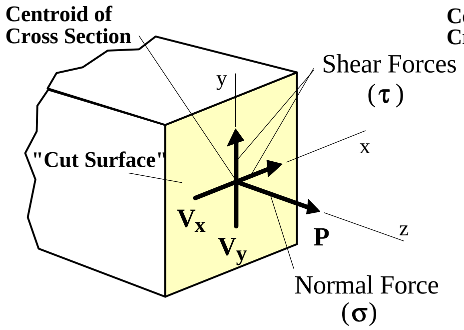
Dr. D. B. Wallace

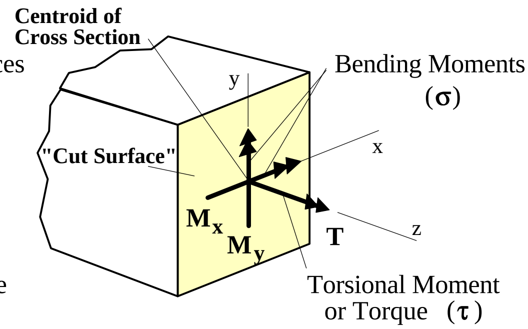
Basic Stress Equations

Internal Reactions:

6 Maximum
(3 Force Components & 3 Moment Components)

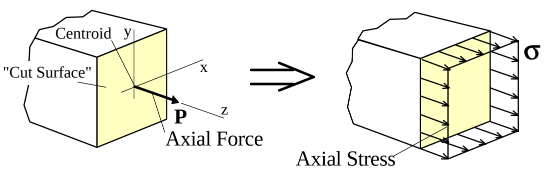


Force Components



Moment Components

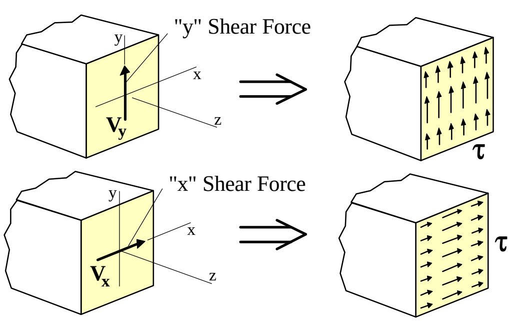
Normal Force:

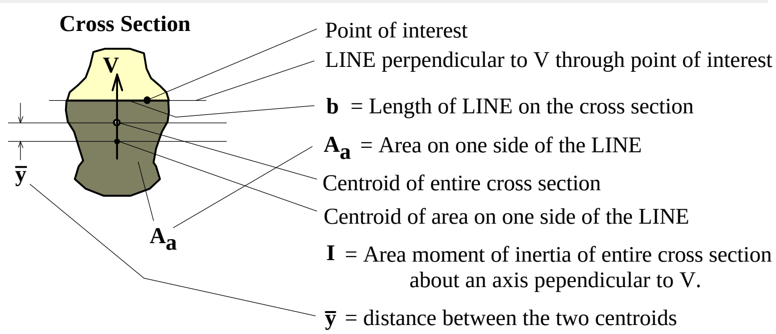


$$\sigma = \frac{P}{A}$$

- Uniform over the entire cross section.
- Axial force must go through centroid.

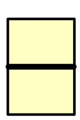
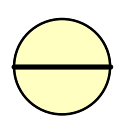
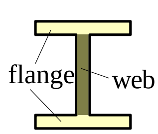
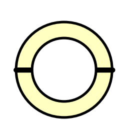
Shear Forces:





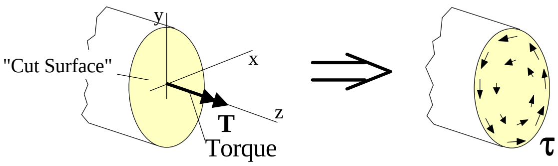
$$\tau = \frac{V \cdot \left(A_a \cdot \bar{y}\right)}{I \cdot b}$$

***Note*:** The <u>maximum</u> shear stress for common cross sections are:

<u>Cross Section</u>:			**<u>Cross Section</u>:**		
Rectangular:		$\tau_{max} = 3/2 \cdot V/A$	Solid Circular:		$\tau_{max} = 4/3 \cdot V/A$
I-Beam or H-Beam:		$\tau_{max} = V/A_{web}$	Thin-walled tube:		$\tau_{max} = 2 \cdot V/A$

Torque or Torsional Moment:

Solid Circular or Tubular Cross Section:



$$\tau = \frac{T \cdot r}{J}$$

r = Distance from shaft axis to point of interest
R = Shaft Radius
D = Shaft Diameter

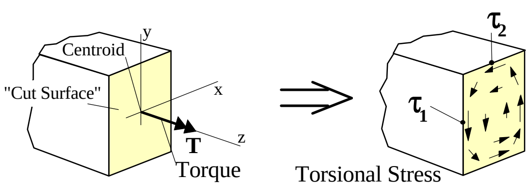
$$J = \frac{\pi \cdot D^4}{32} = \frac{\pi \cdot R^4}{2} \quad \text{for solid circular shafts}$$

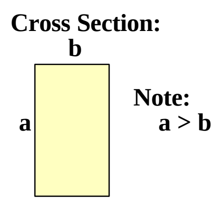
$$J = \frac{\pi \cdot \left(D_o{}^4 - D_i{}^4\right)}{32} \quad \text{for hollow shafts}$$

$$\tau_{max} = \frac{16 \cdot T}{\pi \cdot D^3} \quad \text{for solid circular shafts}$$

$$\tau_{max} = \frac{16 \cdot T \cdot D_o}{\pi \cdot \left(D_o{}^4 - D_i{}^4\right)} \quad \text{for hollow shafts}$$

Rectangular Cross Section:





Method 1:

$$\tau_{max} = \tau_1 = T \cdot (3 \cdot a + 1.8 \cdot b) \big/ \left(a^2 \cdot b^2\right)$$

ONLY applies to the center of the longest side

Method 2:

$$\tau_{1,2} = \frac{T}{\alpha_{1,2} \cdot a \cdot b^2}$$

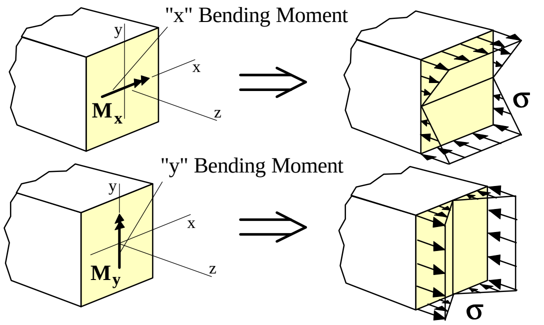
Use the appropriate **α** from the table on the right to get the shear stress at either position **1** or **2**.

a/b	**α_1**	**α_2**
1.0	.208	.208
1.5	.231	.269
2.0	.246	.309
3.0	.267	.355
4.0	.282	.378
6.0	.299	.402
8.0	.307	.414
10.0	.313	.421
∞	.333	----

Other Cross Sections:

Treated in advanced courses.

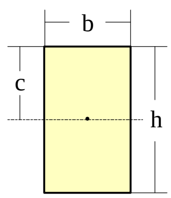
Bending Moment



$$\sigma = \frac{M_x \cdot y}{I_x} \quad \text{and} \quad \sigma = \frac{M_y \cdot x}{I_y}$$

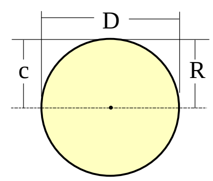
where: $\mathbf{M_x}$ and $\mathbf{M_y}$ are moments about indicated axes
y and **x** are perpendicular from indicated axes
$\mathbf{I_x}$ and $\mathbf{I_y}$ are moments of inertia about indicated axes

Moments of Inertia:



$$\bar{I} = \frac{b \cdot h^3}{12} \quad \text{h is perpendicular to axis}$$

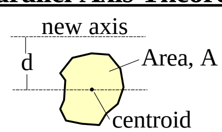
$$Z = \frac{\bar{I}}{c} = \frac{b \cdot h^2}{6}$$



$$\bar{I} = \frac{\pi \cdot D^4}{64} = \frac{\pi \cdot R^4}{4}$$

$$Z = \frac{\bar{I}}{c} = \frac{\pi \cdot D^3}{32} = \frac{\pi \cdot R^3}{4}$$

Parallel Axis Theorem:



$$I = \bar{I} + A \cdot d^2$$

I = Moment of inertia about new axis
$\bar{I}$ = Moment of inertia about the centroidal axis
A = Area of the region
d = perpendicular distance between the two axes.

Maximum Bending Stress Equations:

$$\sigma_{max} = \frac{M \cdot c}{I} = \frac{M}{Z}$$

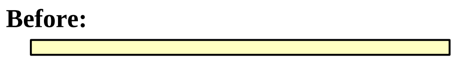
$$\sigma_{max} = \frac{32 \cdot M}{\pi \cdot D^3} \quad (\text{Solid Circular})$$

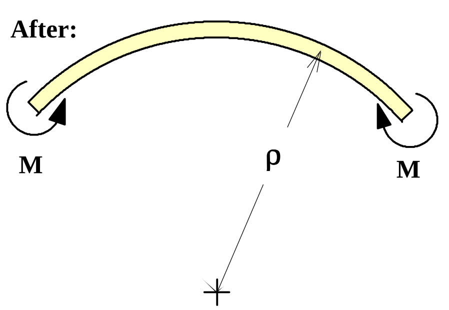
$$\sigma_{max} = \frac{6 \cdot M}{b \cdot h^2} \quad (\text{Rectangular})$$

The section modulus, **Z**, can be found in many tables of properties of common cross sections (i.e., I-beams, channels, angle iron, etc.).

Bending Stress Equation Based on Known Radius of Curvature of Bend, ρ.

The beam is assumed to be initially straight. The applied moment, **M**, causes the beam to assume a radius of curvature, ρ.





$$\sigma = E \cdot \frac{y}{\rho}$$

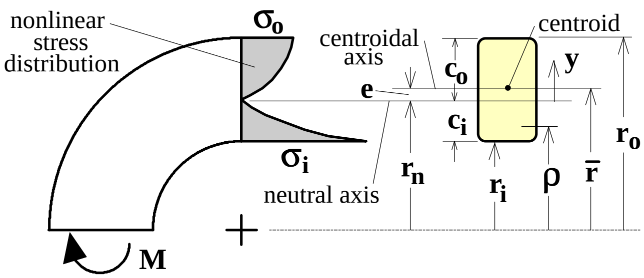
E = Modulus of elasticity of the beam material

y = Perpendicular distance from the centroidal axis to the point of interest (same **y** as with bending of a straight beam with $\mathbf{M_x}$).

ρ = radius of curvature to centroid of cross section

Bending Moment in Curved Beam:

Geometry:



$$r_n = \frac{A}{\int_{area} \frac{dA}{\rho}}$$

$$e = \bar{r} - r_n$$

A = cross sectional area

r_n = radius to neutral axis

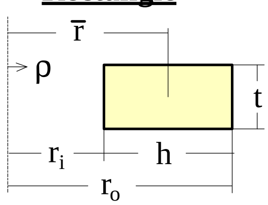
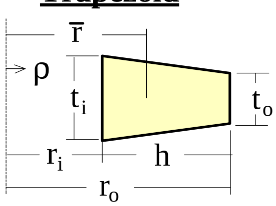
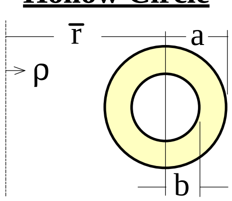
$\bar{r}$ = radius to centroidal axis

e = eccentricity

Stresses:

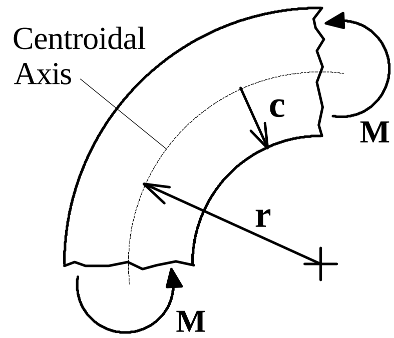
Any Position:	**Inside** *(maximum magnitude)*:	Outside:
$\sigma = \frac{-M \cdot y}{e \cdot A \cdot (r_n + y)}$	$\sigma_i = \frac{M \cdot c_i}{e \cdot A \cdot r_i}$	$\sigma_o = \frac{-M \cdot c_o}{e \cdot A \cdot r_o}$

Area Properties for Various Cross Sections:

Cross Section	$\bar{r}$	$\int_{area} \frac{dA}{\rho}$	A
Rectangle 	$r_i + \frac{h}{2}$	$t \cdot ln\left(\frac{r_o}{r_i}\right)$	$h \cdot t$
Trapezoid 	$r_i + \frac{h \cdot (t_i + 2 \cdot t_o)}{3 \cdot (t_i + t_o)}$ ***For triangle:*** set t_i or t_o to 0	$t_o - t_i + \frac{r_o \cdot t_i - r_i \cdot t_o}{h} \cdot ln\left(\frac{r_o}{r_i}\right)$	$h \cdot \frac{t_i + t_o}{2}$
Hollow Circle 	$\bar{r}$	$2 \cdot \pi \left[\sqrt{\bar{r}^2 - b^2} - \sqrt{\bar{r}^2 - a^2}\right]$	$\pi \cdot (a^2 - b^2)$

Bending Moment in Curved Beam (Inside/Outside Stresses):

Stresses for the inside and outside fibers of a curved beam in pure bending can be approximated from the straight beam equation as modified by an appropriate curvature factor as determined from the graph below [**i** refers to the inside, and **o** refers to the outside]. The curvature factor magnitude depends on the amount of curvature (determined by the ratio **r/c**) and the cross section shape. **r** is the radius of curvature of the beam centroidal axis, and **c** is the distance from the centroidal axis to the inside fiber.



Inside Fiber:

$$\sigma_i = K_i \cdot \frac{M \cdot c}{I}$$

Outside Fiber:

$$\sigma_o = K_o \cdot \frac{M \cdot c}{I}$$