Math 383 { Spring 2010 } Hw 2

§1.4 # 1.22, 1.23

§1.5 # 1.24

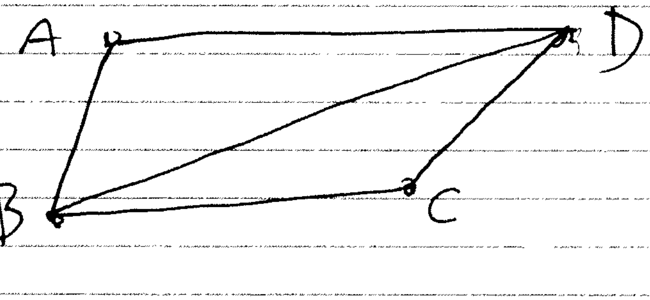
§1.6 # 1.26, 1.27, 1.29, 1.30, 1.31, 1.32

§1.7 # 1.45

§1.8 # 1.50

1.22 The angles of a quadrilateral sum to 360°. Generalize this to an n-sided polygon.

pf:



① Draw BD.

② The interior angles of ΔABD & ΔBDC each sum to 180°.

③ $\angle ABC = \angle ABD + \angle DBC$
and $\angle CDA = \angle CDB + \angle BDA$.

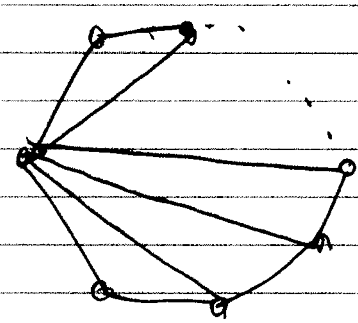
interior $\angle$ sum of ABCD

(4) $\angle A + \angle ABC + \angle C + \angle CDA$

$$= \left\{ \begin{array}{l} \text{sum of interior angles of } \triangle ABD \\ \text{and interior angles of } \triangle BDC \end{array} \right\}$$

$$= 180^\circ + 180^\circ = \boxed{360^\circ}.$$

If we have an n-gon, then divide into $\triangle$'s

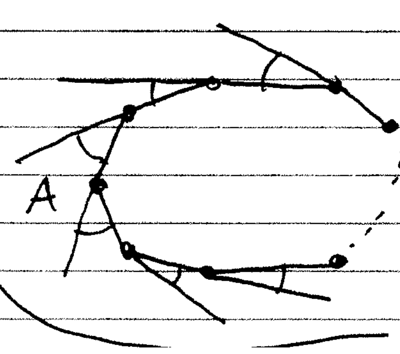


there are (n-2) triangles here

So the ~~sum~~ sum of the n-2 triangle interior $\angle$'s is $(n-2)180^\circ = \boxed{180^\circ n - 360^\circ}$ ∎

(123) What is the sum of the exterior $\angle$'s of an n-gon?

If you start at A and count the ~~turns~~ angles as you travel around the n-gon, then you will have made one complete $\boxed{360^\circ}$ turn by the time you return to A.

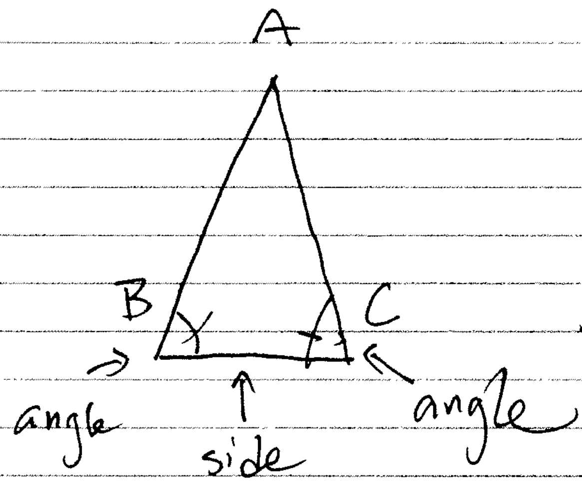


$\boxed{\text{so the exterior angles sum to } 360^\circ.}$

1.24 In ΔABC, if $\angle ABC = \angle BCA$, then $|AB| = |AC|$.

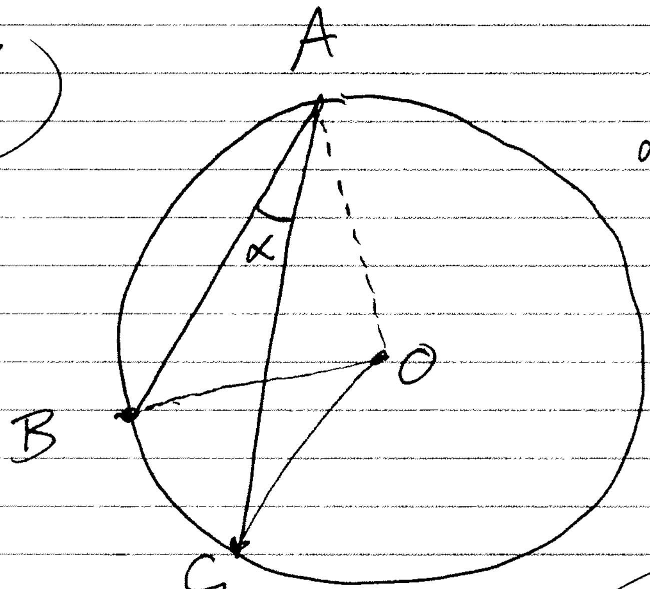
pf: The assumption on the angles means

$$\Delta ABC \equiv \Delta ACB \text{ by ASA}$$



So $|AB| = |AC|$. ☑

1.26



$\alpha = \angle A = \frac{1}{2} \angle BOC$.

pf:

Draw AO. Then ΔAOB and ΔAOC are isosceles triangles

Then $\angle ABO = \angle BAO$ and $\angle ACO = \angle CAO$.

Then $\alpha = \angle BAO - \angle CAO$

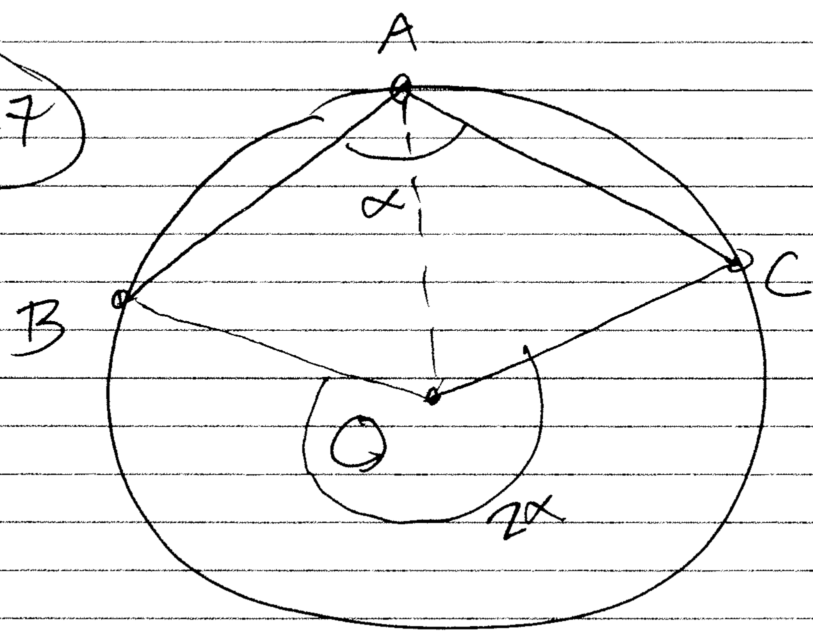
and $\angle BOC = \angle AOC - \angle AOB$

$$= (180^\circ - 2\angle CAO) - (180^\circ - 2\angle BAO)$$

$$= 2\angle BAO - 2\angle CAO = 2\alpha$$

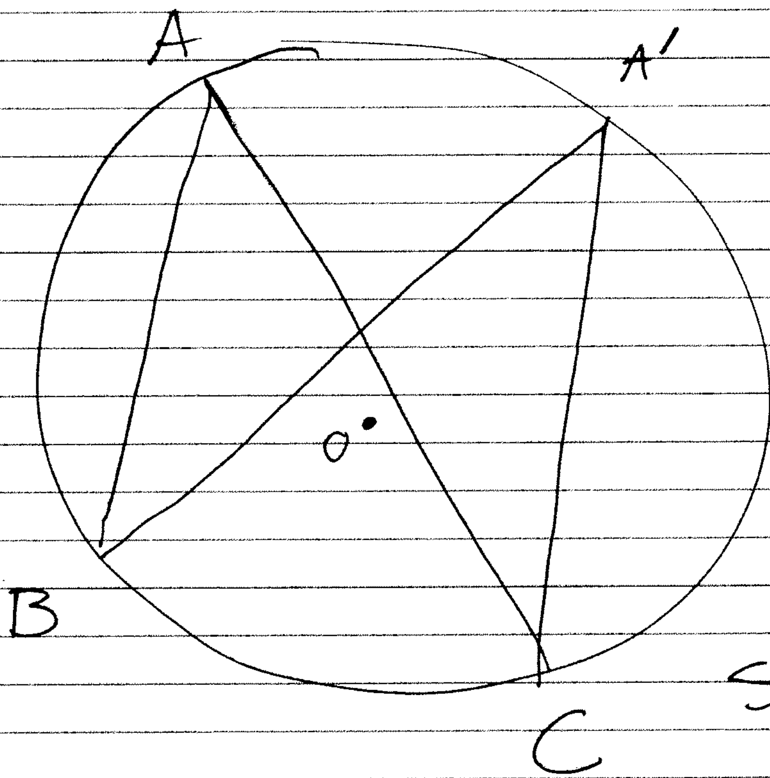
So $\frac{1}{2}\angle BOC = \alpha$. ▣

1.27



Draw AO (again). The proof is similar to 1.26. ▣

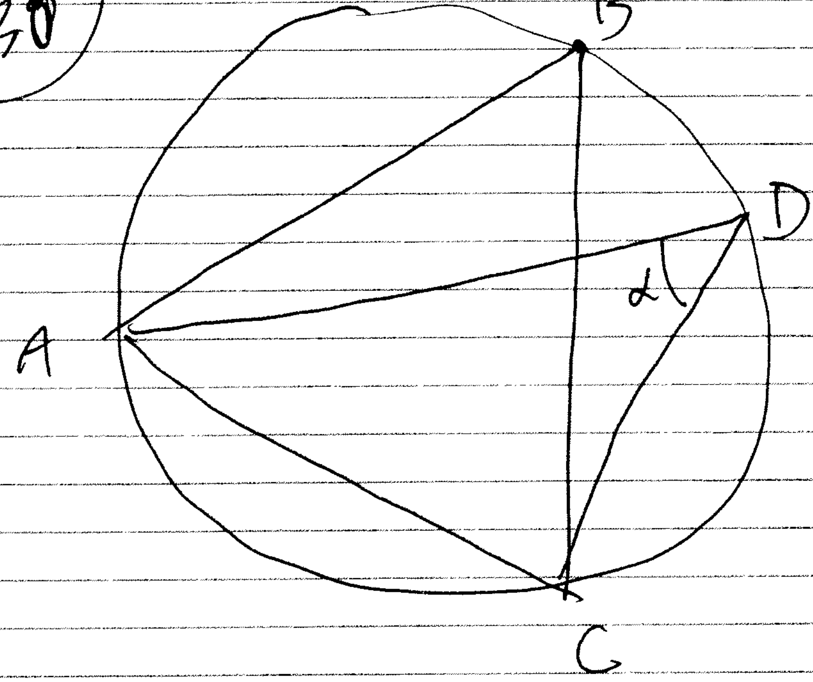
1.29 (Bow tie lemma)



$\angle BAC = \angle BA'C$.

pf: These angles subtend the same arc BC, so they both equal $\frac{1}{2}\angle BOC$. So they are equal. $\boxtimes$

1.30

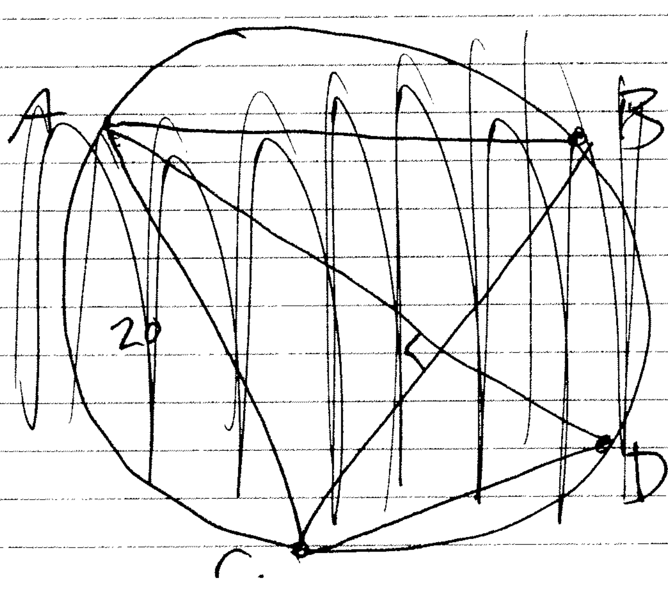


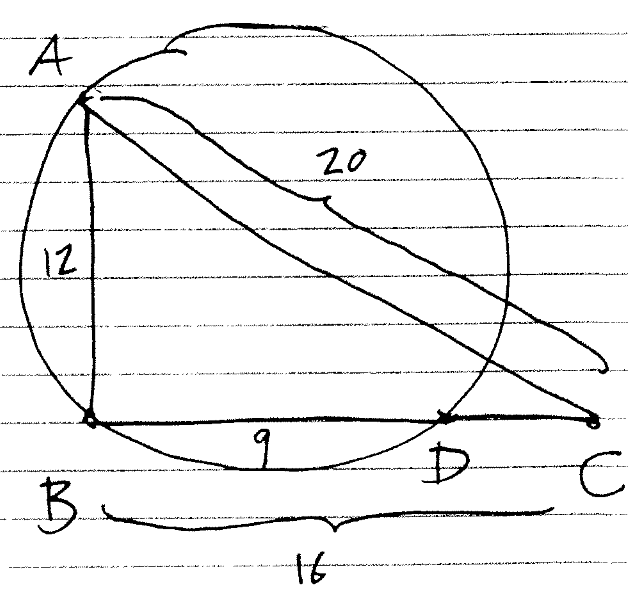
If $|AB| = |BC| = |AC|$ then what is the angle at D?

Sol'n: $\triangle ABC$ is equilateral, so $\angle B = 60°$. Since $\angle D$ subtends the same arc as $\angle B$, we have $\alpha = 60°$. $\blacksquare$

1.31 $|AB| = 12$, $|BD| = 9$, $|BC| = 16$, $|AC| = 20$.

What is the length of the diameter?





Sol'n: Since $12^2 + 16^2 = 400 = 20^2$, ΔABC is a right Δ w/ a right angle at B.

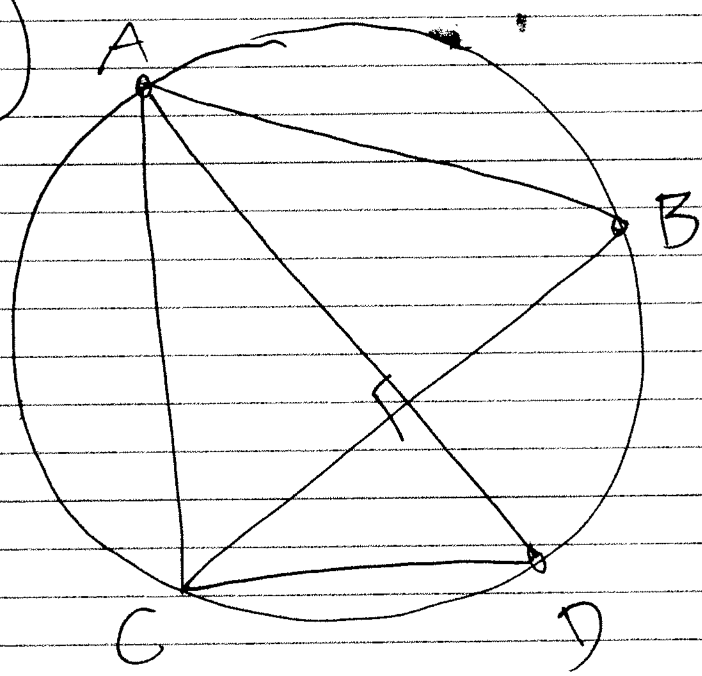
So the arc AD is half the circle, because a $90°$ subtends it. So AD is diameter.

Then by the Pythagorean Theorem

$$|AD|^2 = 12^2 + 9^2 = 225 = 15^2.$$

So $|AD| = 15$. ∎

1.32

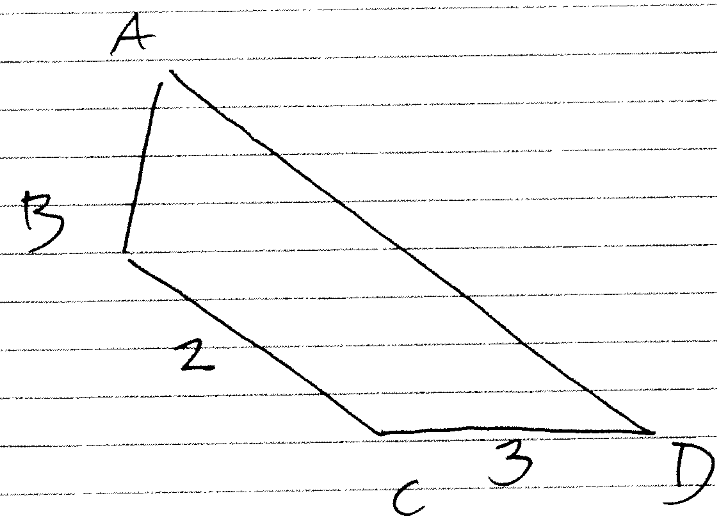


If $|AB| = |AC| = |BC|$ and $AD \perp BC$, then what is $\angle BCD$?

Sol'n: ΔABC is equilateral. So $\angle B = 60°$. Then $\angle D = 60°$ also, because it subtends the same arc as $\angle B$. Then

$$\angle BCD = 180 - 90 - 60 = 30°.$$ ∎

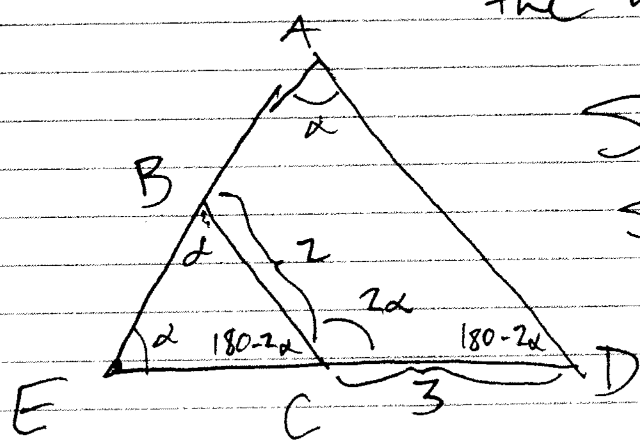
1.45



$AD \parallel BC$, $\angle C = 2\angle A$, $|CD| = 3$, $|BC| = 2$.

What is $|AD|$?

Sol'n: Extend AB and CD to E, and fill in the angles as shown below.



So ΔEBC is isosceles.

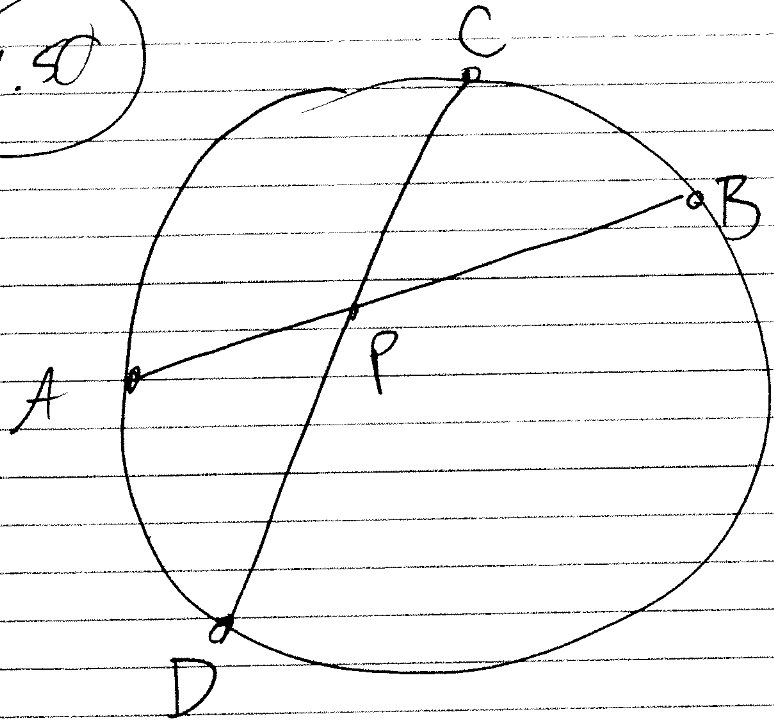
So $|EC| = 2$, so $|ED| = 5$.

Then by similar Δ's, we have

$$\frac{2}{5} = \frac{|EC|}{|ED|} = \frac{|BC|}{|AD|} = \frac{2}{|AD|}.$$

So $|AD| = 5$. $\blacksquare$

1.50



If $|AP| = 2$, $|AB| = 6$, $|PC| = 3$, then what is $|PD|$?

Sol'n: By power of ~~power~~ the point,

$$|AP||PB| = |DP||PC|.$$

So $|AP| \cdot (|AB| - |AP|) = |PD| \cdot |CP|$, so

$$2 \cdot (6-2) = |PD| \cdot 3$$

$$\downarrow$$

$$8/3 = |PD|. \quad \square$$