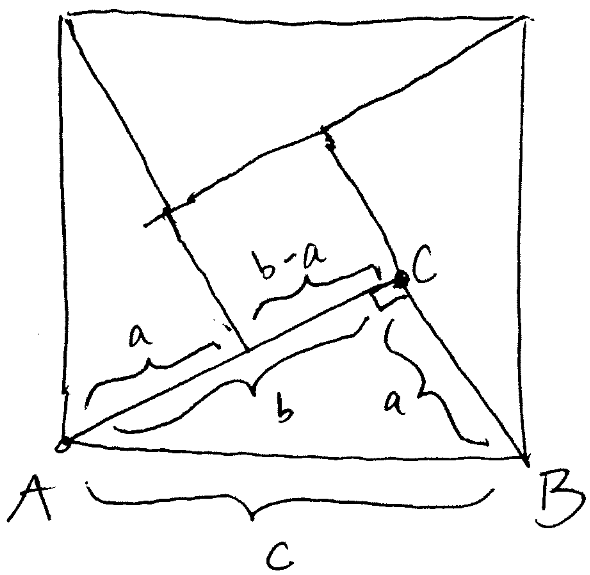
Math 383 | Spring 2010 | Hw 1

1.5



Use this to prove the pythag. Thm.

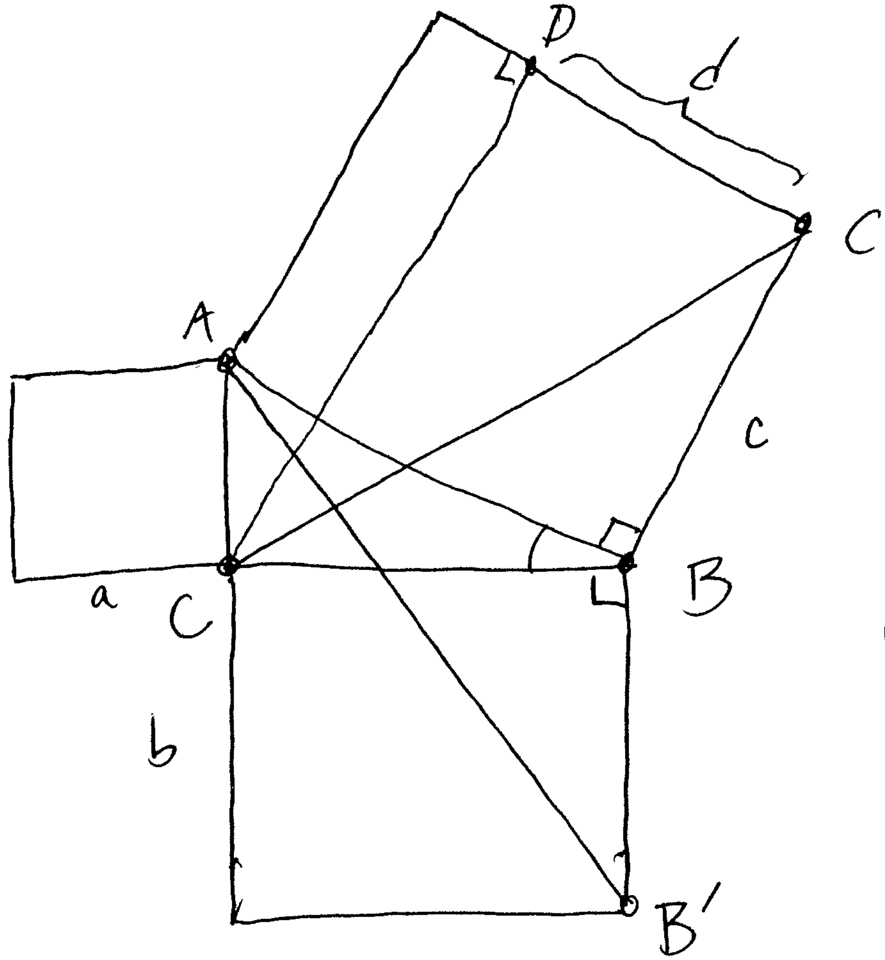
pf: Let $|AC| = b$, $|BC| = a$ and $|AB| = c$ (as in the figure).

Since $\angle CBA + \angle CAB = 90$, the figure above is a square with side length c. So its area is c^2.

But its area is also $4 \cdot \frac{ab}{2} + (b-a)^2$, because the central figure is a square of side length $b-a$.

So $$c^2 = 2ab + (b-a)^2 = a^2 + b^2.$$ $\blacksquare$

1.6



prove the pythagorean theorem.

Sol'n: Assume that the quadrilaterals attached to the sides of ΔABC are squares.

Then $\Delta B'BA \equiv \Delta CBC'$, by SAS because

$$|B'B| = |CB| = b$$

$$\angle B'BA = 90 + \angle CBA = \angle CBC'$$

and $|AB| = c = |BC'|$.

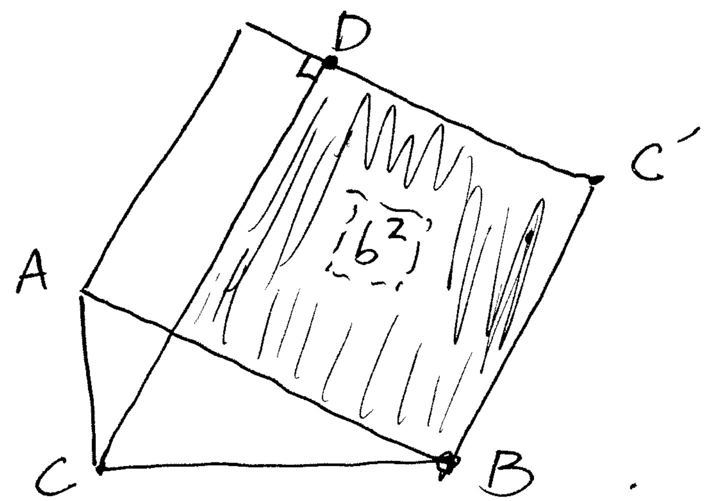
We observe the following:

① $\text{Area}(\Delta B'BA) = \frac{b \cdot b}{2}$ (use $B'B$ as the base and BC as the height)

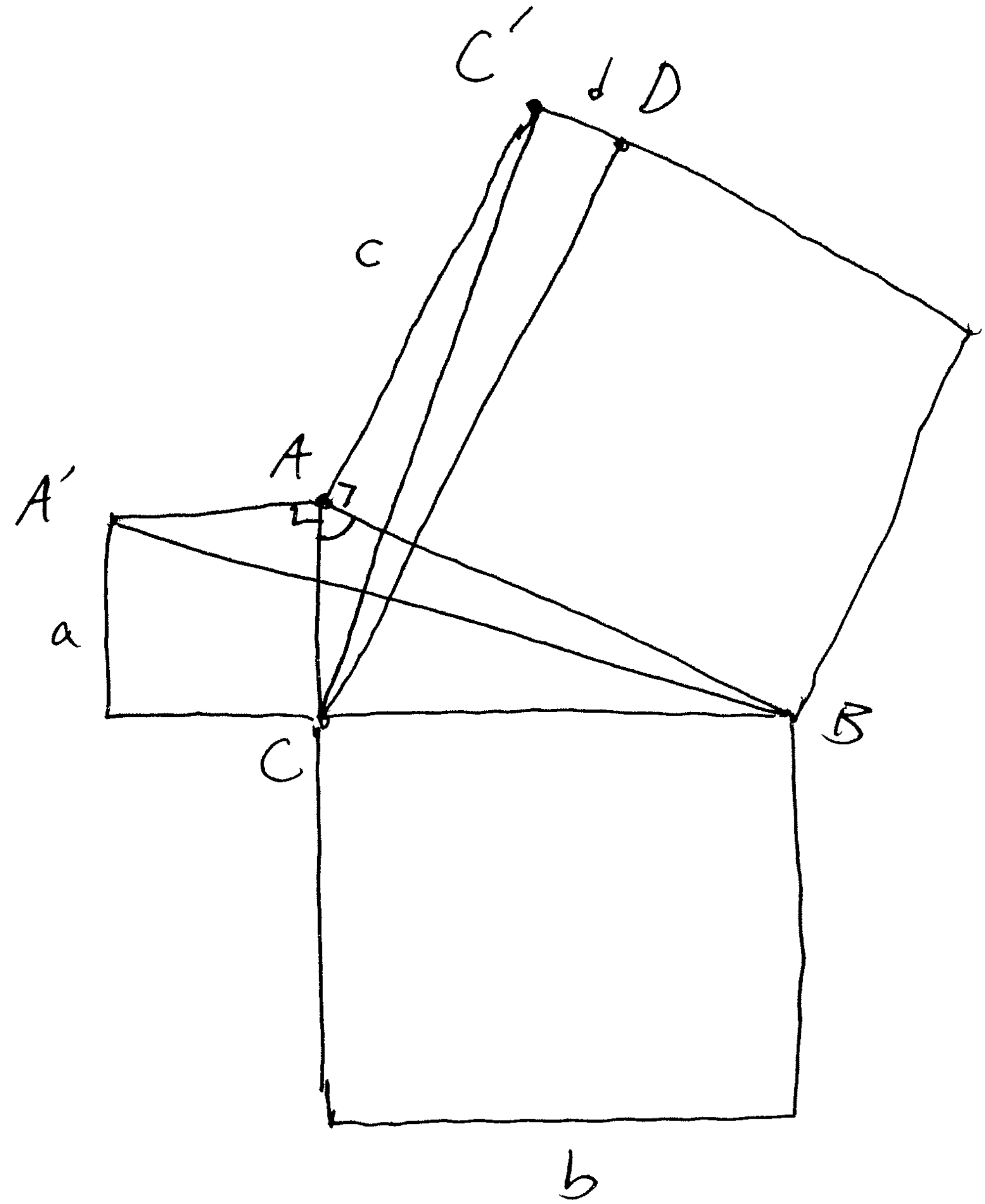
② $\text{Area}(\Delta CBC') = \frac{c \cdot d}{2}$ (use BC' as the base and $C'D$ as the height)

Since these triangles are congruent, their areas are equal; so $b^2 = cd$. But cd is the area of the rectangle inside the square on side AB with sides BC' and $C'D$.

So

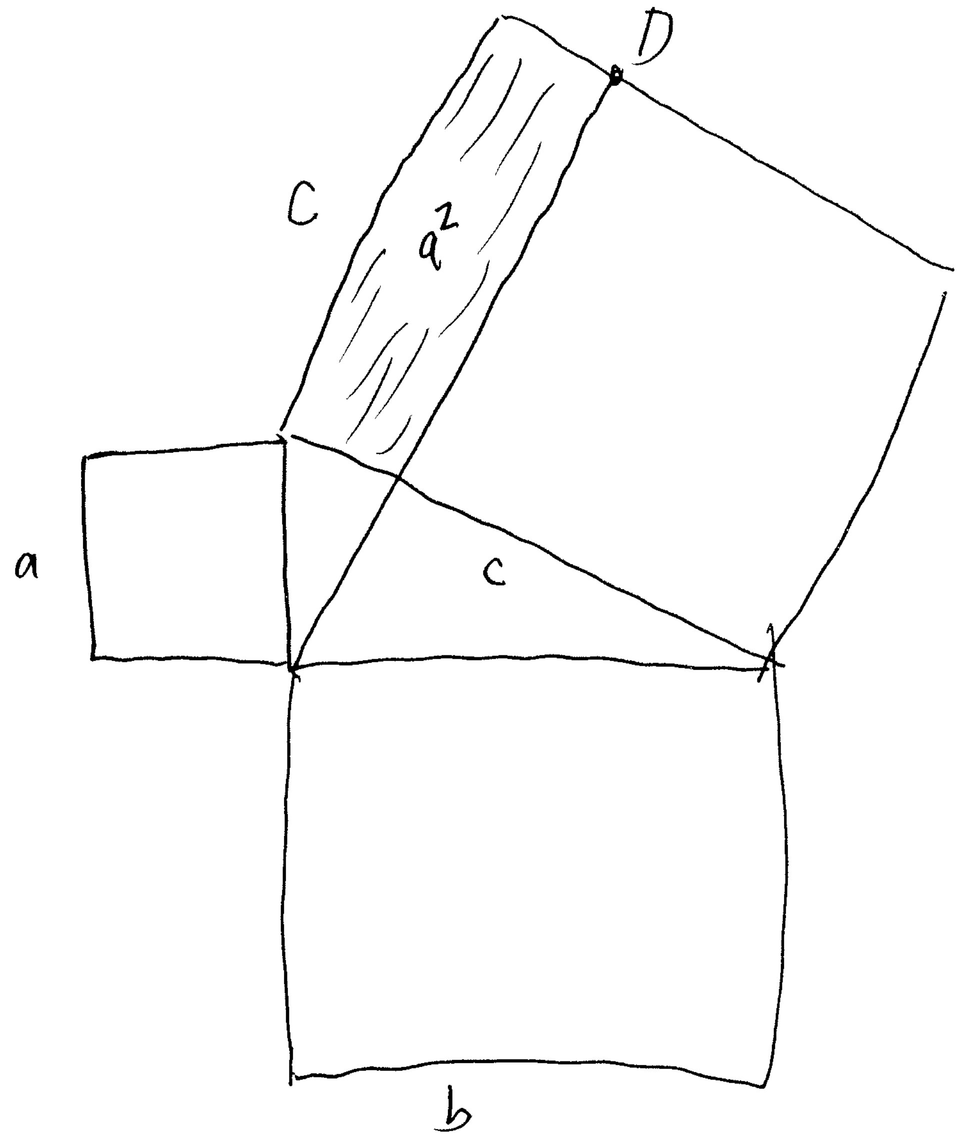


Now we can use the following picture:



To show that $\Delta A'AB \equiv \Delta CAC'$, again by SAS.

The same arguments in the first case can then be used to show:

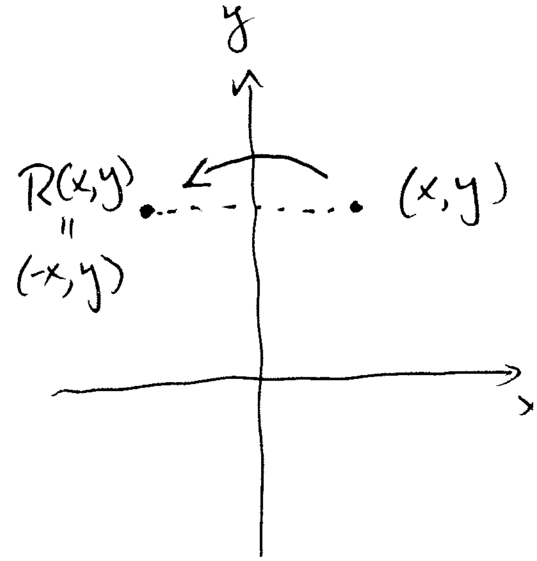


So the square with side c has area a^2+b^2. So $c^2 = a^2+b^2$. $\blacksquare$

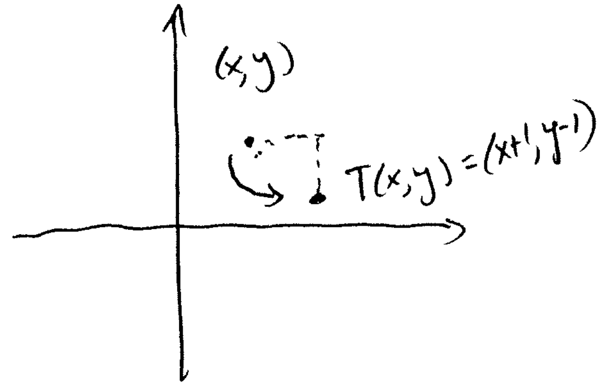
1.9 Find an isometry of the Euclidean plane that has no fixed points and is not a translation.

Sol'n: Composing two isometries produces another isometry, so let's try composing a reflection and a translation.

$R(x,y) = (-x,y)$ reflection in the y-axis



$T(x,y) = (x+1, y-1)$ translation



$$T(R(x,y)) = T(-x,y) = (-x+1, y-1).$$

To prove this has no fixed points we have to show

$$T(R(x,y)) \neq (x,y)$$ (because otherwise (x,y) would be fixed)

Sol:
$$(-x+1, y+1) \stackrel{?}{=} (x,y)$$

$$\Downarrow$$

$$\left.\begin{aligned} -x+1 &= x \\ y+1 &= y \end{aligned}\right\} \longrightarrow \begin{aligned} x &= 1/2 \\ 1 &= 0 \leftarrow \text{wtf?} \end{aligned}$$

Since $1 \neq 0$, there can be no point that gets fixed. $\blacksquare$