Scribe: Elmira Emini

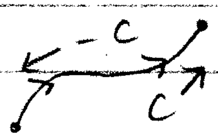
3/4/10

$$\int_C P(x,y)\,dx + Q(x,y)\,dy =$$

$$\int_a^b P(x(t),y(t))\,x'(t)\,dt + \int_a^b Q(x(t),y(t))\,y'(t)\,dt$$

$$C: \vec{r}(t) = \langle x(t), y(t)\rangle \quad a \le t \le b$$

<u>Notation:</u> $\int_{-C} P\,dx + Q\,dy = -\int_C P\,dx + Q\,dy$



$\int_C f(x,y,z)\,ds$ The line integral along C of $f(x,y,z)$ with <u>respect to arc length</u>.

<u>Def:</u> $\int_a^b f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

$$C: \vec{r}(t) = \langle x(t), y(t), z(t)\rangle \quad a \le t \le b$$

<u>Idea:</u> $\int_C f(x,y,z)\,ds$ = mass of curve if f = density

* Note: If $f = 1$ then $\int_C ds = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, d$

$$\| $$

length(C).

$$\int_C \underset{\text{wrt } x}{P(x,y,z)\,dx} + \underset{\text{wrt } y}{Q(x,y,z)\,dy} + \underset{\text{wrt } z}{R(x,y,z)\,dz}$$

$$= \int_a^b P\,x'(t)\,dt + \int_a^b Q\,y'(t)\,dt + \int_a^b R\,z'(t)\,dt$$

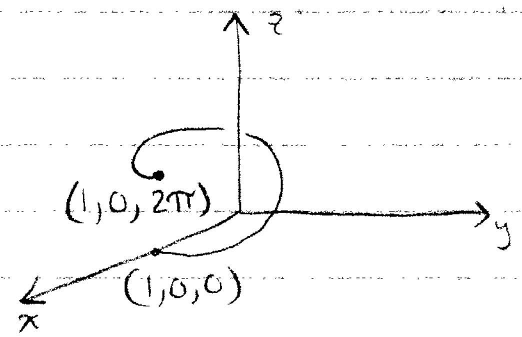
Def $= \int_a^b f(\vec{r}(t)) \, |\vec{r}'(t)| \, dt = \int_C f \, ds$

Ex: $\int_C y \sin z \, ds =$

$C: \; x = \cos t$

$\quad y = \sin t \qquad 0 \le t \le 2\pi$

$\quad z = t$

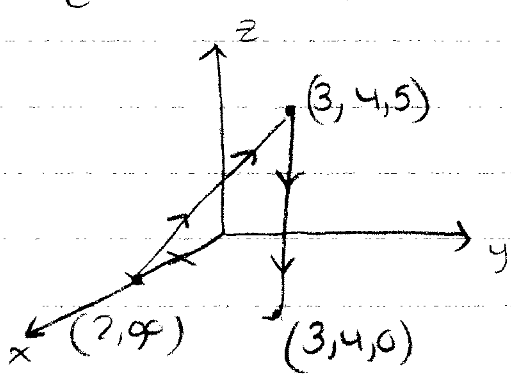


$= \int_0^{2\pi} (\sin t)(\sin t) \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2} \, dt$

$= \sqrt{2} \int_0^{2\pi} \sin^2 t \, dt = \sqrt{2} \int_0^{2\pi} \frac{1}{2}(1 - \cos 2t) \, dt$

$= \ldots$ exercise

Ex: $\int_C y\,dx + z\,dy + x\,dz$



$C = C_1 \cup C_2$

Describe $C_1: \vec{r}_1(t) = \langle 2,0,0 \rangle + t \overbrace{\langle 1,4,5 \rangle}^{\vec{r}_1'}$

$= \langle \underset{x}{2+t}, \underset{y}{4t}, \underset{z}{5t} \rangle$

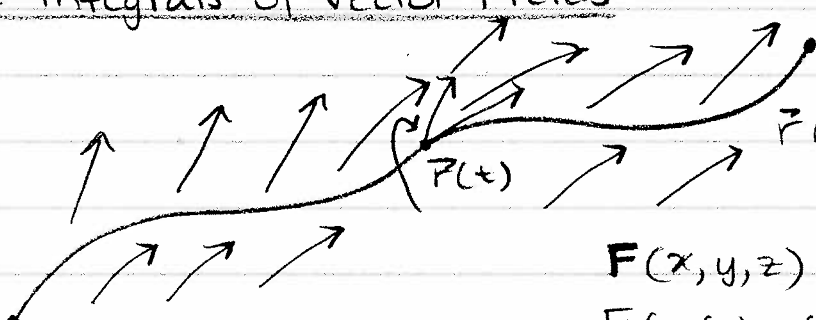
$C_2: \vec{r}_2(t) = \langle 3, 4, 5-5t \rangle \qquad 0 \le t \le 1$

$$\int_{C_1} y\,dx + z\,dy + x\,dz + \int_{C_2} y\,dx + z\,dy + x\,dz =$$

$$\int_0^1 (4t)(1) + (5t)(4) + (2+t)(5)\,dt$$
$$+ \int_0^1 (4)(0) + (5-5t)(0) + (3)(-5)\,dt = \ldots$$

$$\int_C P(x,y,z)\,\underbrace{dx}_{x'(t)\,dt} \qquad x'(t)\,dt = \sqrt{\left(\frac{dx}{dt}\right)^2}\,dt$$

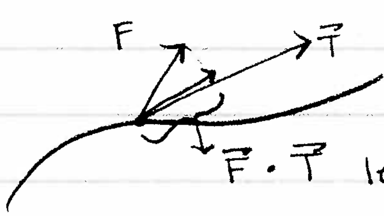
Line Integrals of Vector Fields



C
Curve in Space
$\vec{r}(t) = \langle x(t), y(t), z(t$
$a \le t \le b$

$\mathbf{F}(x,y,z)$ a vector field
$F(x(t), y(t), z(t))$

$\vec{T}(t) = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|}$



$\vec{F} \cdot \vec{T}$ length of projection (component of F pushing in direction of motion along C.)

* $\int_C \vec{F} \cdot \vec{T} \, ds$ = work done by $\vec{F}$ moving a particle along C.

$$\int_a^b \underbrace{\vec{F}(x(t), y(t), z(t))}_{\vec{F}(\vec{r}(t))} \cdot \underbrace{\frac{\vec{r}'(t)}{|r'(t)|}}_{\vec{T}(t)} \underbrace{|\vec{r}'(t)| \, dt}_{ds}$$

$$= \int_a^b \underbrace{(\vec{F}(\vec{r}(t)) \cdot \vec{r}'(t))}_{\text{the line integral of } \vec{F} \text{ along } C.} \, dt$$

$$= \int_C \vec{F} \cdot dr$$