Section 14.2: Line Integrals

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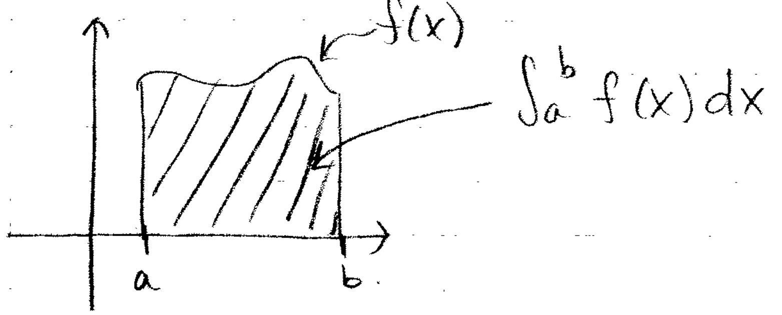
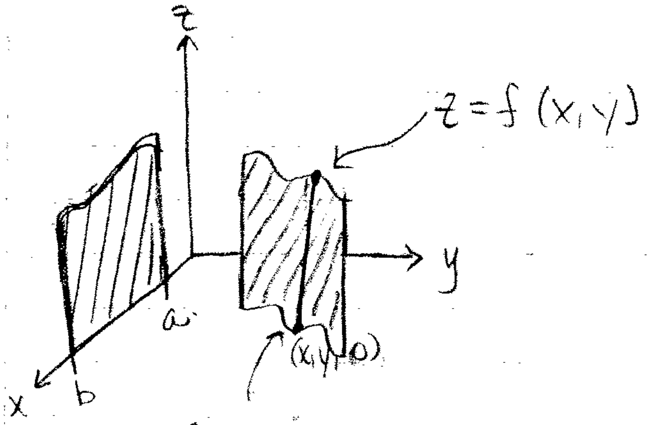




C a curve in the xy-plane

Defn: The line integral of $f(x,y)$ with respect to the arc length is denoted by...
(C is a curve in the xy-plane)

$\int_C f(x,y)\,ds =$

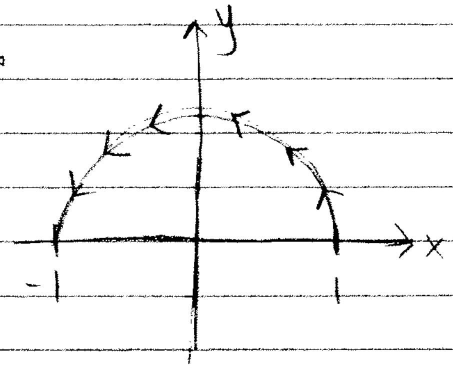
"the signed Area under the curtain in the picture".

$$\int_a^b f(x(t),y(t)) \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}\, dt$$

where,
$\vec{r}(t) = \langle x(t), y(t) \rangle$
$a \le t \le b$
is <ins>any</ins> parameterization of C

ex $\int_C (2+x^2y)\,ds = \underbrace{\int_0^\pi 2+\cos^2 t \cdot \sin t \sqrt{(-\sin t)^2+(\cos t)^2}}_{\text{fill in using a parameterization of } C}$

C:



$\vec{r}(t) = \langle 1\cos(t), 1\sin(t)\rangle$

$0 \le t \le \pi$

$x(t) = \cos(t)$

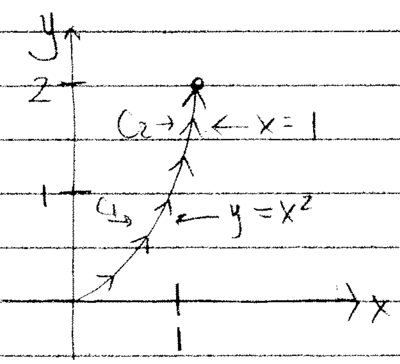
$y(t) = \sin(t)$.

$= \int_0^\pi \left[(2+\cos^2 t \cdot \sin t)\sqrt{1}\right] dt = 2\pi + \int_0^\pi \underbrace{(\cos^2 t \sin t)}\, dt$

$= 2\pi + -\frac{1}{3}\cos^3 t \Big|_0^\pi$

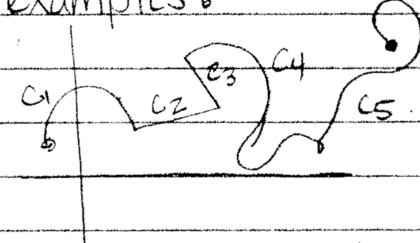
$\boxed{= 2\pi + \frac{2}{3}}$

① C:



$C = C_1 \cup C_2$ is called Piecewise smooth

examples:



$C = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5$

$\int_C 2x\,ds = \int_{C_1} 2x\,ds + \int_{C_2} 2x\,ds$

continued

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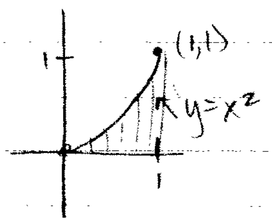
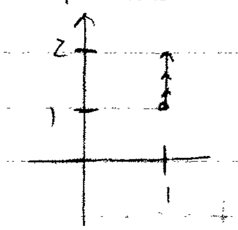
Describe C_1:
$\vec{r}(t) = \langle t, t^2 \rangle$
$0 \le t \le 1$



$\begin{bmatrix} x(t) = t \\ y(t) = t^2 \end{bmatrix}$

Describe C_2:
$\vec{r}_2(t) = \langle 1, t \rangle$
$1 \le t \le 2$



$\begin{bmatrix} x(t) = 1 \\ y(t) = t \end{bmatrix}$

$$\int_C 2x\,ds = \int_{C_1} 2x\,ds + \int_{C_2} 2x\,ds$$

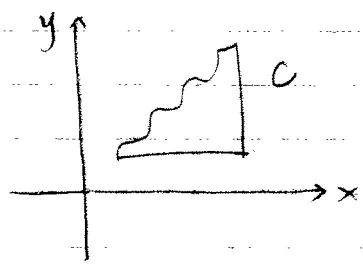
$$= \int_0^1 2\cdot t\sqrt{(1)^2 + (2t)^2}\,dt + \int_1^2 2\cdot 1\sqrt{(0)^2 + (1)^2}$$

$$= \int_0^1 2t\sqrt{1+2t^2}\,dt + \int_1^2 2\,dt = \text{substitution} + 2t\Big|_1^2$$

② $\int_a^b f(x(t), y(t))\, x'(t)\,dt$ & $\int_a^b f(x(t), y(t))\, y'(t)\,dt$

the line integrals along C of $f(x,y)$ with respect to x & y. (as opposed to the arc length)

<u>Motivation</u>: (for diff. types of Integrals)



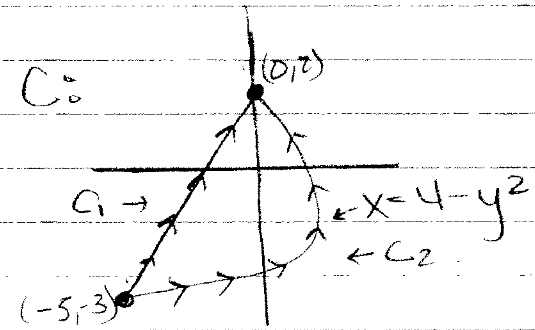
Notation:

We often write

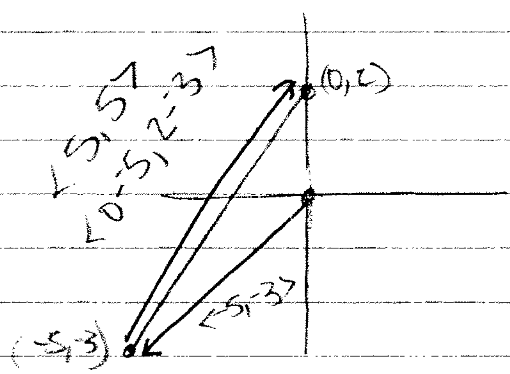
$$\int_C P(x,y)\,dx + Q(x,y)\,dy$$
$$\|$$
$$\int_C P(x,y)\,dx + \int_C Q(x,y)\,dy.$$

examples

① $\int_C y^2\,dx + x\,dy$ C:



Describe C_1:



$$\vec{r}_1(t) = \langle -5,-3\rangle + t\langle 5,5\rangle$$
$$0 \le t \le 1$$
$$= \langle \underbrace{5t-5}_{x(t)},\ \underbrace{5t-3}_{y(t)} \rangle.$$

Describe C_2:

$\vec{r}_2(t) = \langle 4-t^2, t \rangle$ $x = 4 - y^2$

$-3 \le t \le 2$

$$\int_C y^2\,dx + x\,dy = \int_0^1 \left[\underbrace{(5t-3)^2}_{y}\underbrace{(5)}_{x'(t)} + \underbrace{5t-5}_{x}\underbrace{(5)}_{y'(t)}\right] dt$$

$= ex.$

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With Respect to C_2

$$\int_{-3}^{2}\left[\underbrace{(t)}_{y}{}^{2}-\underbrace{(2t)}_{x'(t)}+\underbrace{(4-t^2)}_{x}\underbrace{(1)}_{y'(t)}\right]dt = \underline{\underline{ex}}.$$

Check! Answers arent the same.