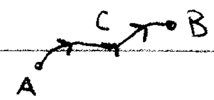
Meghan Grenier Scribe 3/22/10

17.3 cont.

Suppose we know $\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$ is conservative.

$\vec{F} = \nabla f$ for some $f(x,y,z)$

- How do we find this $f(x,y,z)$?
- once we find f, this allows us to solve $\int_C \vec{F} \circ d\vec{r} = f(B) - f(A)$ using the F.T.L.I.



ex: Later on, we will learn a set of criterion for determining whether or not a 3D v.f. is conservative.
For now, assume $\vec{F}(x,y,z) = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$ is conservative.
Then find a potential function $f(x,y,z)$ for $\vec{F}$.

- we know there is an f w/ these properties.

$f_x = y^2$
$f_y = 2xy + e^{3z}$
$f_z = 3ye^{3z}$

integrate w/ respect to x

$\int f_x dx = \int y^2 dx$

$\boxed{f = xy^2 + g(y,z)}$

$\boxed{3ye^{3z} = f_z = \frac{\partial g}{\partial z}}$

$g_z = 3ye^{3z}$

$g_z = \int g_z dz = \int 3ye^{3z} dz = ye^{3z} + h(y)$

so $\boxed{f = xy^2 + \underbrace{ye^{3z} + h(y)}_{g(y,z)}} \xrightarrow{\frac{\partial}{\partial y}} f_y = 2xy + e^{3z} + h'(y)$

$f_y = 2xy + e^{3z}$

$h'(y) = 0$
$h(y) = C$

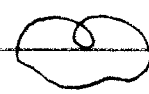
$\boxed{\text{so } f(x,y,z) = xy^2 + ye^{3z} + C}$

end 17.3

17.4 Green's Theorem

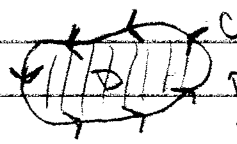
Def: A simple closed curve in the plane is a curve that closes up and doesn't cross itself.


simple

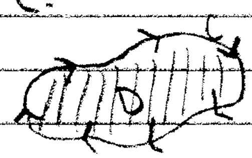

not simple

Fact: Any simple closed curve divides the plane into 2 regions

Def: A simple closed curve C is called oriented positively if the bounded region inside of C is on the left as you travel in the direction of C.

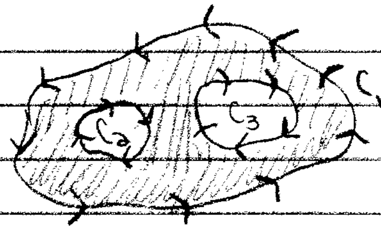


positively oriented



negatively oriented

Also works for a collection of simple closed curves:



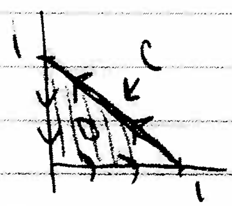
$C = C_1 \cup C_2 \cup C_3$

positively oriented

Green's THM: let C be a (collection of) simple closed curve(s) bounding some region D. Let $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$ be a vector field w/ continuous partial derivatives.

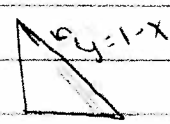
Then $\int_C \vec{F} \cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$.

ex: $\oint_C \overbrace{x^4\,dx + xy\,dy}^{\vec{F}\cdot d\vec{r}} \underset{\text{Green's THM}}{=} \iint_D (y-0)\,dA = \iint_D y\,dA$



$\iint_D y\,dA = \int_0^1 \int_0^{1-x} y\,dy\,dx = \ldots$

$0 \le x \le 1$
$0 \le y \le 1-x$



fyi: you would need 3 integrals to compute this using line integration.
w/ $\vec{F}(x,y) = \langle \underset{P}{x^4}, \underset{Q}{xy} \rangle$

ex: $\oint_C \underbrace{(3y - e^{\sin x})}_{P}\,dx + \underbrace{(7x + \sqrt{y^4+1})}_{Q}\,dy = \iint_D (7-3)\,dA = 4\iint_D dA$

Green's THM

$\boxed{36\pi} = 4\,\text{Area}(D)$

C: $x^2+y^2=9$