3/18/10

Jennifer I's not ready for the quiz.

If $\vec{F}$ is conservative (so $\vec{F} = \nabla f$) then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$\vec{F} = \langle P, Q \rangle$

$\boxed{\text{Not true that } P_y = Q_x \longrightarrow F \text{ is conservative.}}$

<u>Ex</u> $\vec{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

P Q

but not independent of path.

<u>Th^m</u> If $\vec{F}$ is defined on a simply connected region of the plane, <u>then</u> if

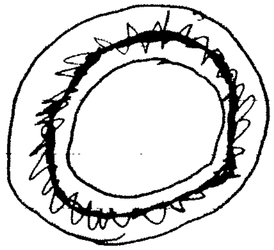
[$P_y = Q_x$ then $\vec{F}$ is conservative.]

<u>Def</u> A simply connected region is a connected region where every closed curve in the region encloses only points that are a part of the region.

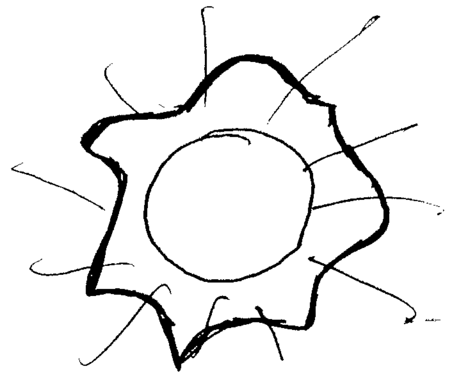
<u>Examples</u>

- $\mathbb{R}^2$ is simply connected

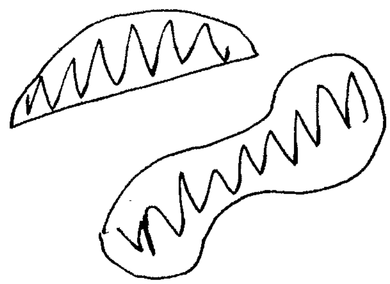
Not simply connected



encloses points that don't belong in the set.



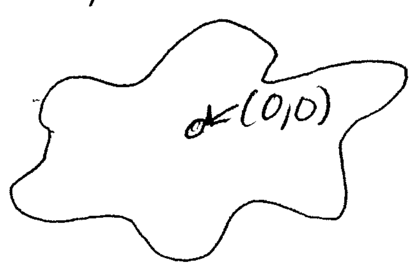
Not Simply Connected



not simply connected

The previous vector field fails to be conservative b/c $\left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ not defined on the origin.

$\mathbb{R}^2-(0,0)$ not simply connected



<u>Examples</u> $\vec{F}(x,y) = \langle x-y, x-2 \rangle$

Defined on $\mathbb{R}^2$ (simply connected)

$\frac{\partial P}{\partial y} = -1 \quad \frac{\partial Q}{\partial x} = 1 \rightarrow$ This fails to be conservative b/c $P_y \neq Q_x$

Ex $\vec{F}(x,y) = \langle 3+2xy,\, x^2-3y^2 \rangle$

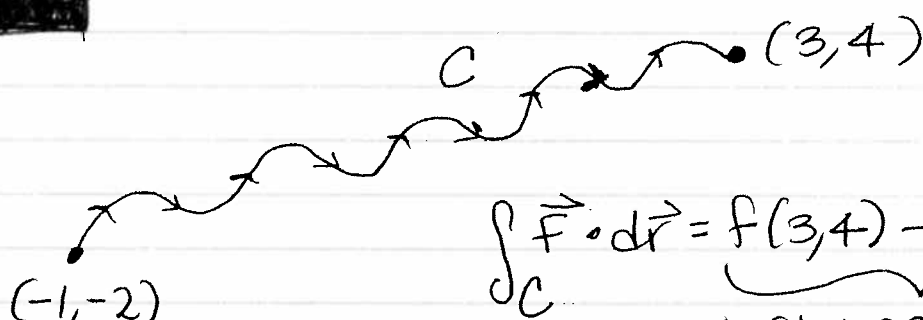
$\frac{\partial P}{\partial y} = 2x$

equal

$\frac{\partial Q}{\partial x} = 2x$

b/c $\vec{F}$ is continuous on $\mathbb{R}^2$ (simply connected)
$\vec{F}$ has to be conservative.

[redacted]



$$\int_C \vec{F}\cdot d\vec{r} = \underbrace{f(3,4) - f(-1,-2)}$$

just need to find the f that satisfies
$\nabla f = \vec{F} = \langle 3+2xy,\, x^2-3y^2$

Strategy to find $f(x,y)$:

$f_x = 3+2xy = P$

$f_y = x^2+3y^2 = Q$

Integrate f_x w.r.t. x

$$f(x,y) = \int (3+2xy)\,dx$$

$$\| \quad 3x + x^2y + \underline{g(y)}$$

← any function of y ... like "plus C"

So $f(x,y) = 3x + x^2y + g(y)$

$\downarrow \frac{\partial}{\partial y}$

$f_y = x^2 + g'(y)$

$\| \; x^2 - 3y^2$

equal $\rightarrow g'(y) = -3y^2$

$g(y) = -y^3 + C$

so $f(x,y)$
$\quad \parallel$
$3x + x^2y - y^3 + C$

$f_x = 3 + 2xy + 0 = P$
$f_y = x^2 - 3y^2 = Q$
$\therefore \vec{F} = \nabla f$

$$\int_C \vec{F} \cdot d\vec{r} = f(3,4) - f(-1,-2)$$
$$\parallel$$
$$3 \cdot 3 + 9 \cdot 4 - 4^3 + C$$
$$-(-3 - 2 + 8 + C) = \ldots$$