Caitlin C. wants her 1 hour of sleep back. Thanks a lot Daylight Savings.

3/15/10

## Announcements

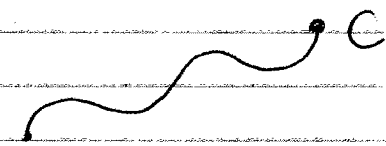
- No HW this Thursday
- Quiz Thursday

- Thursday 5 pm BNW138 "Humans vs. Computers in Ches
  Dr. King
  -XC for attending & learning someth

## Line Integration

Two kinds:

CONTEXT SPACE



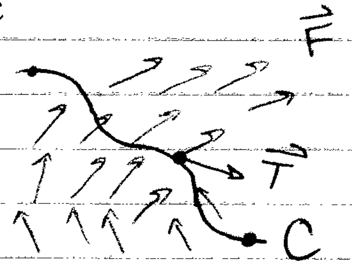

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$a \le t \le b$

(function integral) 1) $\int_C f\, ds = \int_a^b f(\vec{r}(t))\,|\vec{r}\,'(t)|\,dt$

(vector field integral) 2) $\int_C P\,dx + Q\,dy + R\,dz$

$= \int_C \vec{F} \cdot d\vec{r}$



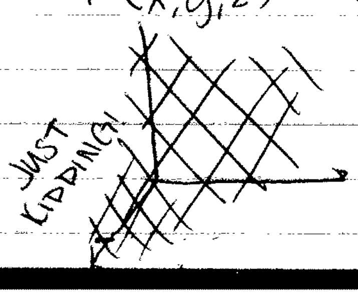

Work of $\vec{F}$ along C $= \int_C \vec{F} \cdot \vec{T}\, ds$

$= \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

definition $= \int_C \vec{F} \cdot d\vec{r}$

ex. $\int_C \vec{F} \cdot d\vec{r}$ C: $x(t) = t$ $y(t) = t^2$ $z(t) = t^3$ $0 \le t \le$

$\vec{F}(x,y,z) = \langle xy, yz, zx \rangle$



$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \vec{F}(x(t),y(t),z(t))\cdot\langle x'(t),y'(t),z'(t)\rangle d$$

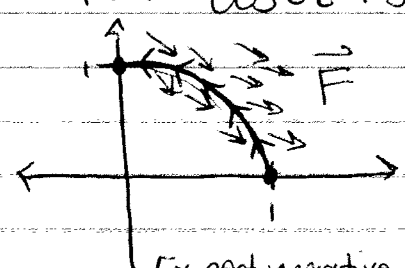
$$=\int_0^1 \langle t\cdot t^2, t^2\cdot t^3, t^3\cdot t\rangle\cdot\langle 1, 2t, 3t^2\rangle dt$$

$$=\int_0^1 (t^3\cdot 1 + t^5\cdot 2t + t^4\cdot 3t^2)\,dt$$

$$=\int_0^1 (t^3 + 2t^6 + 3t^6)\,dt \ldots$$

$\boxed{\text{ANSWER: } \frac{27}{28}}$

ex. $F(x,y) = x^2\vec{i} - xy\vec{j} = \langle x^2, -xy\rangle$
$\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} \quad 0 \le t \le \pi/2$




Expect negative work!

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{\pi/2} \vec{F}(x(t),y(t))\cdot\langle x'(t),y'(t)\rangle dt$$

$$=\int_0^{\pi/2} \langle \cos^2 t, -\cos t\sin t\rangle\cdot\langle -\sin t, \cos t\rangle dt$$

$$=\int_0^{\pi/2} (-\sin t\cos^2 t - \cos^2 t\sin t)\,dt = \int_0^{\pi/2} -2\sin t\cos^2 t\,dt$$

Find new limits by plugging in original limits to $u=\cos t$

$u=\cos t \rightarrow \int_1^0 2u^2\,du = \boxed{-\frac{2}{3} \text{ as we expected}}$

Remark!

$$\vec{F}(x,y,z) = P(x,y,z)\,\vec{i} + Q(x,y,z)\,\vec{j} + R(x,y,z)\,\vec{k}$$

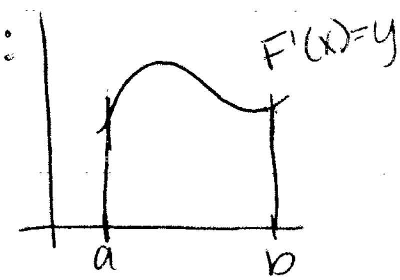
$$\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$$

$$\int_c \vec{F} \cdot d\vec{r} = \int_c \langle P, Q, R \rangle \cdot \langle x', y', z' \rangle = \int_c Px' + Qy' + Rz'$$

where the notation comes from $\longrightarrow$ $= \int_c P\,dx + Q\,dy + R\,dz$

# 17.3 Fundamental Theorem of Line Integrals

Recall:




$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

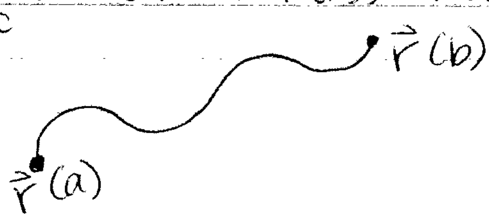
Recall: We called a vector field $\vec{F}(x,y,z)$ <u>Conservative</u> if $\vec{F} = \nabla f$

gradient of f

$\langle f_x, f_y, f_z \rangle$

<u>Fundamental Theorem of Line Integrals</u>

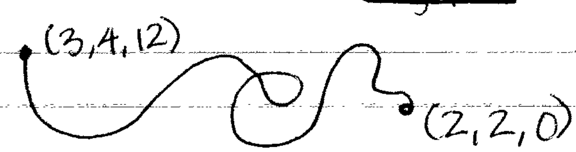
$$\int_c \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$



$$\vec{F}(\vec{x}) = \frac{-mMG}{|\vec{x}|^3}\vec{x} \rightsquigarrow f(x,y,z) = \frac{mMG}{\sqrt{x^2+y^2+z^2}}$$

$$\boxed{\vec{x} = \langle x, y, z\rangle}$$

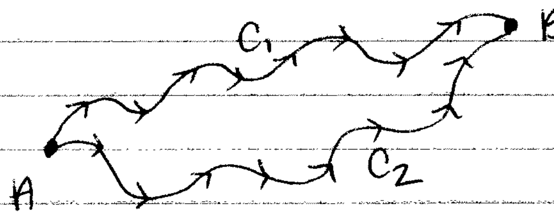
$\int_C \vec{F}(x)\cdot d\vec{r}$ where $C$ is <u>any path</u> from $(3,4,12)$ to $(2,2,0)$




**ANSWER** $= f(2,2,0) - f(3,4,12) = \underbrace{\frac{mMG}{\sqrt{8}} - \frac{mMG}{\sqrt{9+16+1}}}$

work done moving a particle of mass $m$ from $(3,4,12)$ to $(2,2,0)$

$\vec{F}$ is a <u>vector field</u>




Not true in general that

$$\int_{C_1}\vec{F}\cdot d\vec{r} = \int_{C_2}\vec{F}\cdot d\vec{r}$$

<u>Observation 1</u>

if $\vec{F}$ is conservative, then $\vec{F} = \nabla f$, and so

$$\int_{C_1}\vec{F}\cdot d\vec{r} = f(B) - f(A) \quad \& \quad \int_{C_2}\vec{F}\cdot d\vec{r} = f(B) - f(A)$$

b/c $\vec{F} = \nabla f$

(fundamental theorem of line integrals)

So if $\vec{F}$ is conservative then the line integrals are the same!

Definition: $\int_C \vec{F}\cdot d\vec{r}$ is called <u>independent of path</u>

if $\int_{C_1}\vec{F}\cdot d\vec{r} = \int_{C_2}\vec{F}\cdot d\vec{r}$ whenever $C_1$ & $C_2$ have the same endpoints.