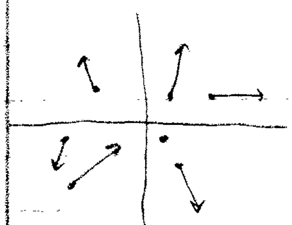
3/1/2010

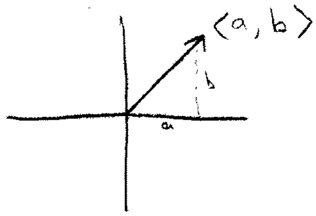
Chapter 17

Vector Fields

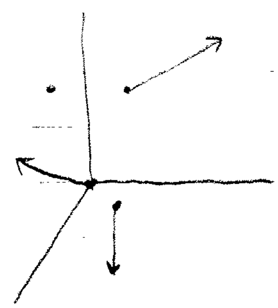
Review 2D:



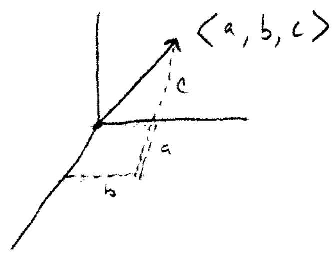
vectors = direction and magnitude



3D:



the point is vector $\langle 0,0 \rangle$ or $\langle 0,0,0 \rangle$



A vector field is an assignment

pt in space $\vec{x}$ $\longrightarrow$ $\vec{F}(\vec{x})$ vector based at the point $\vec{x}$

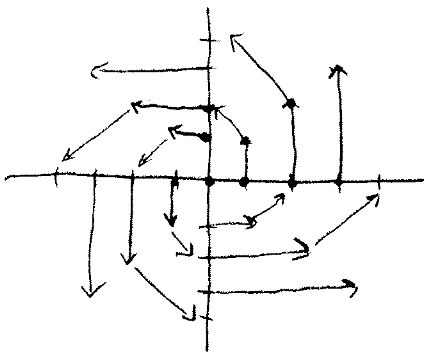
$$\vec{F}(x,y) = P(x,y)\,\vec{i} + Q(x,y)\,\vec{j} \qquad i = \langle 1,0 \rangle,\ j = \langle 0,1 \rangle$$

$$\vec{F}(x,y,z) = P(x,y,z)\,\vec{i} + Q(x,y,z)\,\vec{j} + R(x,y,z)\,\vec{k}$$

$i = \langle 1,0,0 \rangle$
$j = \langle 0,1,0 \rangle$
$k = \langle 0,0,1 \rangle$

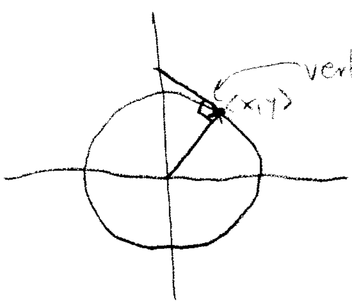
Ex:

$\vec{F}(x,y) = -y\hat{i} + x\hat{j} \longrightarrow \vec{F}(x,y) = \langle -y, x \rangle$



(x,y)	$\vec{F}(x,y)$
$(0,0)$	$\langle 0,0 \rangle$
$(1,0)$	$\langle 0,1 \rangle$
$(2,0)$	$\langle 0,2 \rangle$
$(3,0)$	$\langle 0,3 \rangle$
$(-1,0)$	$\langle 0,-1 \rangle$
$(0,1)$	$\langle -1,0 \rangle$
$(0,2)$	$\langle -2,0 \rangle$
$(2,2)$	$\langle -2,2 \rangle$
$(1,1)$	$\langle -1,1 \rangle$

$\vec{F}(x,y) = \langle -y, x \rangle$ moves a point at (x,y) in the direction tangent to the circle centered on the origin that passes through (x,y):

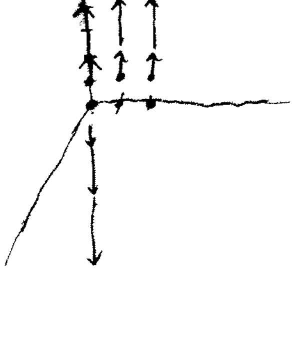


verify This is $\pi/2$ by showing

$\langle x, y \rangle \cdot \langle -y, x \rangle = 0$

Dot product: $x(-y) + y(x) = 0$

Ex: $\vec{F}(x,y,z) = z\vec{k} \longrightarrow \langle 0,0,z \rangle$

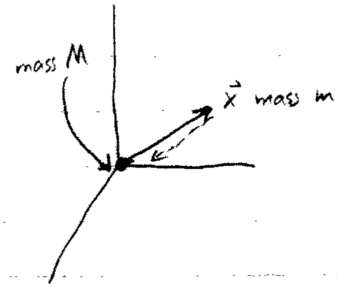


(x,y,z)	$\langle 0,0,z \rangle$
$(0,0,0)$	$\langle 0,0,0 \rangle$
$(0,0,1)$	$\langle 0,0,1 \rangle$
$(0,0,2)$	$\langle 0,0,2 \rangle$
$(0,1,1)$	$\langle 0,0,1 \rangle$

Gravitational Vector Field

If $\vec{x} = (x, y, z)$

$$\vec{F}(\vec{x}) = \left(\frac{-mMG}{|\vec{x}|^3}\right)\vec{x}$$



If $f(x,y)$ is a function, then we can define its gradient vector field

$$\vec{F}(x,y) = \nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$$

$$f(x,y,z) \longrightarrow \nabla f = \langle f_x, f_y, f_z \rangle$$

Ex: $f(x,y) = x^2 + y^2 \longrightarrow \nabla f(x,y) = \langle 2x, 2y \rangle$

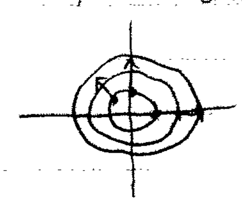
Facts about ∇f:

- ∇f is always $\perp$ level sets of f

level set = set $f(x,y)$ = constant

constant $= f(x,y) = x^2 + y^2$

$x^2 + y^2 =$ constant $\rightarrow$ circle



Definition: $\vec{F}(\vec{x})$ is called conservative if $\vec{F} = \nabla f$ for some function f. f is called a potential function for $\vec{F}$.

Ex: $f(x,y,z) = \dfrac{mMG}{\sqrt{x^2+y^2+z^2}} \rightarrow \nabla f =$ gravitational vector field

$\nabla f(1,0) = \langle 2, 0 \rangle$

$\nabla f(0,1) = \langle 0, 2 \rangle$

$\nabla f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \langle -\sqrt{2}, \sqrt{2} \rangle$