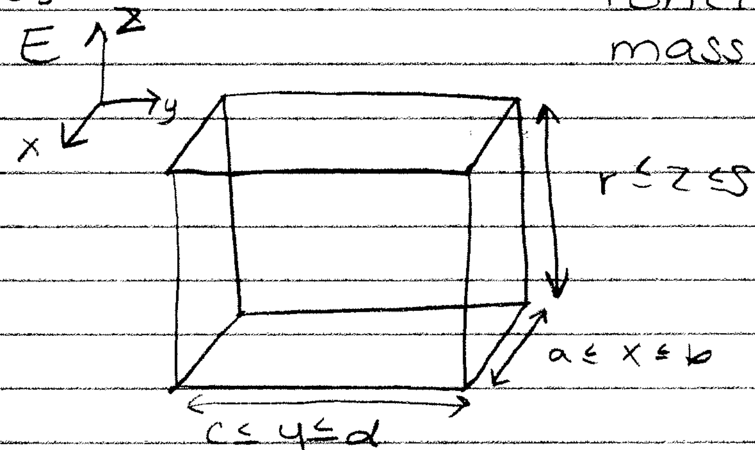
Sheila McCarthy

2/8/10

Sheila M. is happy that it is semi-nice outside today.

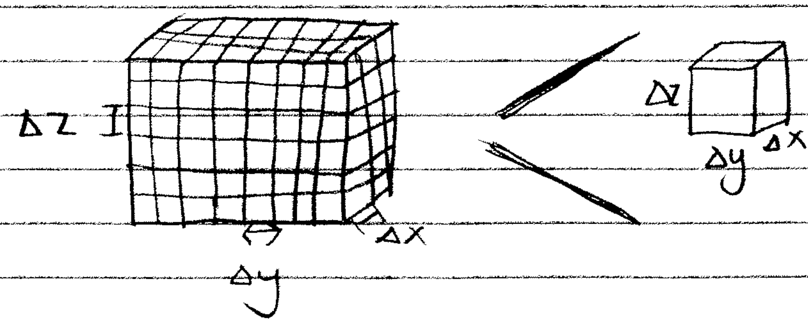
§16.6 Triple Integration

$\iiint_E f(x,y,z)\,dV$ "= If $f(x,y,z)$ is a density function, then this is the mass of the solid E"



First Case: $E = [a,b] \times [c,d] \times [r,s]$ (box)

Idea: cut E into tiny boxes with dimensions $\Delta x \times \Delta y \times \Delta z$



So now pick inside the box with the i^{th} x-coordinate, the j^{th} y-coordinate, and the k^{th} z-coordinate, pick a point (x_i^*, y_j^*, z_k^*).

The density inside this subbox is approximately $f(x_i^*, y_j^*, z_k^*)$ as long as the subbox is tiny.

So the mass of the ijk^th subbox is approximately

$$\boxed{f(x_i^*, y_j^*, z_k^*)\,\underbrace{\Delta x \Delta y \Delta z}_{dx\,dy\,dz}}$$

$$\int_r^s\int_c^d\int_a^b f(x,y,z)\,dx\,dy\,dz \leftarrow \text{mass } (E)$$

Fubini's Theorem =

$$\int_c^d\int_r^s\int_a^b f(x,y,z)\,dx\,dz\,dy$$

$$\|$$

$$\int_c^d\int_a^b\int_r^s f(x,y,z)\,dz\,dx\,dy$$

* only true for boxes. *

$$= \ldots$$

$$\iiint_{[0,1]\times[-1,2]\times[0,3]} xyz^2\,dV = \int_{-1}^{2}\int_0^3\int_0^1 xyz^2\,dx\,dz\,dy$$

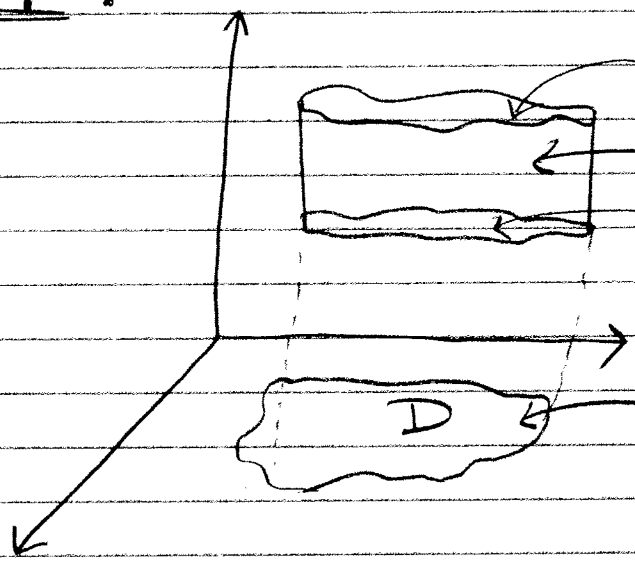
$$= \int_{-1}^{2}\int_0^3 \left[\frac{x^2yz^2}{2}\right]_{x=0}^{x=1} dz\,dy$$

$$= \int_{-1}^{2}\int_0^3 \frac{yz^2}{2}\,dz\,dy$$

$$= \int_{-1}^{2}\left[\frac{yz^3}{6}\right]\Bigg|_{z=0}^{z=3} dy = \int_{-1}^{2}\left(\frac{27y}{6}\right)dy = \ldots$$

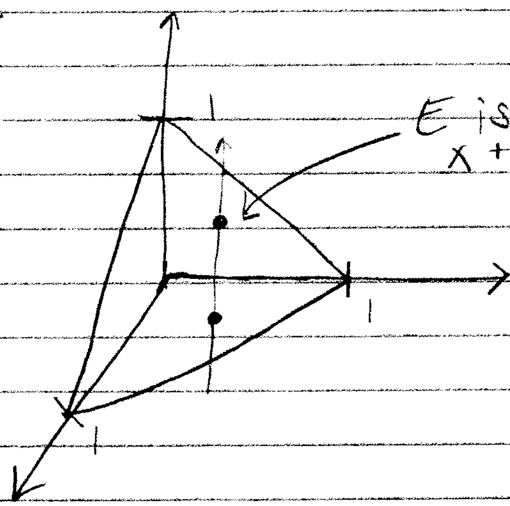
Strategy for more general E's than boxes

Case I:



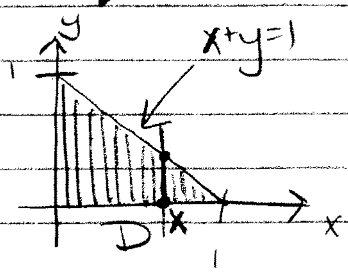
$$\iiint f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\right] dA$$

Ex:



$$\iiint_E z\,dV = ?$$

Shadow of E in the xy-plane is a triangle



D: $0 \le x \le 1$, $0 \le y \le 1-x$ or $0 \le y \le 1$, $0 \le x \le 1-y$

* the one with the numbers always goes on the outside.

$$\int_0^1 \int_0^{1-x} \left[\int_0^{1-x-y} z \quad dz \right] dy\,dx$$