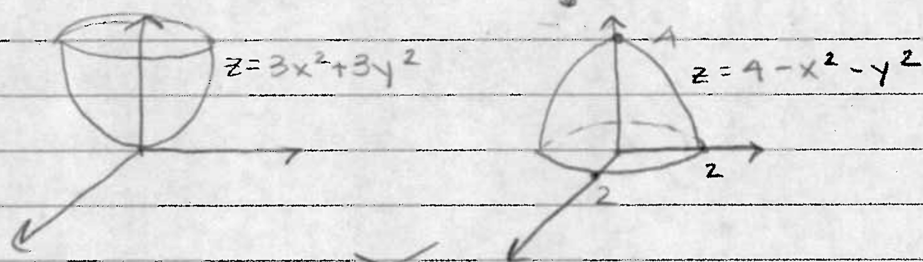
Julie Hudecek

Scribe Notes: 2.4.10

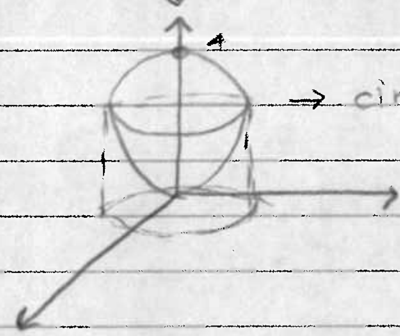
Polar coordinates:

Let's compute the volume of the solid bounded by:

$z = 3x^2 + 3y^2$ } paraboloid ← (steeper bowl)
$z = 4 - x^2 - y^2$ ← (upside down & up by four)



$z=0 \to 0=4-x^2-y^2$
$2^2 = x^2+y^2$
$r=2$



→ circle of intersection, radius ?
(to find region of integration)

set equal: $z = 3x^2+3y^2$ & $z = 4-x^2-y^2$

$$\downarrow$$

$$3x^2+3y^2 = 4-x^2-y^2$$

$$\downarrow$$

$$x^2+y^2=1$$

Strategy to find volume:

$$\iint_D \Big[\underbrace{4-x^2-y^2}_{\text{upper graph}} - \underbrace{(3x^2+3y^2)}_{\text{lower graph}}\Big]\, dA$$

Describe D: $0 \le r \le 1$, $0 \le \theta \le 2\pi$

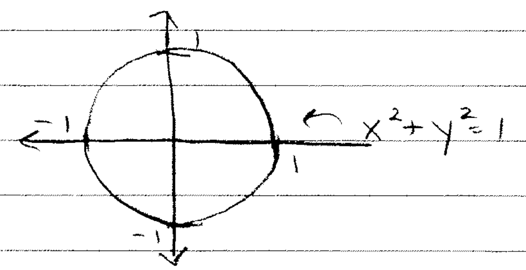
set up integral

$$\downarrow$$

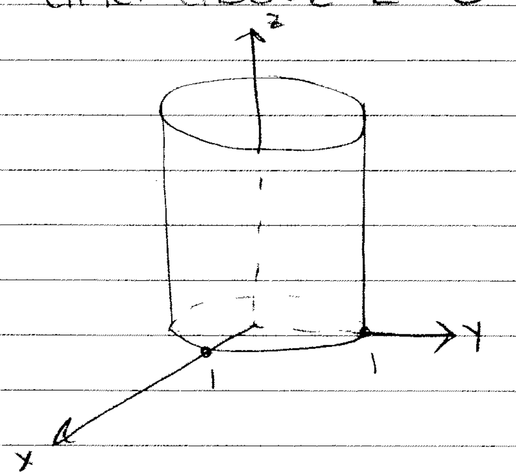
$\int_0^{2\pi}\int_0^1 (4-4r^2)\, r \cdot dr\, d\theta$ ... (should be able to do!)

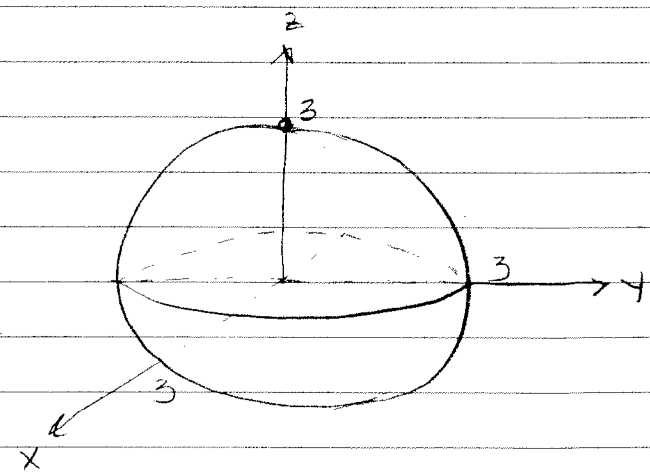
How would you describe in xy-coordinates?

$$\left.\begin{array}{c} -1 \le x \le 1 \\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2} \end{array}\right\} \left\{\begin{array}{c} -1 \le y \le 1 \\ -\sqrt{1-y^2} \le x \le \sqrt{1-y^2} \end{array}\right\}$$ DO POLAR RATHER THAN RECTANGULAR



ex) Find volume of region outside the cylinder $x^2+y^2=1$ but inside sphere: $x^2+y^2+z^2=9$, and above $z=0$





$\longrightarrow$

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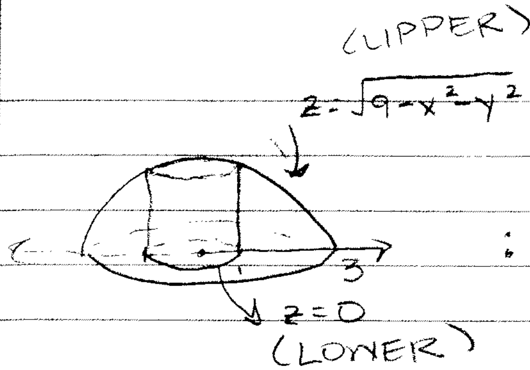
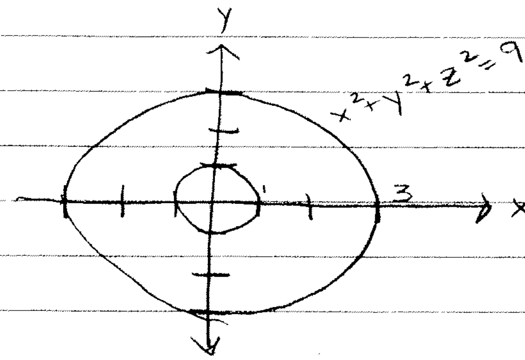


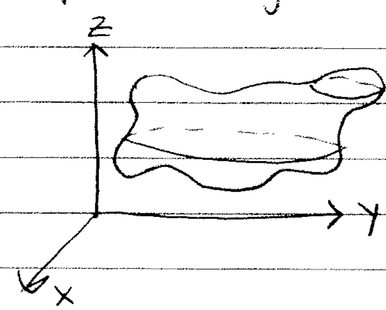


$$\int_0^{2\pi}\int_1^3\left[\sqrt{9-x^2-y^2}\right]r\cdot dr\,d\theta$$

$$\downarrow$$

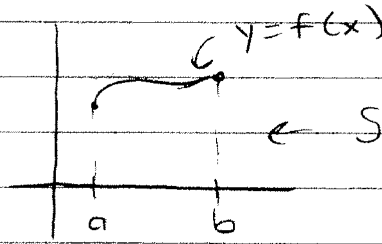
$$\int_0^{2\pi}\int_1^3\left[9-r^2\right]r\cdot dr\,d\theta \quad \ldots \text{ finish}$$

☆ 16.6 Triple Integrals : $\iiint_E f(x,y,z)\,dV$



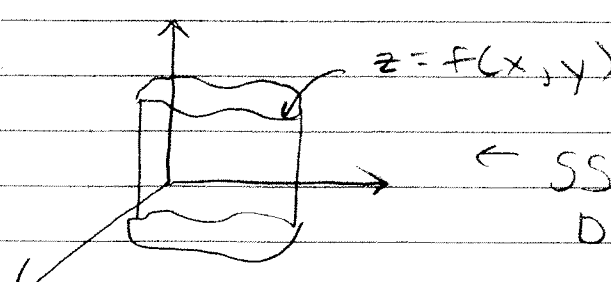
$E \downarrow$ some solid "stuff"

integration: 1D:



$\leftarrow \int_a^b f(x)\,dx$

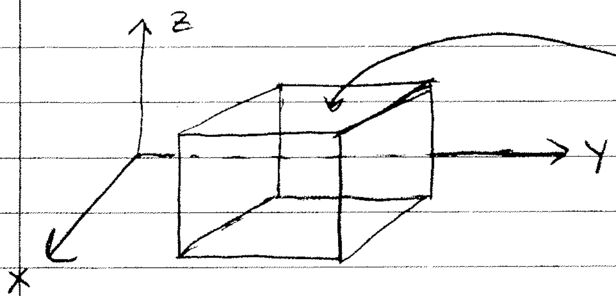
2D:



$\leftarrow \iint_D f(x,y)\,dA$

3D: ? To do this, imagine $f(x,y)$ is a density function.

$\boxed{\text{density: } g/cm^3 \text{ (for example)}}$



E: $[a,b] \times [c,d] \times [r,s]$

$a \le x \le b$

$c \le y \le d$

$r \le z \le s$

$f(x,y,z)$ = density @ the point (x,y,z) in this box

so: $\text{Mass}(E) = \iiint_E f(x,y,z)\, dV$