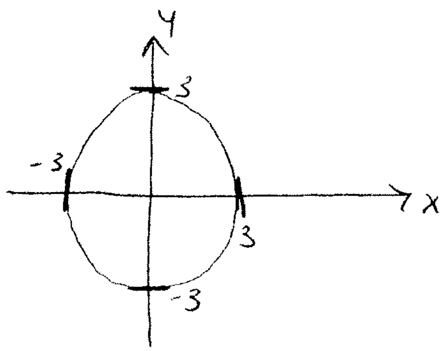
Allison Wigand

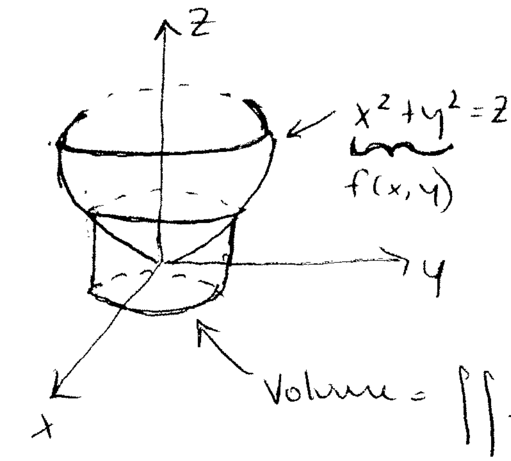
2/3/10

Allison W can't wait for the weekend!! WOOHOO!!!!

⟶ Professor Rafalski showed us some pictures

Example #1) compute the volume under the graph of $z = x^2 + y^2$ above the disk of radius 3 in the xy plane



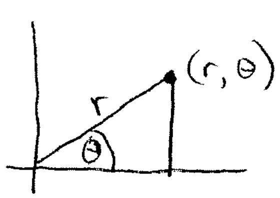


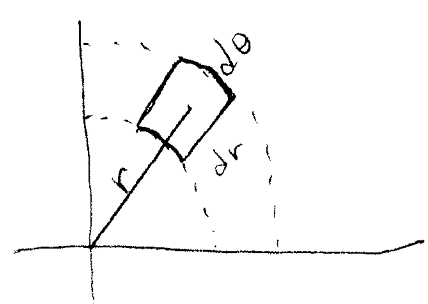
Volume $= \iint_D f(x,y)\,dA$

Recall: In polar coordinates $\iint_D f(x,y)\,dA$ if D is a polar rectangle ($D: \alpha \leq \theta \leq \beta$; $a \leq r \leq b$)

Then: $\iint_D f(x,y)\,dA = \underbrace{\int_\alpha^\beta \int_a^b \left[f(r\cos\theta, r\sin\theta) r \right] dr\,d\theta}$

"why?": $r\,dr\,d\theta$ is approximately the area of a very small polar rectangle at distance r from the origin





$D: 0 \le r \le 3$ $\quad x^2+y^2 = r^2$

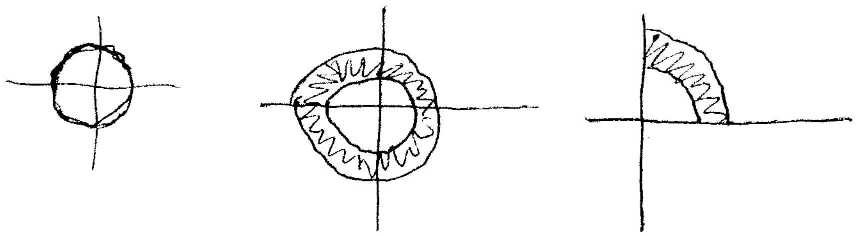
$\quad 0 \le \theta \le 2\pi$

Volume: $\displaystyle\int_0^{2\pi}\int_0^3 (r^2)\,r\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^4}{4}\Big|_0^3\right]d\theta = \int_0^{2\pi}\left(\frac{81}{4}-0\right)d\theta$

$$= \frac{81}{4}\,2\pi = \boxed{\frac{81\pi}{2}}$$

→ Tips on when to use polar coordinates:

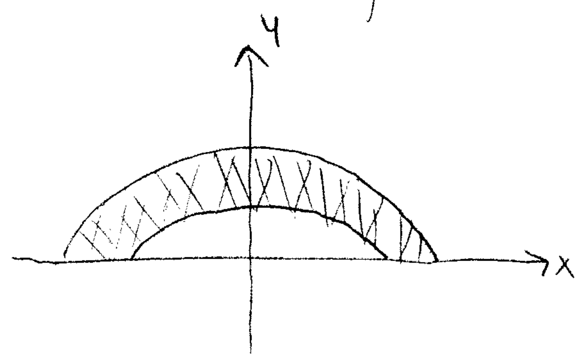
(#1) Anytime D has any circular symmetry of any kind



(#2) If the integrand has x^2+y^2 in it

Consider polar if either of these holds to be true

Example #2) $\displaystyle\iint_D (3x+4y^2)\,dA$ where D:



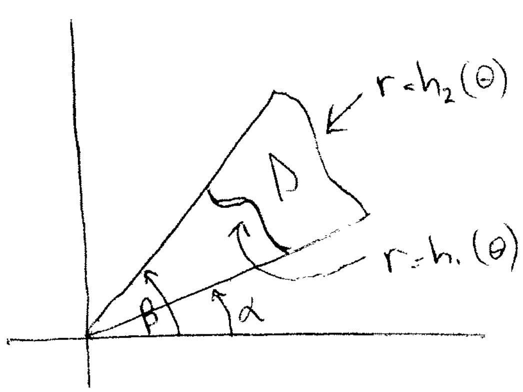
$$\int_0^{\pi}\int_1^2 \quad r\,dr\,d\theta$$

$$\int_0^{\pi}\int_1^2 \left(3r\cos\theta + 4r^2\sin^2\theta\right)r\,dr\,d\theta$$

Exercise: complete the double integral

$* \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

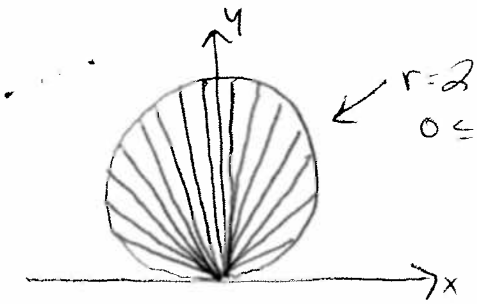
$\quad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$



Then: $\displaystyle\iint_D f(x,y)\,dA$

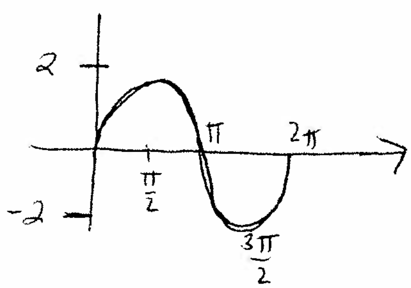
$$\| $$

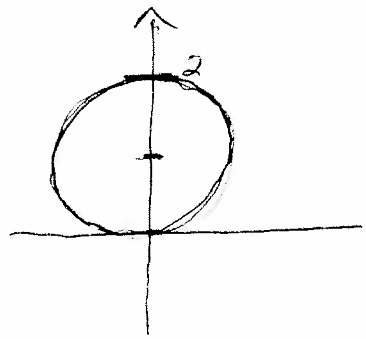
$$\int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)}\left[f(r\cos\theta, r\sin\theta)\,r\right]dr\,d\theta$$



$r = 2\sin\theta$
$0 \le \theta \le \pi$

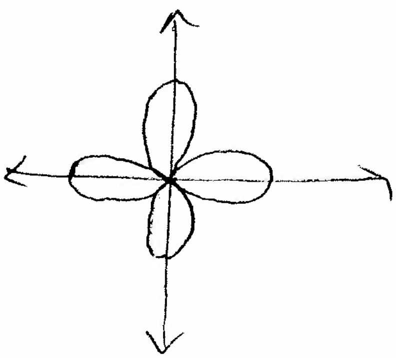
To Justify this a little:



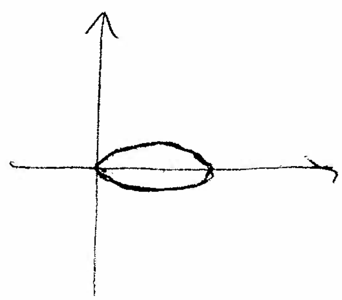


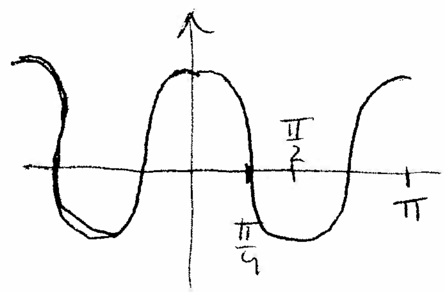
→ In this case $h_1(\theta) = 0$ and $h_2(\theta) = 2\sin\theta$

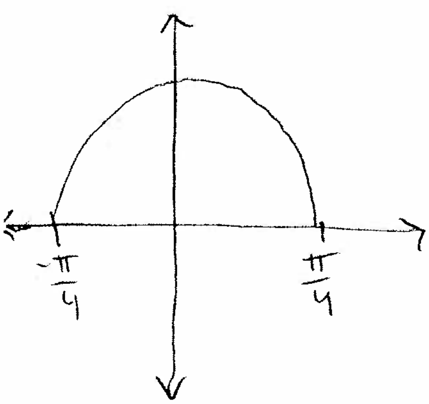
Example #3) $r\cos 2\theta$

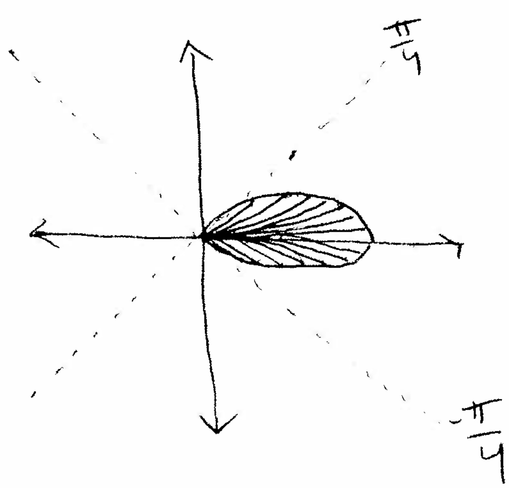


1 leaf









$h_1(\theta) = 0$
$h_2(\theta) = \cos 2\theta$

→ so lets compute the area of 1 leaf

$$\iint_{\text{leaf}} 1\,dA = \text{Area of leaf}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\cos 2\theta} 1\, r\,dr\,d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\frac{r^2}{2} \Big|_0^{\cos 2\theta} \right] d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos^2 2\theta}{2}\, d\theta$$

* $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}(1+\cos 4\theta)\,d\theta = \frac{1}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1+\cos 4\theta)\,d\theta$$

Exercise: complete this integral