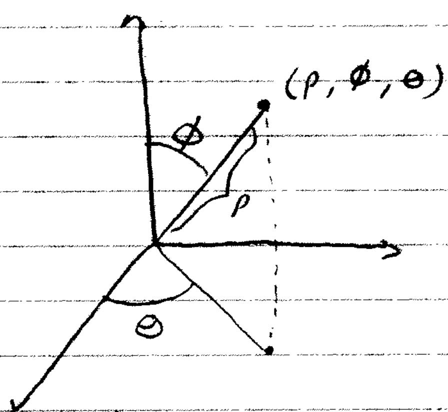
Monday Feb 22, 2010

Notes

1. Review Wednesday - Bring all questions
2. Possible Extra Review Wed (TBA)
3. Exam Thursday 2/25

Spherical Coordinates



$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$x^2 + y^2 + z^2 = \rho^2$

$$\boxed{dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}$$

if ρ = constant : sphere

if ϕ = constant : cone

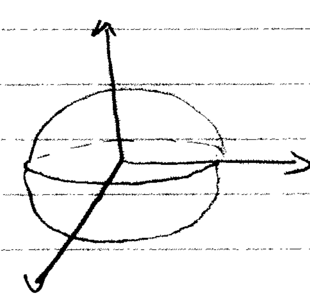
if θ = constant : vertical plane through origin

Integrating Functions

$$\iiint_E f(x,y,z)\, dV = \underbrace{\iiint}_{\text{E in terms of Spherical Coordinates}} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

Ex 1 $\displaystyle\iiint_B e^{(x^2+y^2+z^2)^{3/2}} \, dV$

B: Solid ball of radius 1



$0 \le \rho \le 1$

$\left.\begin{array}{l} 0 \le \phi \le \pi \\ 0 \le \theta \le 2\pi \end{array}\right\}$ or $\begin{array}{l} 0 \le \phi \le 2 \\ 0 \le \theta \le \pi \end{array}$

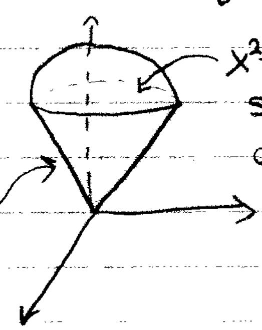
$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 e^{(\rho^2)^{3/2}} \cdot \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi}\int_0^1 \underbrace{\rho^2 e^{\rho^3}}_{\text{substitution}} \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \sin\phi \left[\tfrac{1}{3} e^{\rho^3}\Big|_0^1\right] d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi} \sin\phi \left(\tfrac{e}{3} - \tfrac{1}{3}\right) d\phi\, d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{3}(e-1)\left[-\cos\phi \Big|_0^{\pi}\right] d\theta = \dots$$

Ex(2) $\iiint_E 1\, dV$ - set up the integral that represents the volume of E

E: the region pictured here:



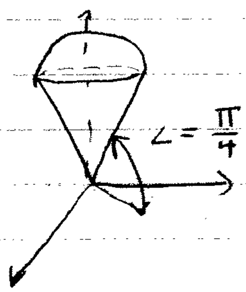
E in spherical coordinates:

$x^2+y^2+z^2 = z$

$\downarrow$

$\rho^2 = \rho\cos\phi$

$\therefore \rho = 0$ or $\rho = \cos\phi$



Why? $z = \sqrt{x^2+y^2}$

$\downarrow$

$$\rho\cos\phi = \sqrt{(\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2}$$

$$= \sqrt{\rho^2\sin^2\phi\,\underbrace{(\cos^2\theta + \sin^2\theta)}_{1}}$$

$\rho\cos\phi = \rho\sin\phi$

$\cos\phi = \sin\phi$ only at $\pi/4$

So...

$0 \le \rho \le \cos\phi$

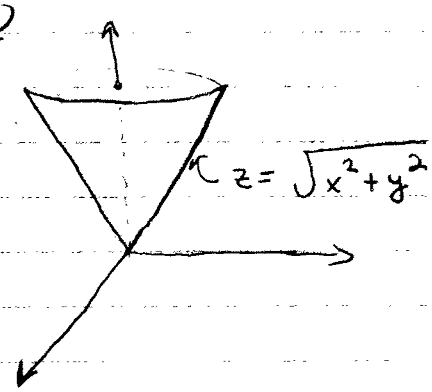
$0 \le \phi \le \pi/4$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\cos\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi/4}\left[\frac{\rho^3}{3}\sin\phi \Big|_0^{\cos\phi}\right] d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \frac{(\cos\phi)^3}{3}\sin\phi\, d\phi\, d\theta = \int_0^{2\pi}\left[-\frac{\cos^4\phi}{12}\Big|_0^{\frac{\pi}{4}}\right] d\theta = \dots$$

Ex ③



Describe E:

Rectangular:

$\le x \le$

$\le y \le$

$\le z \le$

$\Big\}$ Exercize

Cylindrical:

$\le \theta \le$

$\le r \le$

$\le z \le$

$\Big\}$ Exercize

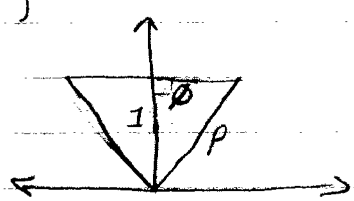
Spherical:

$\le \rho \le$

$\le \phi \le$

$\le \theta \le \frac{1}{\cos\phi}$

$\Big\}$ Exercize



$$\cos\phi = \frac{1}{\rho} \rightarrow \rho = \frac{1}{\cos\phi}$$