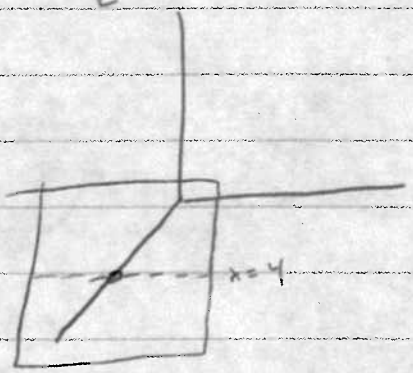
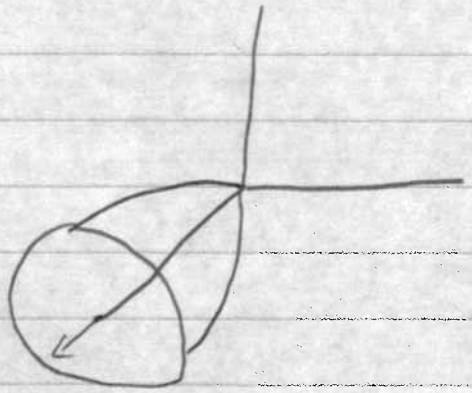
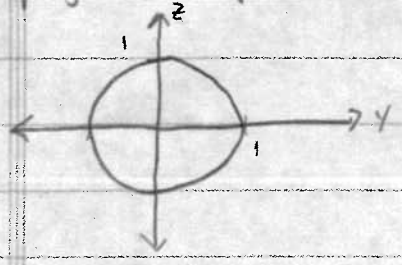
JP Rinfret Scribe Notes

#17 $\iiint_E x\,dV$ $\quad E\begin{cases} x = 4y^2+4z^2 \\ x = 4 \end{cases}$ $\begin{cases} 4 = 4y^2+4z^2 \\ \text{b/c } x=4 \end{cases}$





projection plane



$-1 \le r \le 1$ $\quad y = r\cos\theta$

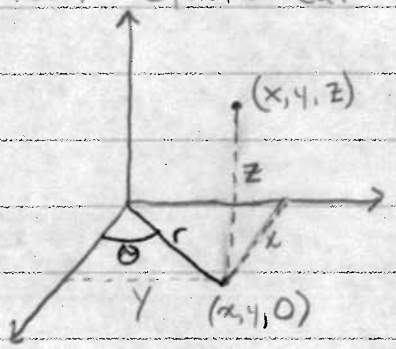
$0 \le \theta \le 2\pi$ $\quad z = r\sin\theta$

$4y^2+4z^2 \le x \le 4$

$\underbrace{\qquad}_{4r^2}$

$$\int_0^{2\pi}\int_0^1 \left(\int_{4r^2}^{4} x\,dx\right) r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 \left(\frac{x^2}{2}\right)\Big|_{x=4r^2}^{x=4} r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 (8-8r^4)\cdot r\,dr\,d\theta =$$

§16.7 Cylindrical Coordinates



(r,θ,z)

$x = r\cos\theta$ $\quad x^2+y^2 = r^2$

$y = r\sin\theta$

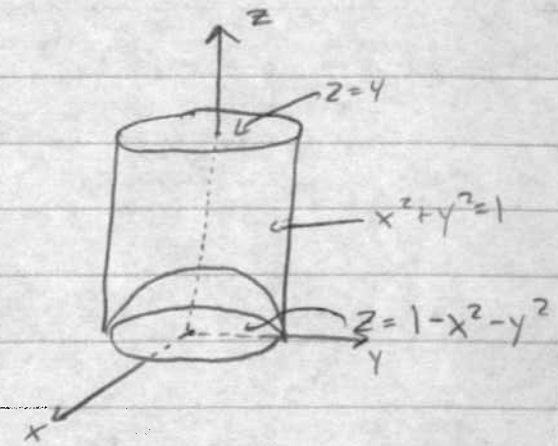
Integrating a triple integral $\iiint_E f(x,y,z)\,dV \Longrightarrow \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} f(r\cos\theta, r\sin\theta, z)\,dz\cdot r$

Example: $\iiint_E \sqrt{x^2+y^2}\, dV$

$0 \le \theta \le 2\pi$

$0 \le r \le 1$

$\underbrace{1-x^2-y^2}_{1-r^2} \le z \le 4$



$$\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r\, dz \cdot r\, dr\, d\theta = \int_0^{2\pi}\int_0^1 r^2\left[\int_{1-r^2}^4 dz\right] dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 r^2 (z)\Big|_{1-r^2}^4 dr\, d\theta = \int_0^{2\pi}\int_0^1 r^2\left(4-(1-r^2)\right) dr\, d\theta = \dots$$

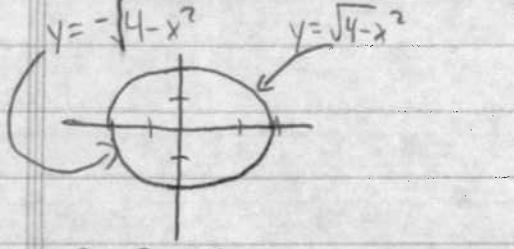
Example: $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\, dz\, dy\, dx$

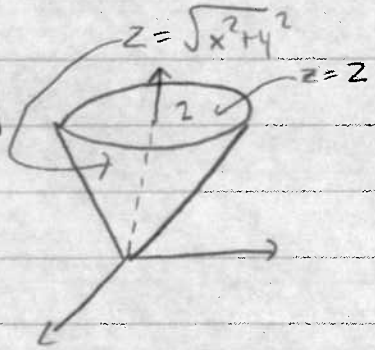
$-2 \le x \le 2$

$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$

$\sqrt{x^2+y^2} \le z \le 2$

} E (read off of triple int.)





$$\int_0^{2\pi}\int_0^2\int_r^2 r^2\, dz \cdot r\, dr\, d\theta = \int_0^{2\pi}\int_0^2 r^3(2-r)\, dr\, d\theta = \dots$$

Task: Interpret as triple integral over solid E and evaluate using cylindrial coords

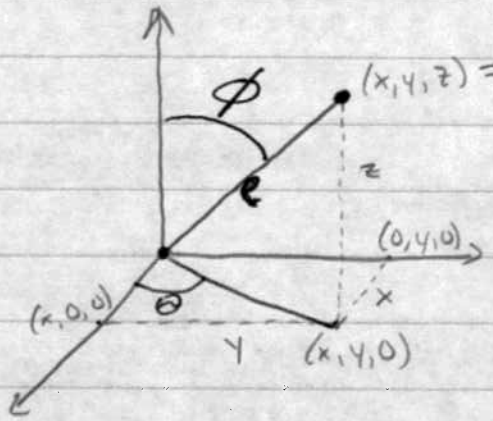
z values always lie btw these 2 surfaces

Verify the projection is circle with radius 2

$z = \sqrt{x^2+y^2}$ $z = 2$

$2 = \sqrt{x^2+y^2} \rightarrow 4 = x^2+y^2$

Spherical Coordinates



$x = \rho \cos\theta \sin\phi$

$y = \rho \sin\theta \sin\phi$

$z = \rho \cos\phi$

$x^2 + y^2 + z^2 = \rho^2$

Examples: ① ρ = Constant → sphere of radius C

② θ = constant → plane w/ angle θ to x-axis

③ ϕ = constant → cone