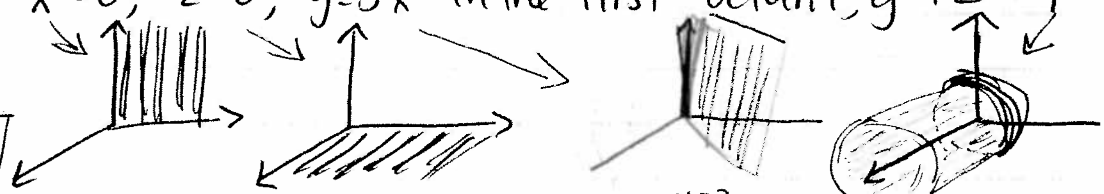
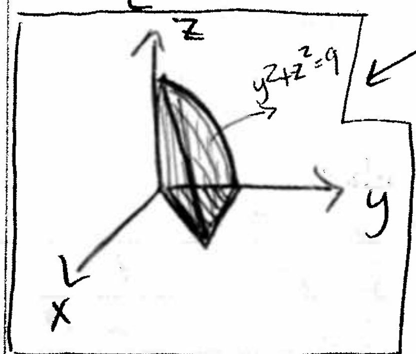
NOTES
- No Quiz Monday

Wednesday
2/17/10

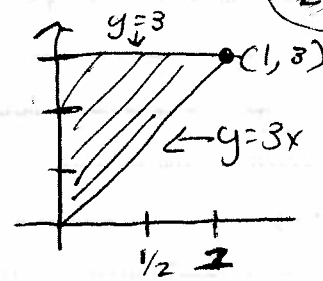
Example: 16.6 #18

$\iiint_E z\,dV$ $\quad x=0,\ z=0,\ y=3x$ in the first octant, $y^2+z^2=9$





① Project it onto the **xy-plane**:



one way to describe the projection

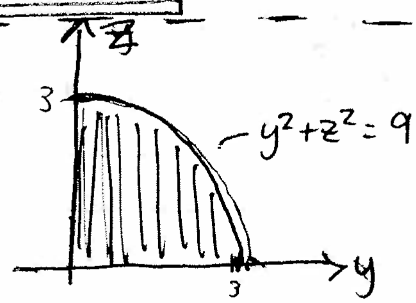
$0 \le x \le 1$
$3x \le y \le 3$
$0 \le z \le \sqrt{9-y^2}$

$\Rightarrow$ The limits of integration when projected onto the xy plane

$$\boxed{\int_0^1 \int_{3x}^3 \int_0^{\sqrt{9-y^2}} z\,dz\,dy\,dx}$$

---

② project it onto the yz-plane:



$0 \le y \le 3$
$0 \le z \le \sqrt{9-y^2}$
$0 \le x \le y/3$
(b/c $y=3x$)

(also, can describe using polars: $y = r\cos\theta$, $z = r\sin\theta$ $\Rightarrow$ $0 \le r \le 3$, $0 \le \theta \le \pi/2$)

$$\boxed{\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{y/3} z\,dx\,dz\,dy}$$

$$\boxed{\int_0^{\pi/2} \int_0^3 \int_0^{\frac{r\cos\theta}{3}} r\sin\theta\, r\,dr\,d\theta}$$ using polar coordinates

Definition

If $\rho(x,y,z)$ is a density function (requires ρ [rho] >0)
then, $m = \iiint_E \rho(x,y,z)\,dV$ is the mass, E

$$M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV$$

$$M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

$$M_{yz} = \iiint_E y\,\rho(x,y,z)\,dV$$

M_{yz}, M_{xy}, M_{yz} : are called the MOMENTS of E about the coordinate planes

$$\boxed{\bar{X} = \frac{M_{yz}}{m}, \quad \bar{y} = \frac{M_{xz}}{m}, \quad \bar{z} = \frac{M_{xy}}{m} \qquad = (\bar{x}, \bar{y}, \bar{z}) = \text{center of mass for } E}$$

E : bounded by $x = y^2$, $x = z$, $z = 0$, $x = 1$
constant density ρ. calculate the center of mass (c.o.m.)

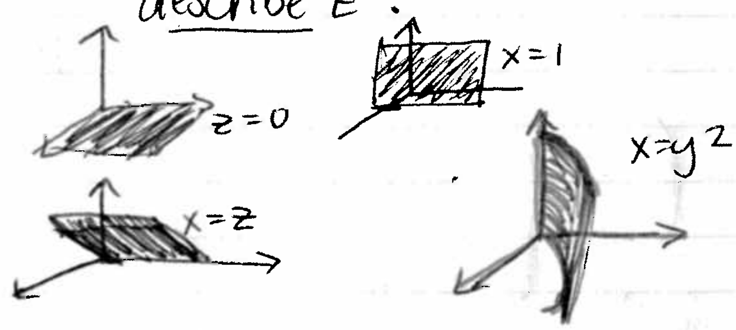
Need: $\iiint_E x\rho\,dV$

$\iiint_E y\rho\,dV$

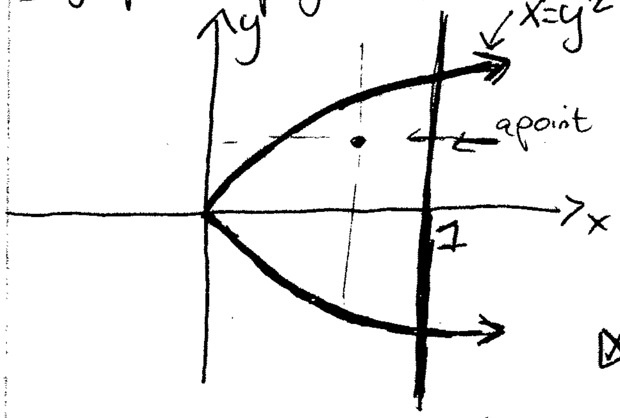
$\iiint_E z\rho\,dV$

$m = \iiint_E \rho\,dV$

describe E :



xy-plane projection:



$$\int_0^1\int_{-\sqrt{x}}^{\sqrt{x}}\int_0^x \rho \, dz\, dy\, dx$$

$0 \le z \le x$

projection $\boxed{\begin{matrix} 0 \le x \le 1 \\ -\sqrt{x} \le y \le \sqrt{x} \end{matrix}}$ or $\boxed{\begin{matrix} -1 \le x \le 1 \\ y^2 \le y \le 1 \end{matrix}}$

solve: $\rho\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \left[z \Big|_0^x \right] dy\, dx$

$= \rho \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} x \, dy\, dx$

$= \rho \int_0^1 \left(xy \Big|_{-\sqrt{x}}^{\sqrt{x}} \right) dx = \rho \int_0^1 (x^{3/2} + x^{3/2})\, dx$

MASS $= 2\rho \dfrac{x^{5/2}}{5/2} \Big|_0^1 = \boxed{\dfrac{4\rho}{5}}$

★ The center of Mass must lie on the xz-plane, so y coordinate $= 0$
therefore, we don't need to calculate $\iiint_E y \rho \, dV$
because: $\bar{y} = 0$ b/c the object has constant density
and is symmetric about the xy-plane

exercize: do the other triple integrals: $\iiint_E x \rho \, dV$

$\iiint_E z \rho \, dV$