2/11/09

Kim totally had a psychic vision about being the scribe today!

Goal: To compute tripple integrals: $\iiint_E f(x,y,z)\,dV$ (E: 3D solid; $f(x,y,z)$: 3 variable function)

$f > 0$, $\iiint_E f(x,y,z)\,dV$ = MASS.

How to do this:

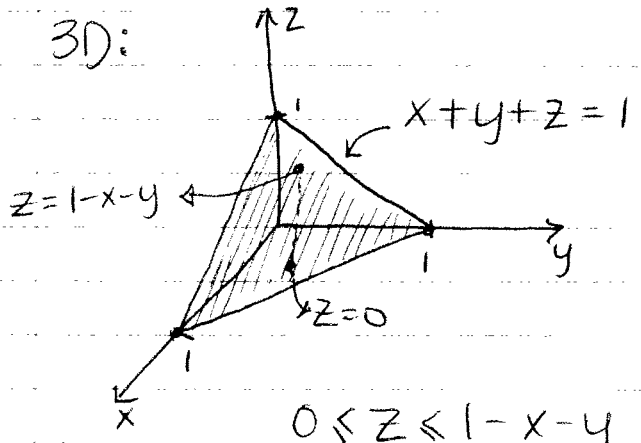
* Describe the solid (E) well.*

Q's: ① What is the plane onto which we should project E's shadow?

② What are the 2 graphs over the shadow that form E?

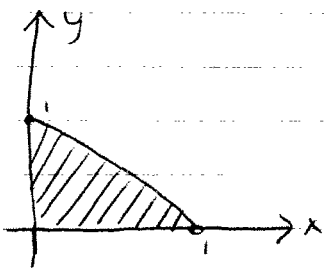
→ Last time:

3D:

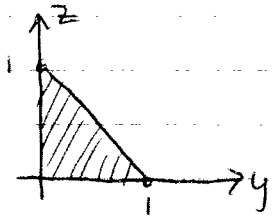


$0 \le z \le 1-x-y$

Shadow on xy-plane:

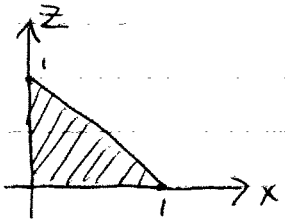


Shadow on yz-plane:



$0 \le x \le 1-z-y$

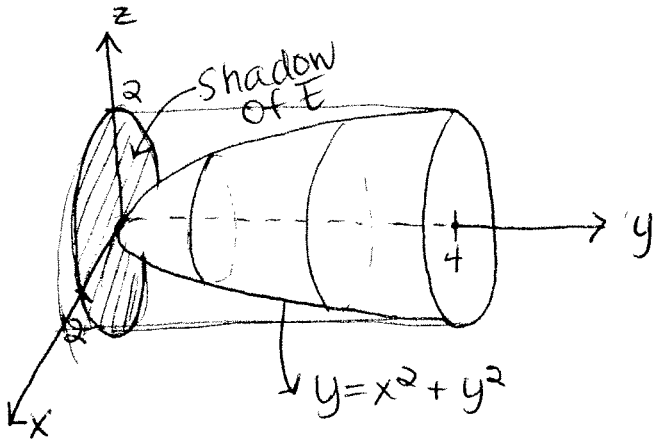
Shadow on xz-plane:



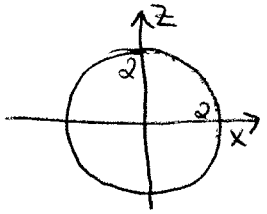
$0 \le y \le 1-x-z$

EX: $\iiint_E \sqrt{x^2+z^2}\, dV$

E is trapped by
$y = x^2 + z^2$ paraboloid
$y = 4$



Step 1: Pick a coordinate plane to look @ the shadow
xz-plane:



* POLAR! *
$0 \le \theta \le 2\pi$
$0 \le r \le 2$

Step 2: $x^2+z^2 \le y \le 4$

$$\iiint_E \sqrt{x^2+z^2}\, dV = \iint_{\text{Shadow of E}} \left(\int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\, dy \right) dA$$

$x^2+z^2 = r^2$

$$= \int_0^{2\pi}\int_0^2 \int_{r^2}^4 \sqrt{r^2} \cdot dy \cdot r \cdot dr \cdot d\theta$$

$$= \int_0^{2\pi}\int_0^2 \left(\int_{r^2}^4 r\, dy \right) r \cdot dr \cdot d\theta$$

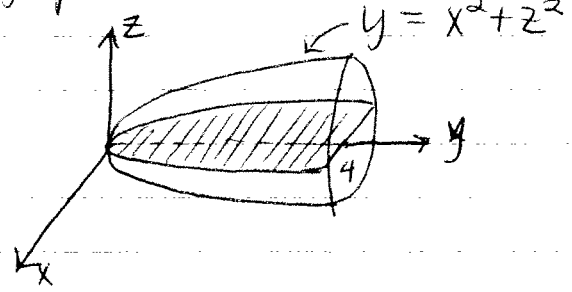
$$= \int_0^{2\pi}\int_0^2 r^2 \left(\int_{r^2}^4 1\, dy \right) dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^2 r^2 \left[y \Big|_{r^2}^4 \right] dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^2 (4r^2 - r^4)\, dr\, d\theta$$

Choose a different plane for the shadow of E:

xy-plane:



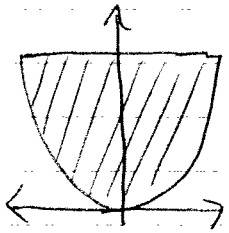
$$z = \pm\sqrt{y - x^2}$$

$$-\sqrt{y-x^2} \le z \le \sqrt{y-x^2}$$

$$\iiint_E \sqrt{x^2+z^2}\, dV$$

$$= \iint_{\substack{\text{shadow}\\ \text{of } E}} \left(\int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2+z^2}\, dz \right) dA$$

Shadow of E using inequalities:



$$= \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2+z^2}\, dz\, dx\, dy$$

$$0 \le y \le 4$$
$$-\sqrt{y} \le x \le \sqrt{y}$$

* STOP! * do not try to do this integral! way 1 (polar) is ~~easiest~~ easiest

Exercise:
write down but do not evaluate the two tripple integrals you get from the shadow of E on the ~~xy~~ yz-plane.

Surfaces in 3D Space:

Planes:
$ax + by + cz = d$
ex: $x = 2$
$x + y = 3$
$z - 2y = 4$
$x - 2y + z = 3$

Most Basic:
$x = K$
$y = K$ K = constant
$z = K$
$ax + by = C$
$ax + bz = C$
$ay + bz = C$

1 missing variable