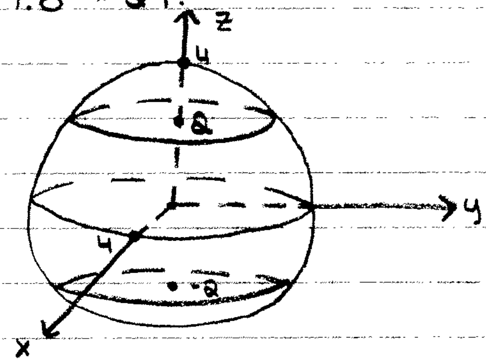
Remarks about § 17.6 # 24:

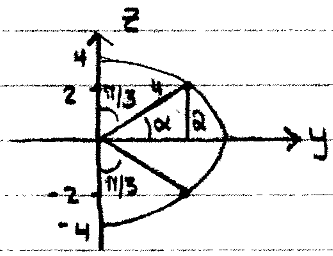


$x = 4\cos\theta\sin\phi$

$y = 4\sin\theta\sin\phi$

$z = 4\cos\phi$

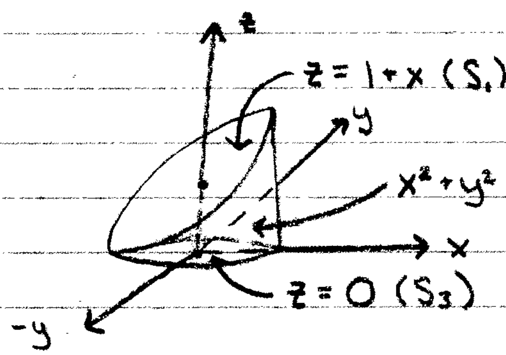
$\left.\begin{array}{l} 0 \le \theta \le 2\pi \\ 0 \le \phi \le \pi \end{array}\right\}$ whole sphere

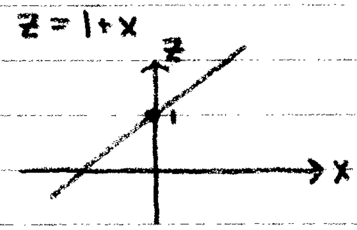


$\frac{\pi}{3} \le \phi \le \frac{2\pi}{3}$

$\sin\alpha = \frac{1}{2}$ $\alpha = \frac{\pi}{6}$

Example:

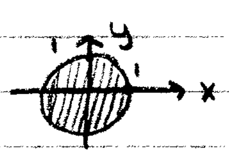




Compute $\iint_S z\,dS$ where $S = S_1 \cup S_2 \cup S_3$

$= \iint_{S_1} z\,dS + \iint_{S_2} z\,dS + \iint_{S_3} z\,dS$

Describe the Parametrization of each surface

S_3: 

$\vec{r}(x,y) = \langle x, y, 0\rangle$ $z = f(x,y) = 0$

or

$\vec{r}(\theta, r) = \langle r\cos\theta, r\sin\theta, 0\rangle$

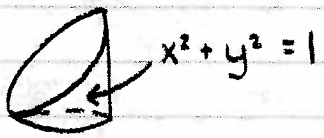
$0 \le r \le 1$ $0 \le \theta \le 2\pi$

$\iint_{S_3} z\,dS = \iint 0\,dS = 0$

Recall: $\iint_S f(x,y,z)\,dS = \iint_D f(x(u,v), y(u,v), z(u,v))\,|\vec{r}_u \times \vec{r}_v|\,dA$

*no matter where you are

S_2:



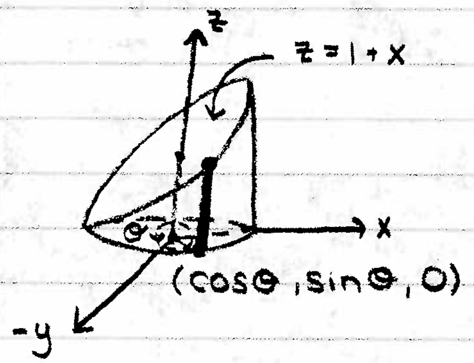
$x = 1\cos\theta$ x,y need to be on

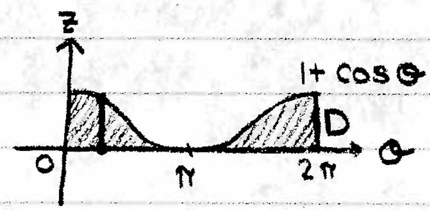
$y = 1\sin\theta$ the unit circle

$0 \le \theta \le 2\pi$

$z = z$

$0 \le z \le \underbrace{1+x}_{1+\cos\theta}$





$\vec{r}(\theta, z) = \langle \cos\theta, \sin\theta, z\rangle$

$0 \le \theta \le 2\pi$

$0 \le z \le 1 + \cos\theta$

$\vec{r}_\theta = \langle -\sin\theta, \cos\theta, 0\rangle$

$\vec{r}_z = \langle 0, 0, 1\rangle$

$|\vec{r}_\theta \times \vec{r}_z|$:

$$\begin{array}{cccccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & \vec{k} \\ -\sin\theta & \cos\theta & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$

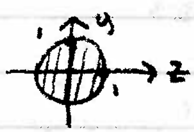
$= \langle \cos\theta, 0, \sin\theta\rangle$

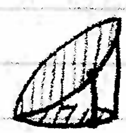
has length 1

$$\iint_{S_2} z\,dS = \int_0^{2\pi}\int_0^{1+\cos\theta} z\cdot 1\,dz\,d\theta$$

Compute as an exercise

S_1: S_1 is a graph of the function $z = 1 + x$ with region D





$\vec{r}(\theta, r) = \langle r\cos\theta, r\sin\theta, 1 + r\cos\theta\rangle$

$0 \le \theta \le 2\pi$

$0 \le r \le 1$

$\vec{r}_\theta = \langle -r\sin\theta, r\cos\theta, -r\sin\theta \rangle$

$\vec{r}_r = \langle \cos\theta, \sin\theta, \cos\theta \rangle$

$$\vec{r}_\theta \times \vec{r}_r = \begin{matrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & \vec{k} \\ -r\sin\theta & r\cos\theta & -r\sin\theta & -r\sin\theta & r\cos\theta & -r\sin\theta \\ \cos\theta & \sin\theta & \cos\theta & \cos\theta & \sin\theta & \cos\theta \end{matrix}$$

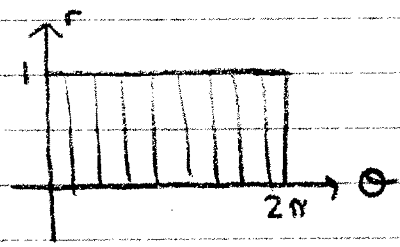
$$= \langle r\cos^2\theta + r\sin^2\theta, -r\sin\theta\cos\theta + r\sin\theta\cos\theta, -r\sin^2\theta - r\cos^2\theta$$

$$= \langle r, 0, -r \rangle$$

$|\vec{r}_\theta \times \vec{r}_r| = \sqrt{r^2 + r^2} = \sqrt{2}\, r$

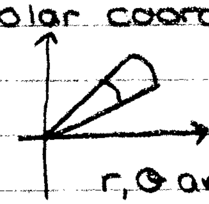
$$\iint_{S_1} z\,dS = \int_0^{2\pi}\int_0^1 \underbrace{(1 + r\cos\theta)}_{z} \cdot \sqrt{2}\, r \; dr\, d\theta$$ * no r here

Compute for practice



use $dr\,d\theta$ (no r)

polar coordinates:



r, θ are warped

use $r\,dr\,d\theta$