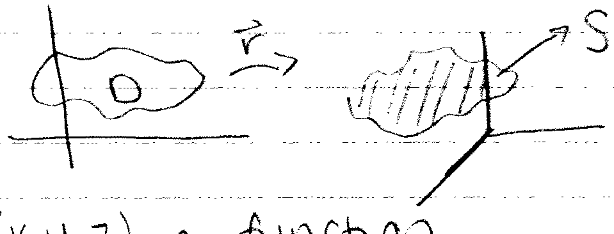
Lauren Balmert

Class Notes

Computing surface integrals... 4/15/10

$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$ describes the surface (u,v) come from a region D in the plane



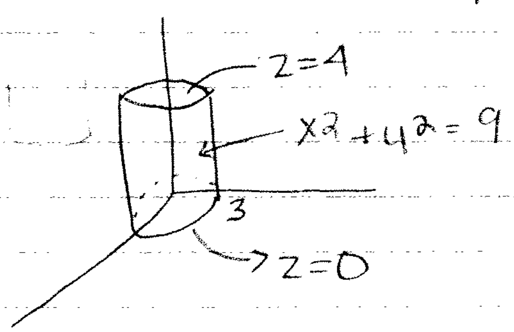
$f(x,y,z)$ a function

$$\iint_S f(x,y,z)\, ds = \iint_D f(\vec{r}(u,v))\, |\vec{r}_u \times \vec{r}_v|\, dA$$

Ex $\iint_S z\, ds$

Let S be the boundary of the solid cylinder cut out by $x^2+y^2=9$ and the planes $z=4$ and $z=0$

pic:



S_1 : $z=4$ part of S
S_2 : cylinder part
S_3 : $z=0$ part

$$S = S_1 \cup S_2 \cup S_3$$

$$\iint_S z\, ds = \iint_{S_1} z\, ds + \iint_{S_2} z\, ds + \iint_{S_3} z\, ds$$

$\iint_{S_1} z\, ds$

Describe S_1 $\quad \vec{r}(x,y) = \langle x, y, 4 \rangle$, $z = f(x,y) = 4$ $\quad$ D is the radius 3 circle centered at (0,0)

$$\iint_{S_1} z\,ds = \iint_D 4\,|\vec{r}_x \times \vec{r}_y|\,dA = \underbrace{\iint 4\sqrt{(f_x)^2+(f_y)^2+1}\,dA}$$

$\vec{r}_x = \langle 1, 0, 0 \rangle$

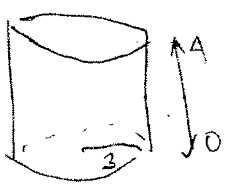
$\vec{r}_y = \langle 0, 1, 0 \rangle$

$$= \iint_D 4\,dA = \boxed{36\pi}$$

$$\boxed{\text{So } \iint_{S_1} z\,ds = 36\pi}$$

$$\iint_{S_2} z\,ds =$$

describe S_2



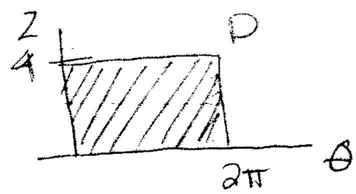
$\vec{r}(\theta, z) = \langle 3\cos\theta, 3\sin\theta, z \rangle$

$0 \le z \le 4 \qquad 0 \le \theta \le 2\pi$

$x = 3\cos\theta$

$y = 3\sin\theta$

$z = z$



$$\iint_{S_2} z\,ds = \iint_D z\,|\vec{r}_\theta \times \vec{r}_z|\,dA$$

$\vec{r}_\theta = \langle -3\sin\theta, 3\cos\theta, 0 \rangle$

$\vec{r}_z = \langle 0, 0, 1 \rangle$

$$\begin{array}{cccccc} i & j & k & i & j & k \\ -3\sin\theta & 3\cos\theta & 0 & -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} = \langle 3\cos\theta, 3\sin\theta, 0 \rangle$$

$$\iint_D z\,|\langle 3\cos\theta, 3\sin\theta, 0 \rangle|\,dA = \int_0^4 \int_0^{2\pi} 3z\,d\theta\,dz$$

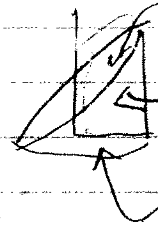
$$= 2\pi \frac{3z^2}{2}\Big|_0^4 = 48\pi$$

$$\boxed{\iint_{S_2} z\,dS = 48\pi}$$

$$\boxed{\iint_{S_3} z\,dS = 0}$$

b/c $z=0$ on S_3

Exercise



$S_1 : z = 1 + x$

$S_2 : x^2 + y^2 = 1$

$S_3 : z = 0$