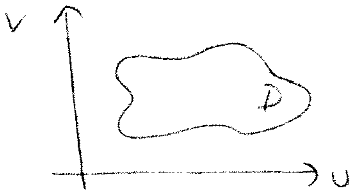
Dorothy M.
Scribe Notes

4/14/10

S is a surface given by a parametrization

$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$
(u,v) come from a region D in the plane

Quiz Tomorrow
(Everything since exam)
Friday morn by noon HW from last week due.



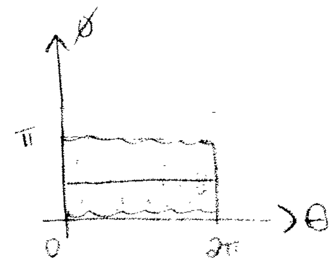
Last time:

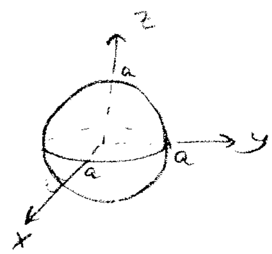
$$A(S) = \iint_S dS = \iint_D |\vec{r}_u \times \vec{r}_v| \, dA$$

ex) Sphere of radius a:

$x(\theta, \phi) = a\cos\theta\sin\phi$ $\qquad 0 \le \theta \le 2\pi$
$y(\theta, \phi) = a\sin\theta\sin\phi$ $\qquad 0 \le \phi \le \pi$
$z(\theta, \phi) = a\cos\phi$

$$|\vec{r}_\theta \times \vec{r}_\phi| = a\sin\phi$$





$$\int_0^{\pi}\int_0^{2\pi} a\sin\phi \, d\theta \, d\phi$$

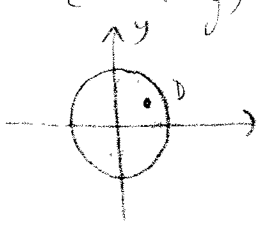
this rectangle gets pressed around this sphere

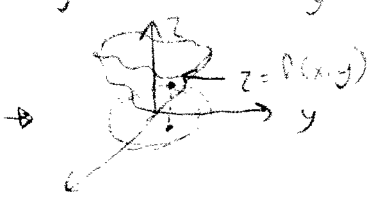
ex) $(u,v) = (x,y)$
$x = x$
$y = y$
$z = f(x,y)$

$\vec{r}(x,y) = \langle x, y, f(x,y) \rangle$
$|\vec{r}_x \times \vec{r}_y| = ?$
$\vec{r}_x = \langle 1, 0, f_x \rangle$
$\vec{r}_y = \langle 0, 1, f_y \rangle$

$\vec{r}_x \times \vec{r}_y =$

$$\begin{array}{cccccc} i & j & k & i & j & k \\ 1 & 0 & f_x & 1 & 0 & f_x \\ 0 & 1 & f_y & 0 & 1 & f_y \end{array}$$





$\langle 0 - f_x, 0 - f_y, 1 - 0 \rangle$
$= \langle -f_x, -f_y, 1 \rangle$

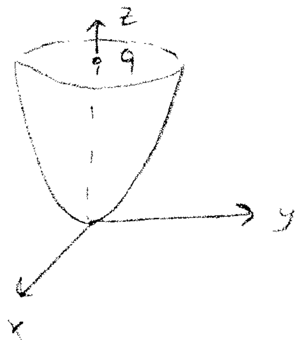
$$|\vec{r}_x \times \vec{r}_y| = \sqrt{f_x^2 + f_y^2 + 1}$$

So

$$A(S) = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

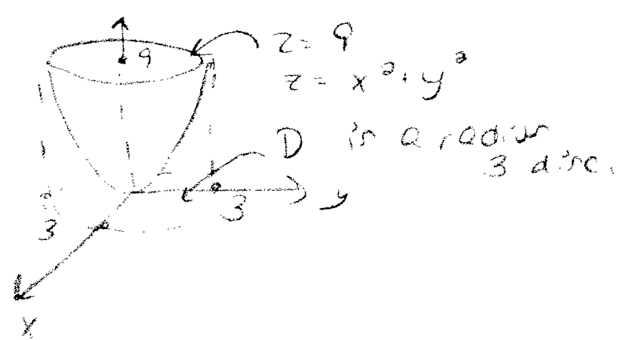
ex) $z = x^2 + y^2 = f(x,y)$

compute the area of this surface underneath the plane $z = 9$.



Solution: Find D.

shadow:



$$\iint_D \sqrt{1 + (2x)^2 + (2y)^2}\, dA$$

$$=$$

$$\iint_D \sqrt{1 + 4x^2 + 4y^2}\, dA$$

$$=$$

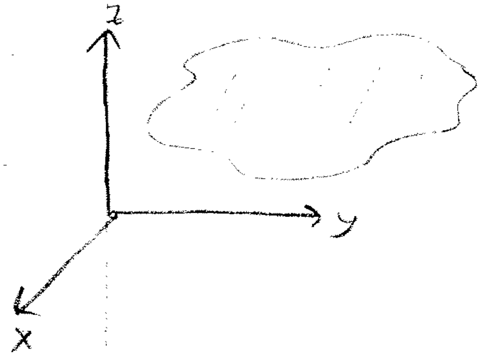
$$\int_0^{2\pi}\int_0^3 \sqrt{1 + \underbrace{4\cos^2\theta + 4\sin^2\theta}_{\cos^2\theta + \sin^2\theta = r^2}}\; r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^3 \sqrt{1 + 4r^2}\; r\,dr\,d\theta$$

$$= \int_0^{2\pi} \tfrac{2}{3 \cdot 8}(1 + 4r^2)^{3/2}\Big|_0^3\, d\theta$$

$= \ldots$ exercise

§17.7 Surface integrals



S surface in space

$f(x,y,z)$ is a function
(if you want, think of f as a density function on S)

The surface integral of $f(x,y,z)$ over S is written:

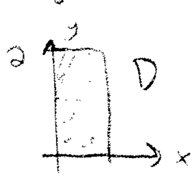
$$\iint_S f(x,y,z)\,dS \quad \text{and equals:}$$

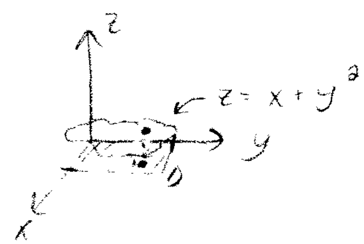
$$\iint_D f\big(x(u,v),\ y(u,v),\ z(u,v)\big)\,|\vec{r}_u \times \vec{r}_v|\,dA$$

Recall: $\int_C f(x,y,z)\,ds \qquad \vec{r}(t) = \langle x(t), y(t), z(t)\rangle$

$$= \int_a^b f\big(x(t), y(t), z(t)\big)\,|\vec{r}\,'(t)|\,dt$$

$\iint_S y\,dS$ where S is the surface $z = x + y^2$ $\quad 0 \le x \le 1$, $0 \le y \le 2$





$\vec{r}(x,y) = \langle x,\ y,\ x+y^2 \rangle$
$\quad (x) \quad (y) \quad (f(x,y))$

$$\iint_S y\,dS = \int_0^2\int_0^1 y\sqrt{(1)^2 + (2y)^2 + 1}\,dx\,dy$$
$$= \int_0^2\int_0^1 y\sqrt{2+4y^2}\,dx\,dy = \int_0^2 y\sqrt{2+4y^2}\,dy \ \ldots \text{(substitution)}$$

exercise

$\iint_S x^2\,dS$ $\quad$ S is the unit sphere.

$\vec{r}(\theta,\phi)$: $\quad x=\cos\theta\sin\phi$

$y=\sin\theta\sin\phi$

$z=\cos\phi$

$|\vec{r}_\theta \times \vec{r}_\phi| = \sin\phi$ $\quad 0\le\phi\le\pi$

$0\le\theta\le 2\pi$

$$\iint_D (\cos\theta\sin\phi)^2 \cdot \sin\phi\, dA = \int_0^{2\pi}\int_0^{\pi} \cos^2\theta\sin^3\phi\, d\phi\, d\theta$$

$f(x,y,z)=x^2$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

<u>One fact about surface integrals</u>: If S is a piecewise smooth surface

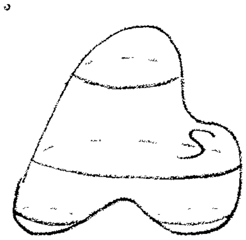
$S = S_1 \cup S_2 \cup S_3 \cup \dots \cup S_k$

ex):

$S = S_1 \cup S_2 \cup S_3$

then $\iint_S f\,dS = \iint_{S_1} f\,dS + \iint_{S_2} f\,dS + \dots + \iint_{S_k} f\,dS$

<u>Motivation</u>:



Solid blob

analogy → tea bag