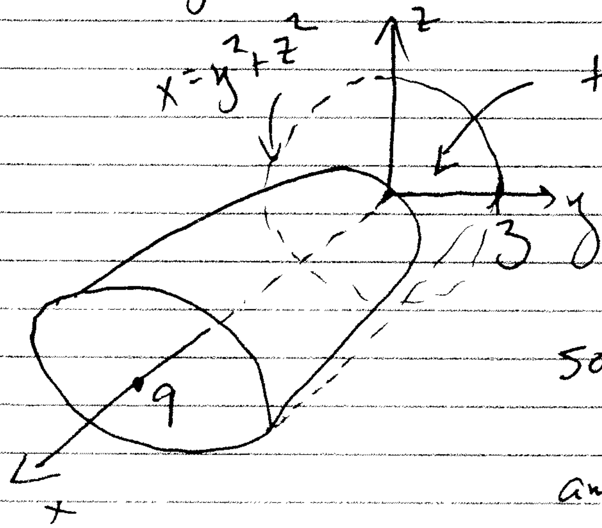
Last HW | Math 272 | Spring 2010

§17.6 #44

§17.7 #10, 14

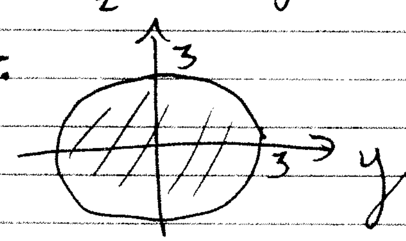
17.6 **44** Find the area of the part of the paraboloid $x = y^2 + z^2$ that lies inside $y^2 + z^2 = 9$.



the surface is the graph over the disk radius 3 in the yz-plane

so $\vec{r}(y,z) = \langle y^2+z^2, y, z \rangle$

and D:



Surface area =

$$\iint_S d\vec{S} = \iint_D |\vec{r}_y \times \vec{r}_z| \, dA$$

17.7 #10 Compute $\iint_S \sqrt{1+x^2+y^2}\, d\vec{S}$

where S : $\langle u\cos v, u\sin v, v\rangle = \vec{r}(u,v)$

$0 \le u \le 1$

$0 \le v \le \pi$

Sol'n: $\vec{r}_u \times \vec{r}_v$: $\langle \sin v, -\cos v, u\rangle$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{1+u^2}.$$

$$\int_0^{\pi}\int_0^1 \sqrt{1+u^2\cos^2 v+u^2\sin^2 v}\cdot\sqrt{1+u^2}\, du\, dv$$

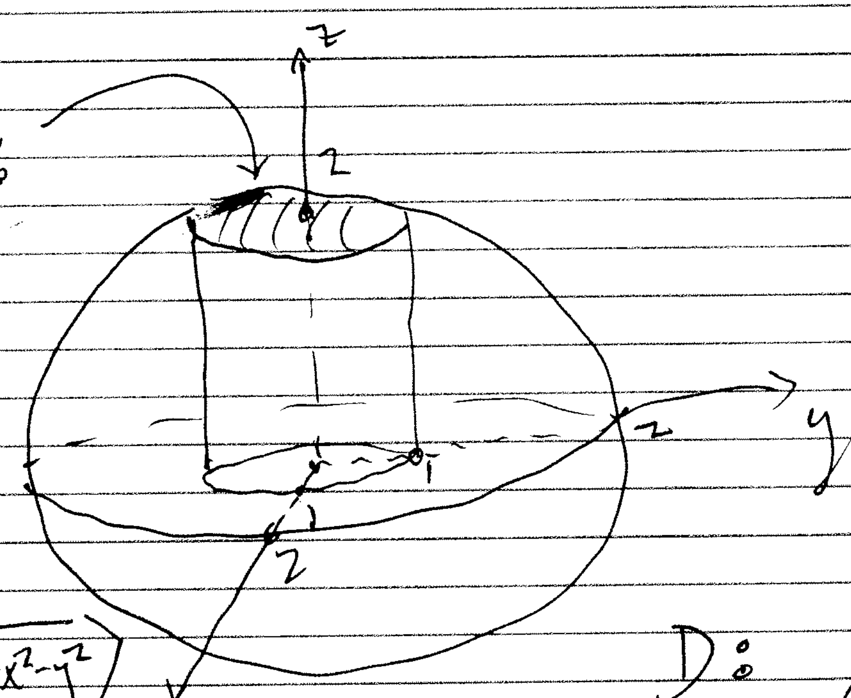
$$= \int_0^{\pi}\int_0^1 \left(\sqrt{1+u^2}\right)^2 du\, dv = \pi\cdot\left(u+\frac{u^3}{3}\right)\Big|_0^1$$

$$= \boxed{\frac{4}{3}\pi}$$

17.7 #14 $\displaystyle\iint_S y^2 \, dS$

S: the portion of $x^2+y^2+z^2=4$ inside $x^2+y^2=1$ and above the xy-plane

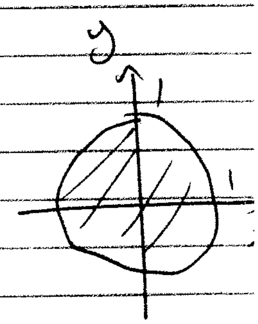
Sol'n: S:



$z=\sqrt{4-x^2-y^2}$

$\vec{r}(x,y)=\langle x, y, \sqrt{4-x^2-y^2}\rangle$

D:



$$\vec{r}_x \times \vec{r}_y = \left\langle \frac{x}{\sqrt{4-x^2-y^2}}, \frac{y}{\sqrt{4-x^2-y^2}}, 1 \right\rangle$$

$$|\vec{r}_x \times \vec{r}_y| = \sqrt{\frac{x^2+y^2+4-x^2-y^2}{4-x^2-y^2}} = \frac{2}{\sqrt{4-x^2-y^2}}$$

$$\iint_S y^2\, dS = \iint_D y^2 \cdot \frac{2}{\sqrt{4-x^2-y^2}}\, dA$$

$$= \int_0^{2\pi}\int_0^1 \frac{2r^2\sin^2\theta}{\sqrt{4-r^2}}\, r\, dr\, d\theta$$

ok to leave like this

Alternate solution:

Parametrize S as

$$x = 2\cos\theta\sin\phi \qquad 0 \le \phi \le \pi/6$$
$$y = 2\sin\theta\sin\phi \qquad 0 \le \theta \le 2\pi$$
$$z = 2\cos\phi$$

This should be easier! :)