Calculus IV | Spring 2010 | HW 5 Solutions

§17.5 #4 §17.6 #4, 24

17.5 #4 Find the curl & divergence of

$$\vec{F} = \cos xz\,\vec{j} - \sin xy\,\vec{k}$$

$\text{curl}\,\vec{F} = \nabla \times \vec{F}$:

$$\begin{matrix} i & j & k & i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z & \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & \cos xz & -\sin xy & 0 & \cos xz & -\sin xy \end{matrix}$$

$$= \langle -x\cos xy + x\sin xz,\ +y\cos xy,\ -z\sin xz \rangle$$

$$\text{div}\,\vec{F} = \nabla \cdot \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \vec{F}$$

$$= 0 + 0 + 0 = 0.$$

(17.6 #4) Identify the surface

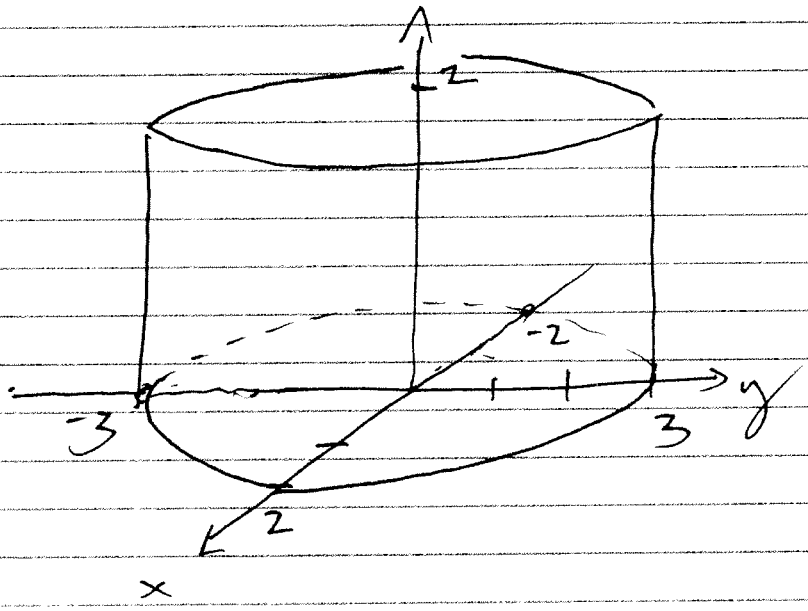
$$\vec{r}(u,v) = 2\sin u\,\vec{i} + 3\cos u\,\vec{j} + v\,\vec{k}$$

$$0 \le v \le 2$$

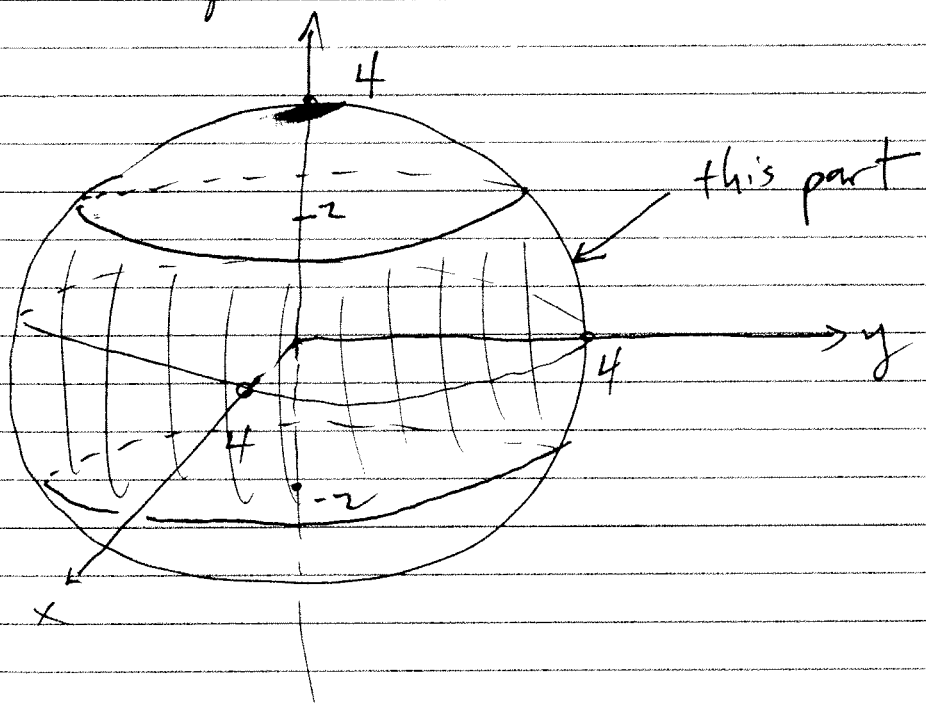
$$\left.\begin{aligned} x &= 2\sin u \\ y &= 3\cos u \end{aligned}\right\} \quad \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = \sin^2 u + \cos^2 u = 1$$

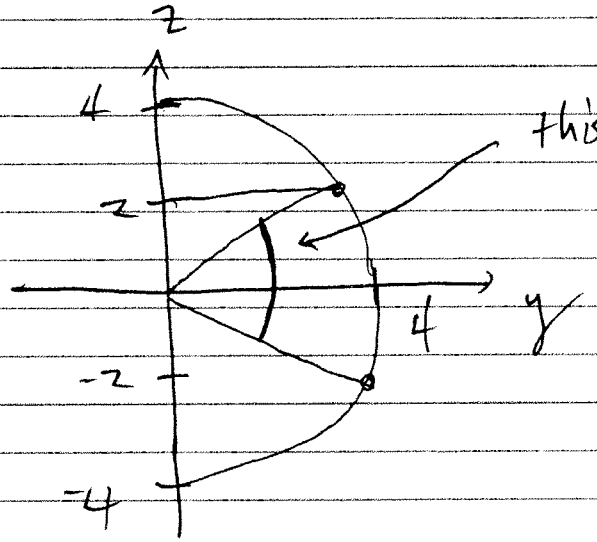
$$z = v$$

so this is an ellipse-shaped cylinder of height 2.



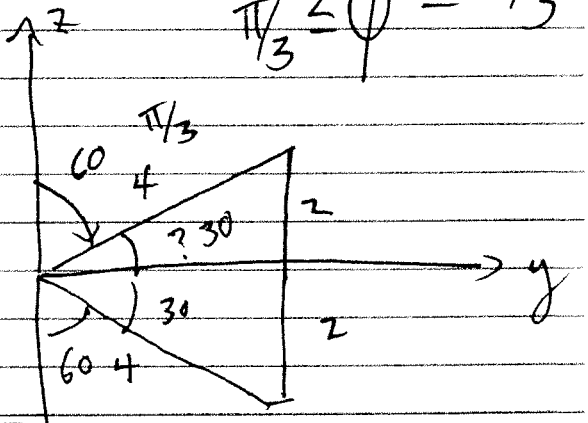
17.6 #24 Find a parametric representation for the part of the sphere $x^2+y^2+z^2=16$ between $z=-2$ & $z=2$.





this is the ϕ range

$\pi/3 \le \phi \le 2\pi/3$



$$\vec{r}(\phi,\theta) = \langle 4\cos\theta\sin\phi,\ 4\sin\theta\sin\phi,\ 4\cos\phi \rangle$$

$0 \le \theta \le 2\pi$

$\pi/3 \le \phi \le 2\pi/3$