

M 272

Spring 2010

Hw 4

§ 17.2 # 32a

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F}(x,y) = \langle x^2, xy \rangle$

and $C \stackrel{\circlearrowleft}{=} x^2 + y^2 = 4$ counter clockwise loop

Sol'n: $x = 2 \cos t$

$0 \leq t \leq 2\pi$

$x' = -2 \sin t$

$y = 2 \sin t$

$y' = 2 \cos t$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \langle 4 \cos^2 t, 4 \cos t \sin t \rangle \cdot \langle -2 \sin t, 2 \cos t \rangle dt$$

$$= \int_0^{2\pi} 0 dt = 0$$

§ 17.3 #16

Find a potential function for

$$\vec{F}(x, y, z) = \langle 2xz + y^2, 2xy, x^2 + 3z^2 \rangle$$

and use this to evaluate

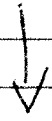
$$\int_C \vec{F} \cdot d\vec{r} \quad \text{where } C: \begin{cases} x = t^2 \\ y = t+1 \quad 0 \leq t \leq 1 \\ z = 2t-1 \end{cases}$$

Soln: Let $\vec{F} = \nabla f$. So

$f_x = 2xz + y^2$	$f_x = 2xz + y^2 \rightarrow$	$f = x^2z + y^2x$
$f_y = 2xy$		$+ g(y, z)$
$f_z = x^2 + 3z^2$	So $f_y = 2yx + \frac{\partial g}{\partial y} = 2xy$.	

So $\frac{\partial g}{\partial y} = 0$. So $g = h(z)$.

Therefore $f = x^2 z + y^2 x + h(z)$.



$$\frac{f}{z} = x^2 + 3z^2 = x^2 + h'(z)$$



$$h'(z) = 3z^2 \rightarrow h(z) = z^3 + C.$$

So $f(x, y, z) = x^2 z + y^2 x + z^3 + C$

$$\int_C \vec{F} \cdot d\vec{r} = f(1, \overset{t=1}{\downarrow} 2, 1) - f(\overset{t=0}{\downarrow} 0, 1, -1)$$

$$= 1 + 4 + 1 + C - (-1 + C)$$

$$= \boxed{7}$$