M 272 | Spring 2010 | Hw 4

§17.2 #32a

$$\int_C \vec{F} \cdot d\vec{r} \quad \text{where } \vec{F}(x,y) = \langle x^2, xy \rangle$$

and C: $x^2+y^2=4$ 1 counter clockwise loop

Sol'n: $x = 2\cos t$ $\quad 0 \le t \le 2\pi$ $\quad x' = -2\sin t$

$y = 2\sin t$ $\quad y' = 2\cos t$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \langle 4\cos^2 t, 4\cos t \sin t \rangle \cdot \langle -2\sin t, 2\cos t \rangle \, d$$

$$= \int_0^{2\pi} 0 \, dt = 0.$$

§17.3 #16

Find a potential function for

$\vec{F}(x,y,z) = \langle 2xz + y^2, 2xy, x^2 + 3z^2 \rangle$

and use this to evaluate

$$\int_C \vec{F} \cdot d\vec{r} \quad \text{where } C: \begin{array}{l} x = t^2 \\ y = t+1 \quad 0 \le t \le 1 \\ z = 2t - 1 \end{array}$$

Soln: Let $\vec{F} = \nabla f$. So

$$\begin{array}{l} f_x = 2xz + y^2 \\ f_y = 2xy \\ f_z = x^2 + 3z^2 \end{array}$$

$f_x = 2xz + y^2 \rightarrow \boxed{f = x^2 z + y^2 x + g(y,z)}$

So $f_y = 2yx + \frac{\partial g}{\partial y} = 2xy$.

So $\frac{\partial g}{\partial y} = 0$. So $g = h(z)$.

Therefore $f = x^2 z + y^2 x + h(z)$.

$$\downarrow$$

$$f_z = x^2 + 3z^2 = x^2 + h'(z)$$

$$\downarrow$$

$$h'(z) = 3z^2 \rightarrow h(z) = z^3 + C.$$

So $f(x,y,z) = x^2 z + y^2 x + z^3 + C$

$$\int_C \vec{F} \cdot d\vec{r} = f(\underset{\overset{\downarrow}{t=1}}{1,2,1}) - f(\underset{\overset{\downarrow}{t=0}}{0,1,-1})$$

$$= 1 + 4 + 1 + C - (-1 + C)$$

$$= \boxed{7}$$