Math 272 | Spring 2010 | Hw 3

§16.6: #6, 13

(6) $\int_0^1 \int_0^z \int_0^y z e^{-y^2} \, dx \, dy \, dz$

$= \int_0^1 \int_0^z xz e^{-y^2} \Big|_{x=0}^{x=y} dy \, dz$

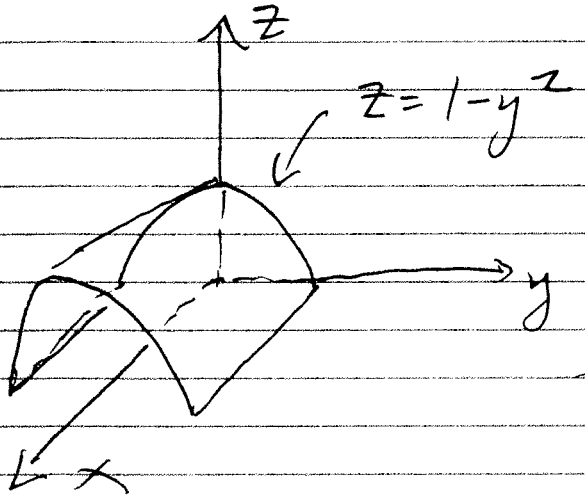
$= \int_0^1 \int_0^z yz e^{-y^2} \, dy \, dz$

$= \int_0^1 -\frac{1}{2} z e^{-y^2} \Big|_{y=0}^{y=z} dz = \int_0^1 \left(-\frac{1}{2} z e^{-z^2} + \frac{1}{2} z \right) dz$

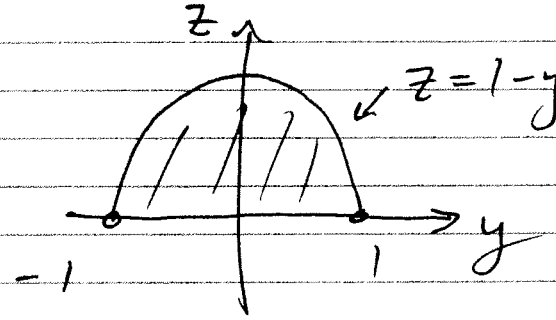
$= \left(\frac{1}{4} e^{-z^2} + \frac{1}{4} z^2 \right) \Big|_0^1 = \frac{1}{4}(e^{-1} + 1) - \frac{1}{4} = \boxed{\frac{1}{4e}}$

(13) $\iiint_E x^2 e^y \, dV$

E: bounded by $z = 1 - y^2$, $z = 0$, $x = 1$, $x = -1$



project to yz-plane:



$$\begin{cases} -1 \le y \le 1 \\ 0 \le z \le 1 - y^2 \end{cases} \text{ projection "shadow"}$$

$$-1 \le x \le 1 \quad \text{x-range}$$

$$\int_{-1}^{1} \int_{0}^{1-y^2} \int_{-1}^{1} x^2 e^y \, dx \, dz \, dy = \frac{8}{3e}$$

This integral can also be written as:

$$\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-y^2} x^2 e^y \, dz\, dy\, dx$$

or

$$\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-y^2} x^2 e^y \, dz\, dx\, dy$$

or

$$\int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{-1}^{1} x^2 e^y \, dx\, dy\, dz$$

or

$$\int_{0}^{1}\int_{-1}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}} x^2 e^y \, dy\, dx\, dz$$

or

$$\int_{-1}^{1}\int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}} x^2 e^y \, dy\, dz\, dx$$