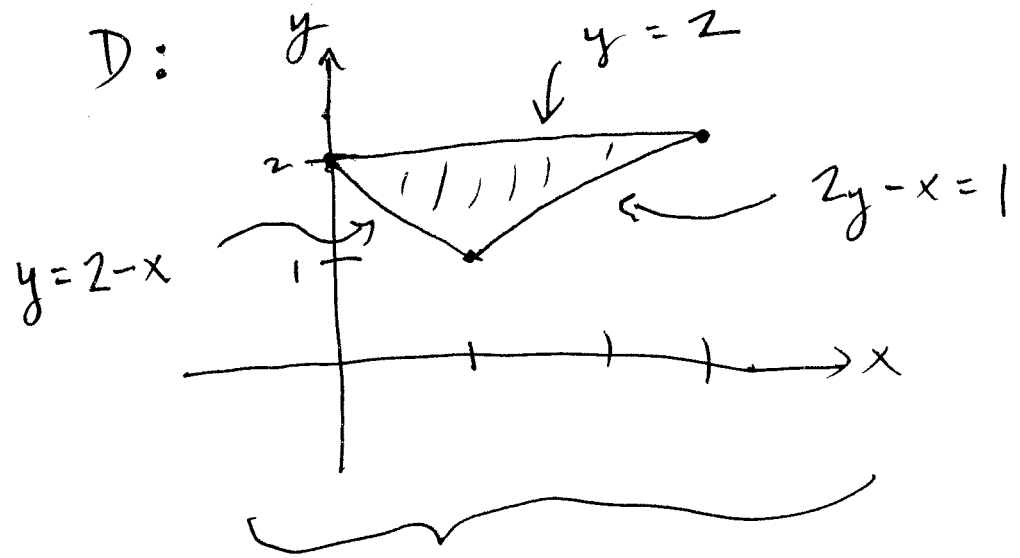
Math 272 | Spring 2010 | HW 2

§16.3 #15, 40

(15) Evaluate $\iint_D y^3\,dA$ D = triangle with vertices $(0,2)$, $(1,1)$, $(3,2)$

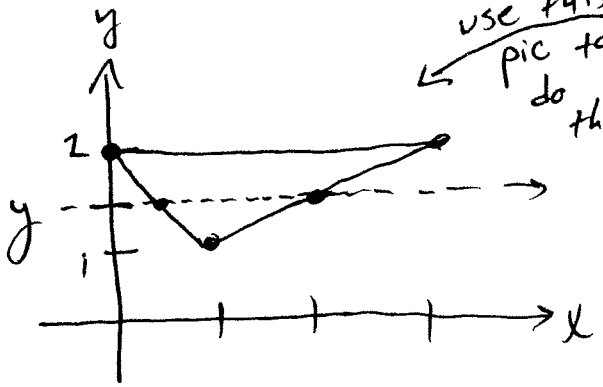
Soln:

D:



Describe as $\boxed{\begin{array}{l} 1 \le y \le 2 \\ 2-y \le x \le 2y-1 \end{array}}$

use this pic to do this



So $$\iint_D y^3\,dA = \int_1^2 \int_{2-y}^{2y-1} y^3\,dx\,dy$$

$$= \int_1^2 \left[y^3 x \Big|_{x=2-y}^{x=2y-1} \right] dy$$

$$= \int_1^2 \left[y^3(2y-1) - y^3(2-y) \right] dy$$

$$= \int_1^2 (3y^4 - 3y^3)\,dy = \left(\frac{3}{5}y^5 - \frac{3}{4}y^4 \right)\Big|_1^2$$

$$= \frac{3}{5}\cdot 2^5 - \frac{3}{4}\cdot 2^4 - \left(\frac{3}{5} - \frac{3}{4} \right)$$

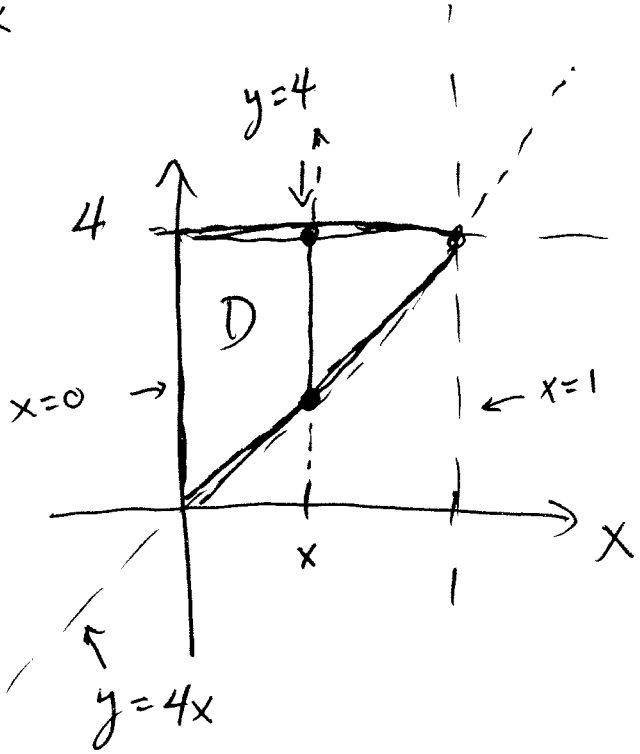
$$= \frac{147}{20} \quad (= 7.35)$$

(40) Sketch region and reverse the order of integration:

$$\int_0^1 \int_{4x}^{4} f(x,y)\, dy\, dx$$

Sol'n:

$$\left.\begin{array}{l} 0 \le x \le 1 \\ 4x \le y \le 4 \end{array}\right\} \longrightarrow$$



Redescribe:

$$\left.\begin{array}{l} 0 \le y \le 4 \\ 0 \le x \le \frac{1}{4} y \end{array}\right\} \longrightarrow \int_0^4 \int_0^{\frac{1}{4}y} f(x,y)\, dx\, dy$$