Math 272 Spring 2010 Hw 1

Graded: § 16.2 #4, 28

(4) $\int_0^1 \int_1^2 4x^3 - 9x^2y^2 \, dy \, dx$

$= \int_0^1 \left[4x^3y - 3x^2y^3 \right]\Big|_{y=1}^{y=2} dx$

$= \int_0^1 \left(8x^3 - 24x^2 - (4x^3 - 3x^2) \right) dx$

$= \int_0^1 (4x^3 - 21x^2) \, dx = (x^4 - 7x^3)\Big|_0^1$

$= 1 - 7 - (0 - 0) = \boxed{-6}$

(28) Find the volume enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$ and $z = 0$.

Soln:

$$\int_{-1}^{1}\int_{0}^{\pi} (1 + e^x \sin y)\, dy\, dx$$

$$= \int_{-1}^{1} \Big[y - e^x \cos y \Big]\Big|_{y=0}^{y=\pi} dx$$

$$= \int_{-1}^{1} \Big[\pi - e^x \cos(\pi) - \big(0 - e^x \cos(0)\big) \Big] dx$$

$$= \int_{-1}^{1} (\pi + 2e^x)\, dx = \Big[\pi x + 2e^x \Big]\Big|_{-1}^{1}$$

$$= \pi + 2e - \left(-\pi + \frac{2}{e}\right) = \boxed{2\pi + 2e - \frac{2}{e}}$$