Monday:

Sep 28 2009

Ali has to stop sleeping a lot :(

outline
- Dot products
- Cross products

Announcement

Exam Monday Oct 5.
study section.

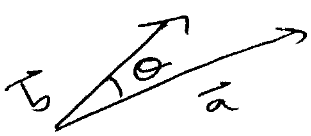
Dot Product:

Recall: $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

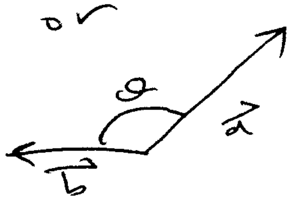
$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$|a| \cdot |b| \cdot \cos\theta$



or



$\vec{a} = \langle 1, 2, 3 \rangle$
$\vec{b} = \langle 2, -1, 0 \rangle$

$\vec{a} \cdot \vec{b} = 1 \cdot 2 + 2 \cdot (-1) + 3 \cdot 0$
$= 0$

Either $\vec{a}$ or $\vec{b}$ (or both) have length zero or $\cos\theta = 0$
$\theta = \frac{\pi}{2}$

$\vec{a} \cdot \vec{b} = 0 \longleftrightarrow \vec{a} \perp \vec{b}$ (perpendicular)

Properties:

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

$(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$

(k scalar) $(k\vec{a})\cdot\vec{b} = k(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(k\vec{b})$

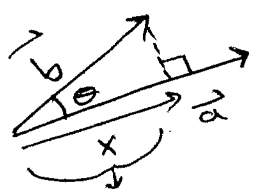
$\vec{0}\cdot\vec{a} = 0$ $\quad$ $(\vec{0} = \langle 0,0,0\rangle$

$\vec{a}\cdot\vec{a} = |\vec{a}|^2$

$\vec{a} = \langle a_1, a_2, a_3\rangle$

$\vec{a}\cdot\vec{a} = a_1^2 + a_2^2 + a_3^2 \Rightarrow \left(\sqrt{a_1^2 + a_2^2 + a_3^2}\right)^2 = |\vec{a}|^2$

Projections



we care a lot about this vector

$$\text{Proj}_{\vec{a}}(\vec{b}) = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\left[\frac{\vec{a}}{|\vec{a}|}\right]$$

$\frac{\vec{a}}{|\vec{a}|}$ is unit length & points in the direction of $\vec{a}$

$$\cos\theta = \frac{x}{|\vec{b}|}$$

$$x\,\frac{\vec{a}}{|\vec{a}|} = (|\vec{b}|\cos\theta)\,\frac{\vec{a}}{|\vec{a}|}$$

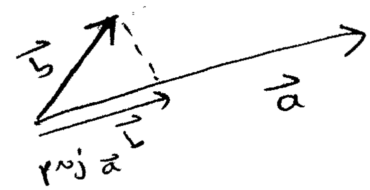
$$\boxed{= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}}$$

$\text{proj}_{\vec{a}}(\vec{b})$ $\quad$ vector projection of $\vec{b}$ on $\vec{a}$

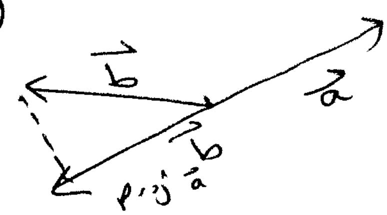
$$\boxed{\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \text{comp}_{\vec{a}}(\vec{b})}$$ scalar projection of $\vec{b}$ onto $\vec{a}$

Projections can take two forms.

I



II



Ex: $\vec{a} = \langle 1, 2, -3 \rangle$
$\vec{b} = \langle 0, 7, 2 \rangle$

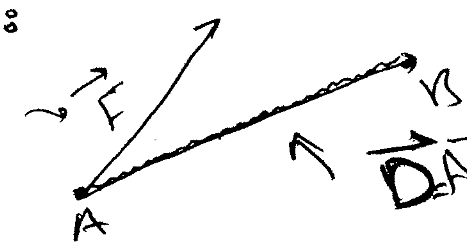
$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$$

$$= \left(\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \left(\frac{8}{14}\right)\langle 1, 2, -3 \rangle$$

$$= \left\langle \frac{8}{14}, \frac{16}{14}, \frac{-24}{14} \right\rangle$$

Work:

constant force



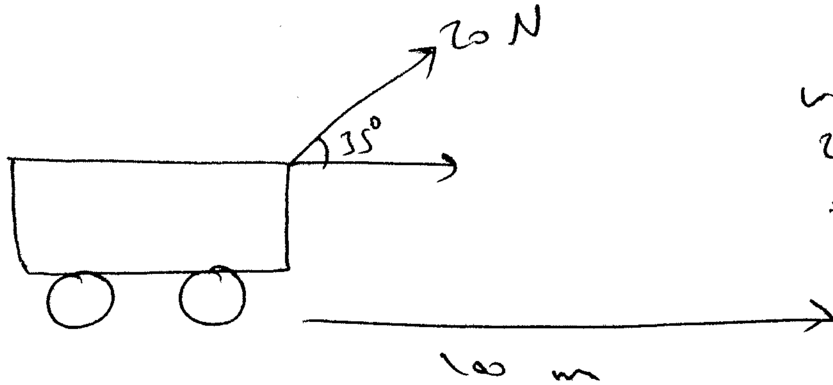
$\overrightarrow{D_{AB}}$ (displacement vector)

Def: the work done by the constant force $\vec{F}$ to move a particle along the displacement vector D.

is

$$|\vec{F}| \cdot |\vec{D}| \cos\theta = \langle \vec{F} \cdot \vec{D} \rangle$$

Ex:



work done:
$20 \cdot 100 \cdot \cos 35^\circ$
$=$ (calc) Joules (N.m)

53. p. 822

$\vec{a} = \langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle$
$\vec{b} = \langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle$

$|\vec{a}| \cdot |\vec{b}| \cos\theta = \vec{a} \cdot \vec{b}$.

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cos\theta = \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} - \frac{1}{4}$

$\cos\theta = \frac{-1}{3}$

$\theta \approx 109^\circ$