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9/24/09

<u>Outline</u>

- vectors (13.2)
- Dot product (13.3)

Exam M. 10/5

Recall: vectors = magnitude & direction

<u>Rules</u>

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$

$\vec{a} + (\vec{b} + \vec{v}) = (\vec{a} + \vec{b}) + \vec{v}$

$\vec{a} + 0 = \vec{a}$

$1\vec{a} = \vec{a}$

$0\vec{a} = \vec{0}$

$(c+d)\vec{a} = c\vec{a} + d\vec{a}$

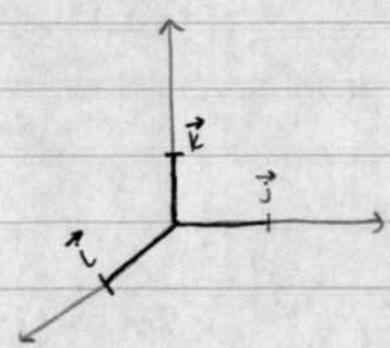
$\vec{a} + (-\vec{a}) = \vec{0}$

Standard basis vectors for $\mathbb{R}^3$ (all real #s)

$\vec{i} = \langle 1,0,0 \rangle$

$\vec{j} = \langle 0,1,0 \rangle$

$\vec{k} = \langle 0,0,1 \rangle$



$|\vec{i}| = \sqrt{1^2 + 0^2 + 0^2} = 1$

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$= a_1\langle 1,0,0 \rangle + a_2\langle 0,1,0 \rangle + a_3\langle 0,0,1 \rangle$

$\langle 1,2,0\rangle = \vec{i} + 2\vec{j} (+ 0\vec{k})$

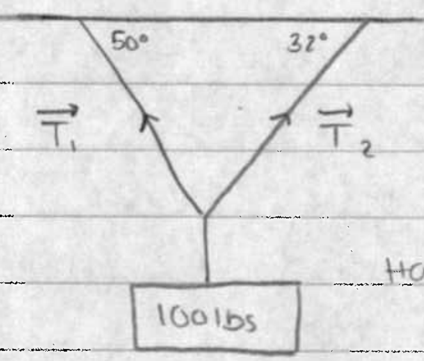
2 ways of writing same vector

Can always make a vector $\vec{a}$ into a <u>unit</u> vector. (a vector of length 1) by...

$$\frac{1}{|\vec{a}|}\vec{a} = \left\langle \frac{a_1}{\sqrt{a_1^2+a_2^2+a_3^2}}, \frac{a^2}{\sqrt{a_1^2+a_2^2+a_2^2}}, \frac{a^2}{\sqrt{a_1^2+a_2^2+a_2^2}} \right\rangle$$

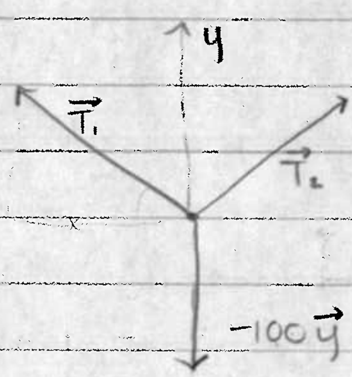
Check: $\left| \frac{1}{|\vec{a}|}\vec{a} \right| = 1$

Ex: (in the plane, so $\vec{i} = \langle 1,0\rangle$, $\vec{j} = \langle 0,1\rangle$



Hanging weight of 100lbs

Q: Find the tension vectors $\vec{T}_1$ & $\vec{T}_2$.



$-100\vec{y}$ = vector for downward weight 100lbs

→

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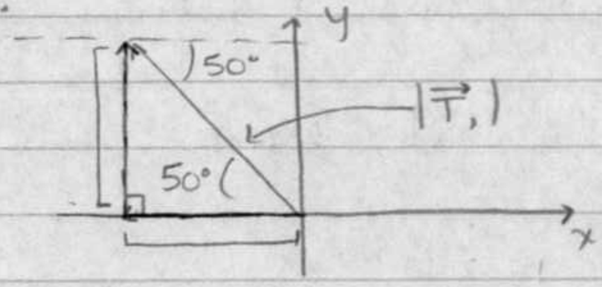
$\vec{T}_1 + \vec{T}_2 + (-100\vec{j}) = \vec{0}$ (b/c object is static)

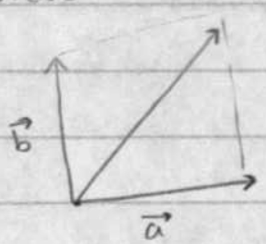
We want to find $\vec{T}_1$ and $\vec{T}_2$

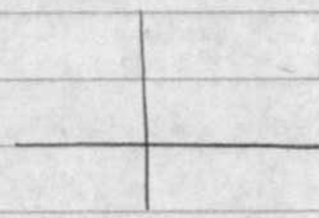
$\vec{T}_1$:



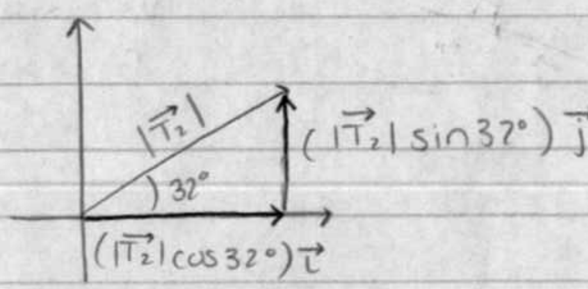
$$\vec{T}_1 = (-|\vec{T}_1| \cos 50°)\vec{i} + (|\vec{T}_1| \sin 50°)\vec{j}$$

Review





$\vec{T}_2$:



$\vec{T}_1 + \vec{T}_2 = 100\vec{j}$

$(-|\vec{T}_1| \cos 50°)\vec{i} + (|\vec{T}_1| \sin 50°)\vec{j} +$

$(|\vec{T}_2| \cos 32)\vec{i} + (|\vec{T}_2| \sin 32)\vec{j} = 100\vec{j}$

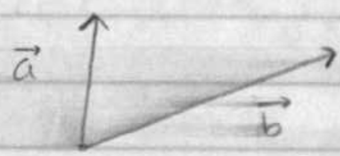
$\longrightarrow$

$$= \left[-|\vec{T}_1|\cos 50° + |\vec{T}_2|\cos 32°\right]\vec{i} \; +$$
$$\left[|\vec{T}_1|\sin 50° + |\vec{T}_2|\sin 32°\right]\vec{j} = 0\vec{i} + 100\vec{j}$$

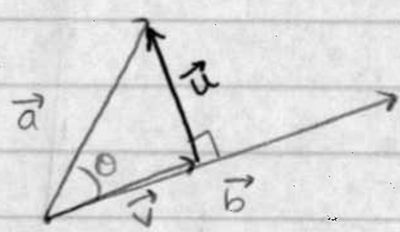
$$-|\vec{T}_1|\cos 50° + |\vec{T}_2|\cos 32° = 0$$
$$|T_1|\sin 50° + |\vec{T}_2|\sin 32° = 100$$

$|\vec{T}_1| \approx 85$ lbs $\qquad |\vec{T}_2| \approx 64$ lbs

Dot Product 13.3



Q: What is the component of the vector $\vec{a}$ that points in the $\vec{b}$ direction?



$\vec{a} = \vec{u} + \vec{v}$

$\vec{v}$ pts in the $\vec{b}$ direction

$$|\vec{v}| = |\vec{a}|\cos\theta$$
$$\downarrow$$

$\dfrac{\vec{b}}{|\vec{b}|}$ = vector pts in $\vec{b}$-direction w/ unit length

$$\downarrow$$

$$(|\vec{a}|\cos\theta)\frac{\vec{b}}{|\vec{b}|} = \vec{v}$$ } vector component of $\vec{a}$ in the $\vec{b}$-direction

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· The Dot Product of $\vec{a}$ and $\vec{b}$ is the number...

geometric definition

$$\vec{a} \cdot \vec{b} := |\vec{a}||\vec{b}|\cos\theta \quad , \quad \theta \text{ is the } \measuredangle \text{ between } \vec{a} \text{ \& } \vec{b}$$

algebraic definition

If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ & $\vec{b} = \langle b_1, b_2, b_3 \rangle$,

then $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

Ex: Lets find the $\measuredangle$ btw. $\vec{a} = \langle 2, 2, -1 \rangle$ &

$\vec{b} = \langle 5, 3, 2 \rangle$

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

$$= \sqrt{4+4+1} \cdot \sqrt{25+9+4} \cos\theta$$

$$= 3\sqrt{38} \cos\theta = 2$$

$10 - 6 - 2$

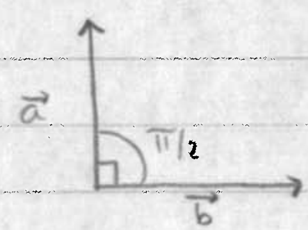
$= 2$

$$\cos\theta = \frac{2}{3\sqrt{38}}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{38}}\right)$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = 0 \ ?$$

$\cos\theta = 0$ when $\theta = \frac{\pi}{2}$



* $\vec{a} \cdot \vec{b}$ iff. $\vec{a} \perp \vec{b}$

Ex: Find & read box of properties of the dot product

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a}\vec{b} + \vec{a}\vec{c}$

$(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a}\vec{c} + \vec{b}\vec{c}$

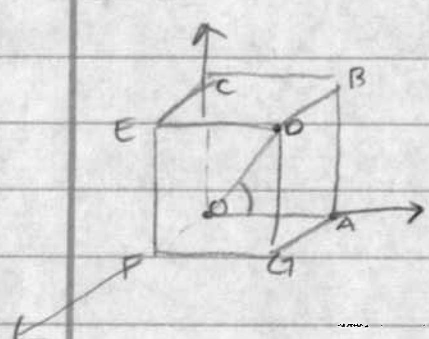
(k scalar) $(k\vec{a}) \cdot \vec{b} = k(\vec{a}\vec{b}) = \vec{a}(k\vec{b})$

$\vec{0} \cdot \vec{a} = 0 \quad (\vec{0} = \langle 0,0,0 \rangle)$

$\vec{a} \cdot \vec{a} = |\vec{a}|^2$

Example Problem: 13.3 p. 821 # 51

Find the angle between a diagonal of a cube and one of its edges.



Angle between OD and OA

O(0,0,0) D(1,1,1) (a,a,a) A(0,1,0) (0,a,0

$\vec{AO} \cdot \vec{OD} = |\vec{OA}||\vec{OD}| \cos\theta$

$\sqrt{0^2 + a^2 + 0^2} \cdot \sqrt{a^2 + a^2 + a^2} \quad \cos\theta = 0 + a^2 + 0$

$a^2 = a\sqrt{3}a \cos\theta = \sqrt{3}a^2 \cos\theta$

$1 = \sqrt{3} \cos\theta$

$\cos\theta = \frac{1}{3}$

$\theta = \cos^{-1} \frac{1}{\sqrt{3}}$